Lesson 4: Dilations on a Square Grid

Goals

- Create a dilation of a polygon on a square grid given a scale factor and center of dilation.
- Identify the image of a figure on a coordinate grid given a scale factor and center of dilation.

Learning Targets

- I can apply dilations to figures on a square grid.
- If I know the angle measures and side lengths of a polygon, I know the angles measures and side lengths of the polygon if I apply a dilation with a certain scale factor.

Lesson Narrative

In this lesson, students apply dilations to polygons on a grid, both with and without coordinates. The grid offers a way of measuring distances between points, especially points that lie at the intersection of grid lines. If point Q is three grid squares to the right and two grid squares up from P then the dilation with center P of Q with scale factor 4 can be found by counting grid squares: it will be twelve grid squares to the right of P and eight grid squares up from P. The coordinate grid gives a more concise way to describe this dilation. If the center P is $(0,0)$ then Q has coordinates $(3,2)$. The image of Q after this dilation is $(12,8)$.

Students continue to find dilations of polygons, providing additional evidence that dilations map line segments to line segments and hence polygons to polygons. The scale factor of the dilation determines the factor by which the length of those segments increases or decreases. Using coordinates to describe points in the plane helps students develop language for precisely communicating figures in the plane and their images under dilations (MP6). Strategically using coordinates to perform and describe dilations is also a good example of MP7.

Alignments

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.
- 8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print and cut one copy of the blackline master for each student.

Student Learning Goals

Let's dilate figures on a square grid.

4.1 Estimating a Scale Factor

Warm Up: 5 minutes
In this warm-up, students estimate a scale factor based on a picture showing the center of the dilation, a point, and its image under the dilation.

Addressing

- 8.G.A

Launch

Tell students they will estimate the scale factor for a dilation. Clarify that "estimate" doesn't mean "guess." Encourage students to use any tools available to make a precise estimate. Provide access to geometry toolkits.

Student Task Statement

Point C is the dilation of point B with center of dilation A and scale factor s. Estimate s. Be prepared to explain your reasoning.

Student Response

Answers vary. Sample response: about 2.3.

Activity Synthesis

Check with students to find what methods they used to compare distances. Likely methods include using a ruler and division or using an index card and marking off multiples of the distance from A to B.

Ask students:

- “Is the scale factor greater than 1?” (Yes.) “How do you know?” (the point C is further from A than B)
- “Is the scale factor greater than 2?” (Yes.) “How do you know?” (the distance from C to A is more than twice the distance from B to A)
- “Is the scale factor greater than 3?” (No.) “How do you know?” (The distance from C to A is less than 3 times the distance from B to A.)
- “Is the scale factor greater or less than 2.5?” (It is less.) “How do you know?” (The distance from C to A is less than 2.5 times the distance from B to A.)

4.2 Dilations on a Grid

10 minutes

In previous lessons, students perform dilations on a circular grid and with no grid. In this activity, they perform dilations on a square grid. A square grid is particularly helpful if the center of dilation and the points being dilated are grid points. When the extra structure of coordinates is added, as in the next activity, the grid provides an extremely convenient tool for naming points and describing the effects of dilations using coordinates. As in previous lessons, students will again see that scale factors greater than 1 produce larger copies while scale factors less than 1 produce smaller copies.

Monitor for how students find the dilated points and the language they use to describe the process. In particular:

- using a ruler or index card to measure distances along the rays emanating from the center of dilation
- taking advantage of the grid and counting how many squares to the left or right, up or down

Addressing

- 8.G.A

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

Launch

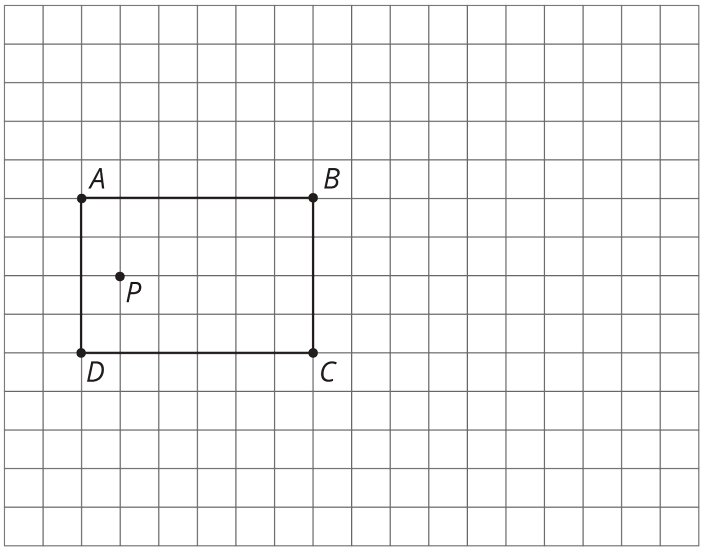
Provide access to geometry toolkits.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Connect a new concept to one with which students have experienced success. For example, reference examples from the previous lessons on methods for dilating points on a circular grid and on no grid to provide an entry point into this activity.
Supports accessibility for: Social-emotional skills; Conceptual processing

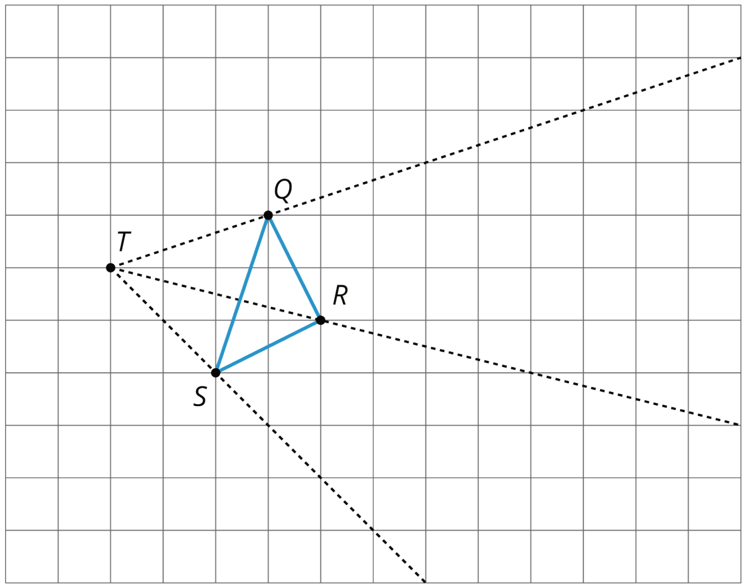
Student Task Statement

1. Find the dilation of quadrilateral $ABCD$ with center P and scale factor 2.

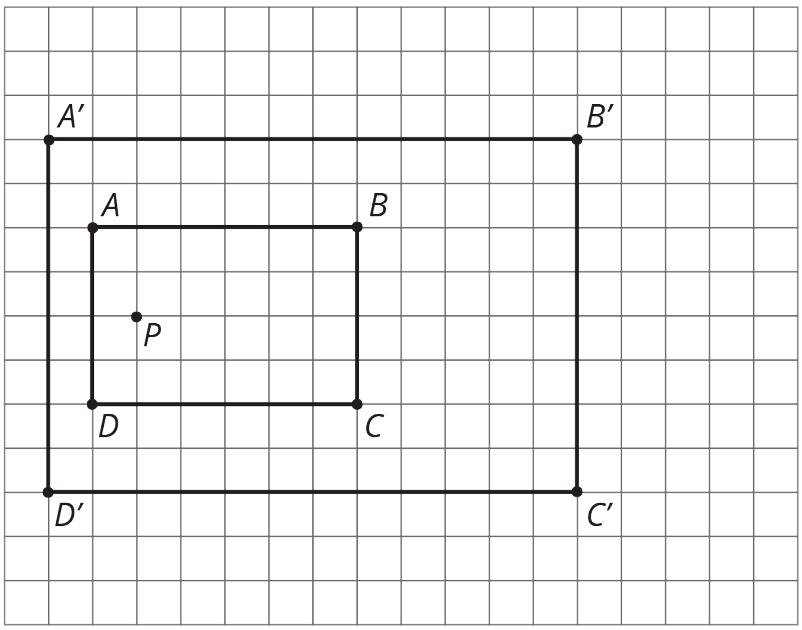


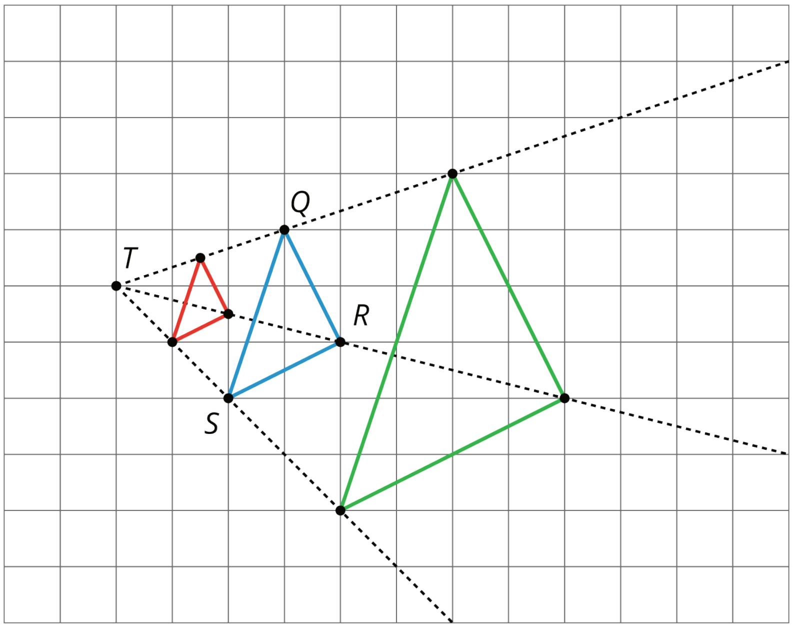
2. Find the dilation of triangle QRS with center T and scale factor 2.

3. Find the dilation of triangle QRS with center T and scale factor $\frac{1}{2}$.



Student Response





1.2-3.

Activity Synthesis

Select students to show how they found the dilations. First, select any students who used the same methods as when there was no grid, followed by students who noticed they could use the structure of the grid. Draw connections between these two methods—show that when you measure with a ruler or by making markings on an index card, the dilated point ends up in the same place as by reasoning about the grid.

Expect students to use expressions like “moving over two and up one.” These measurements can be multiplied by the scale factor in order to find the location of the dilated point.

Tell students that moving forward they will do work on the grid with the added structure of coordinates. The method of performing dilations is the same. The only change is that the coordinates give a concise way to *name* points.

4.3 Card Sort: Matching Dilations on a Coordinate Grid

15 minutes

In the previous task, students worked on a square grid without coordinates. This activity adds the structure of coordinates and this extra structure plays a key role, allowing students to name points. Students match figures with their dilated images, using coordinates to describe the center of dilation and the vertices. The same strategies that were used previously in dilating images,on a circular grid and with no grid, will be useful here.

Monitor for students who identify that the dilation of a circle is a circle and similarly for triangles and quadrilaterals. This will help them eliminate certain possibilities for each match. Because there is one card that does not match, students should verify the other matches by performing the dilations. Once the card without a match has been identified, reasoning based on eliminating possibilities (without performing the dilations) is correct. Monitor for students who systematically perform the dilations to help identify a match versus those who reason by structure and elimination of possibilities. Invite both to share during the discussion.

Addressing

- 8.G.A.3

Instructional Routines

- MLR2: Collect and Display

Launch

Students practice matching an original figure and dilation description to information about the dilated images using the coordinate plane. Distribute one set (numbers 1 through 6 and letters A through F) of cards to each student.

There is one extra option that does not have a match. Students should draw the dilated image for that option themselves.

Access for Students with Disabilities

Representation: Internalize Comprehension. Chunk this task into more manageable parts to differentiate the degree of difficulty or complexity. For example, give students a subset of the cards to start with and introduce the remaining cards once students have completed their initial set of matches.
Supports accessibility for: Conceptual processing; Organization

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Access for English Language Learners

Conversing, Reading: MLR2 Collect and Display. As students work in pairs on the task, circulate and listen to pairs as they decide whether two cards match. Write down the words and phrases students use to justify why an original figure card matches with a dilated figure card. As students review the language collected in the visual display, encourage students to clarify the meaning of a word or phrase. For example, a phrase such as "Card 1 matches with Card C because they are both trapezoids" can be clarified by asking students to explain why Card 1 does not match with Card A even though both are trapezoids. Listen for students who state that the scale factor and center of dilation must also be considered when matching the cards. Write down the language students use to describe how the scale factor and center of dilation affect the dilated figure. This routine will provide feedback to students in a way that supports sense-making while simultaneously increasing meta-awareness of language.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

If students are having trouble finding accurate matches, suggest that they identify the center of dilation and consider if the dilation will result in a smaller or larger sized image.

Student Task Statement

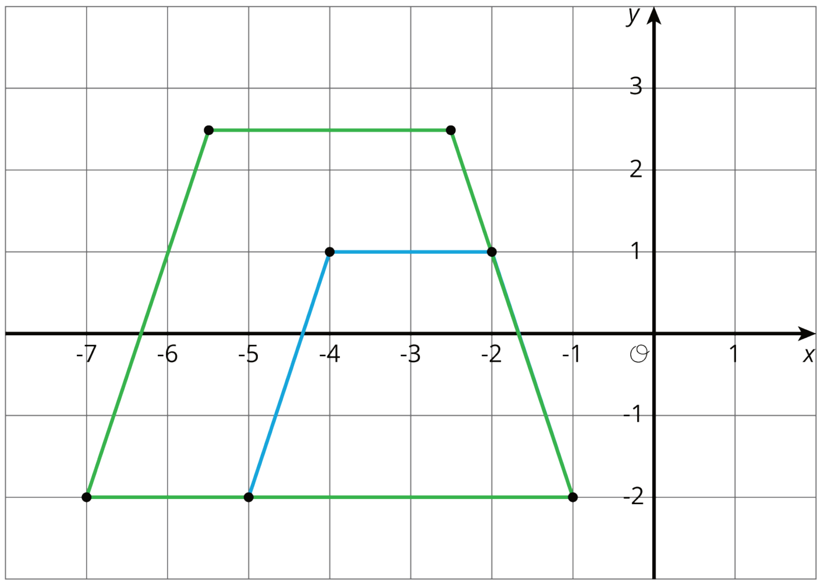
Your teacher will give you some cards. Each of Cards 1 through 6 shows a figure in the coordinate plane and describes a dilation.

Each of Cards A through E describes the image of the dilation for one of the numbered cards.

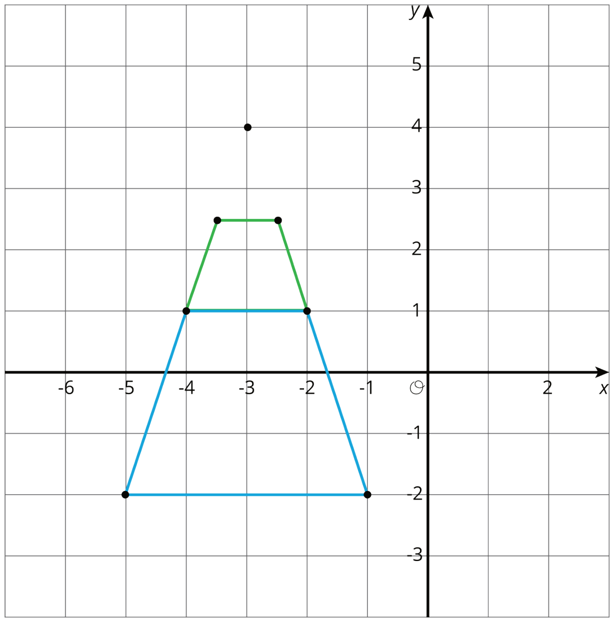
Match number cards with letter cards. One of the number cards will not have a match. For this card, you'll need to draw an image.

Student Response

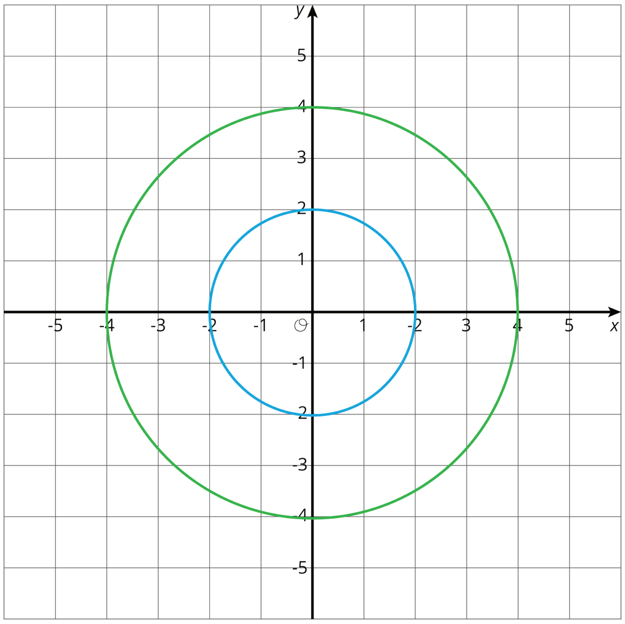
1. C. Answers vary. Sample response: The center of dilation is the point B, so the dilation also contains point B, suggesting this card. The scale factor of $\frac{3}{2}$ works for the two trapezoids which are plotted together.



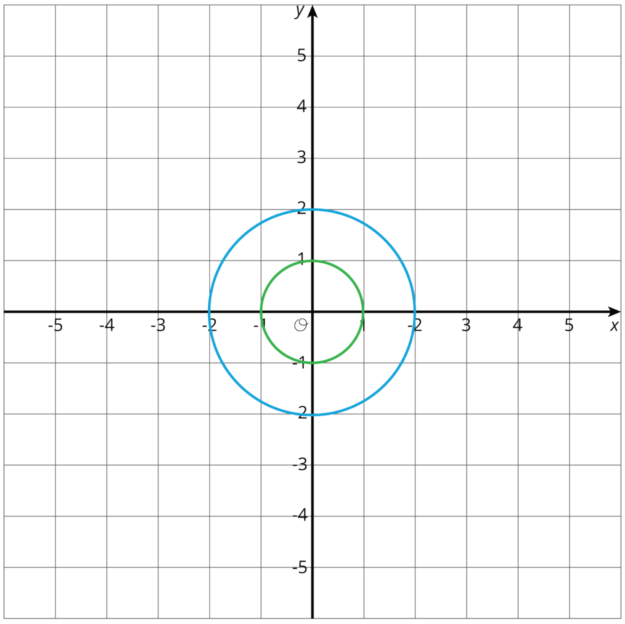
2. A. Answers vary. Sample response: The scale factor was less than one, so the dilation will be closer to the center of dilation. Card A is plotted and shows the dilation since each vertex on the green trapezoid is the midpoint between the center of dilation and the corresponding vertex on the blue trapezoid.



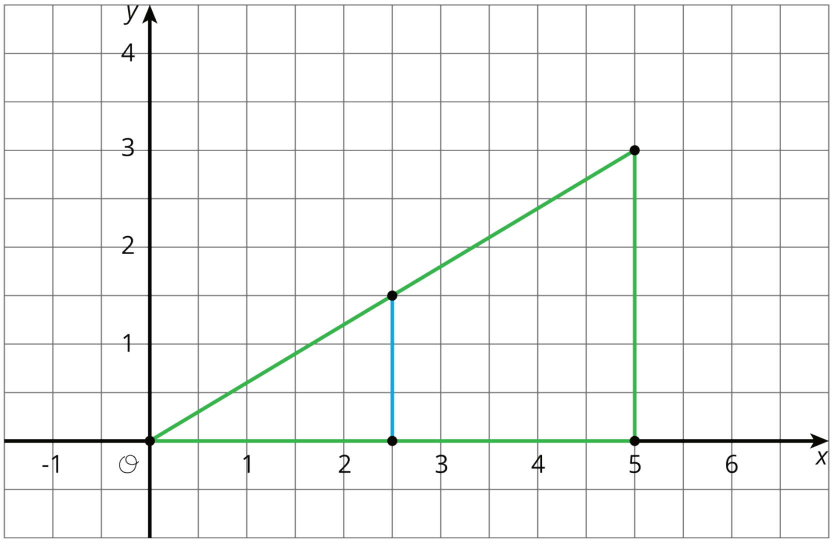
3. B. Answers vary. Sample response: The dilation scale factor was greater than one, so the dilated image will be a larger circle. The image is correct as both circles have the same center and the radius of the green circle is twice the radius of the blue circle.



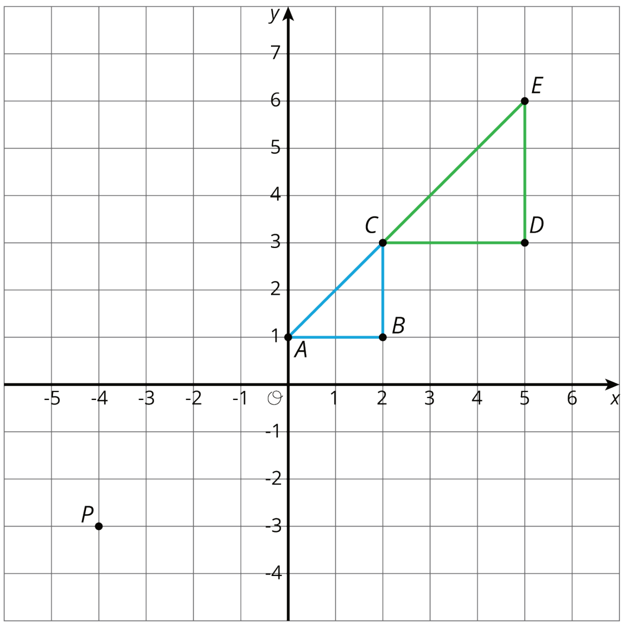
4. E. Answers vary. Sample response: This scale factor is less than one, so the image of the dilation is a circle that is smaller than the original one. The image is correct because the circles have the same center and the radius of the green circle is half the radius of the blue circle.



5. F. The center of dilation is (0,0) so the dilated image is a triangle containing (0,0). This does not match any of the lettered cards.

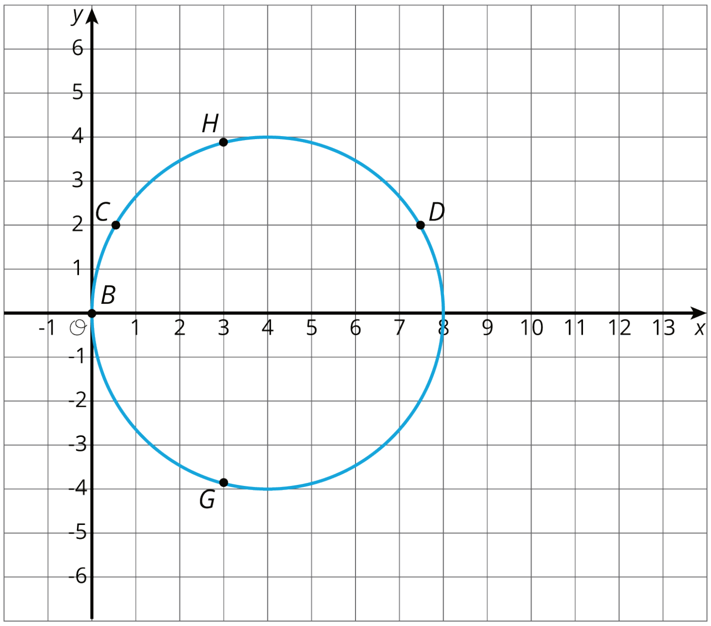


6. D. Answers vary. Sample response: the dilation of the triangle will be a triangle and it will be larger than $\triangle ABC$ since the scale factor is larger than 1. This suggests card D. The two are plotted together and $\triangle CDE$ is the dilation of $\triangle ABC$ with center P and scale factor $\frac{3}{2}$.



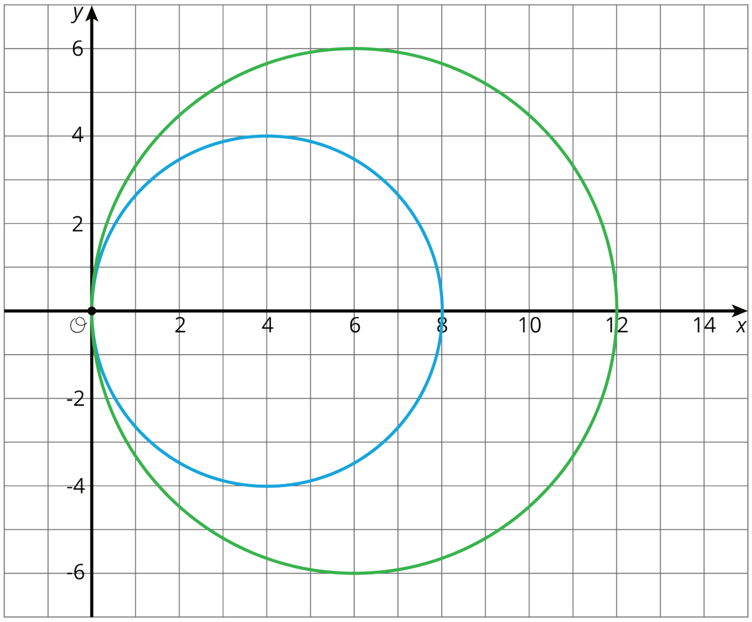
Are You Ready for More?

The image of a circle under dilation is a circle when the center of the dilation is the center of the circle. What happens if the center of dilation is a point on the circle? Using center of dilation $(0, 0)$ and scale factor 1.5, dilate the circle shown on the diagram. This diagram shows some points to try dilating.



Student Response

Original has center (4,0) and radius 4. Image has center (6,0) and radius 6.



Activity Synthesis

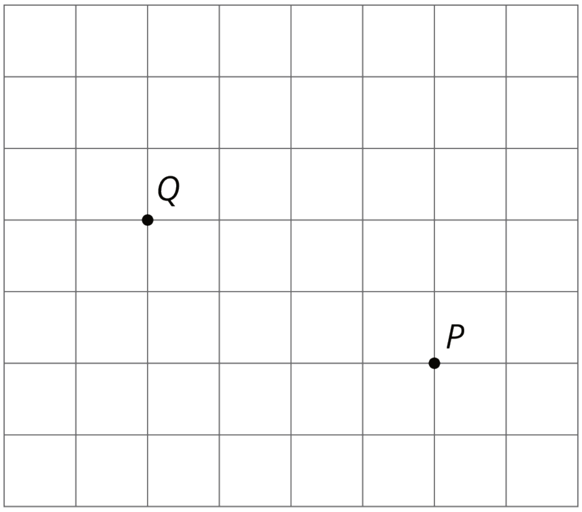
Share the correct answers and invite selected students to share the strategies they used to solve the problems. This is a matching problem, so students may not have dilated the entire image to find the correct answer among the choices. Important points to bring out include:

- A dilation maps a circle to a circle, a quadrilateral to a quadrilateral, and a triangle to a triangle.

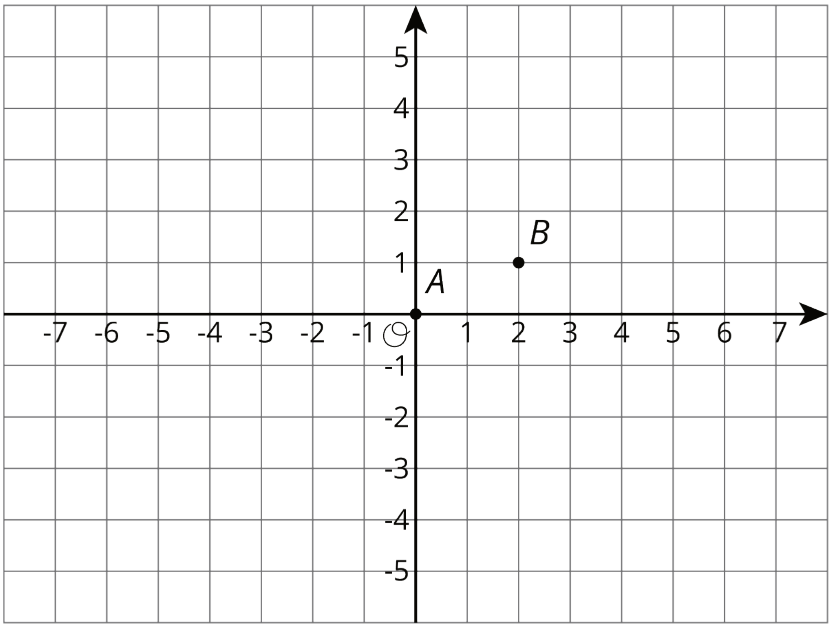
- If the center of dilation for a polygon is one of the vertices, then that vertex is on the dilated polygon.
- If the scale factor is less than 1 then the dilated image is smaller than the original figure.
- If the scale factor is larger than 1 then the dilated image is larger than the original figure.

Lesson Synthesis

- "How do we perform dilations on a square grid?"
- "How do coordinates help describe and perform dilations?"



Just like the circular grid, a square grid is useful for performing dilations. The grid lines can be used as a way to measure distance and direction between points. How can you dilate Q with center P and scale factor $\frac{1}{2}$?The image of Q will be half as many grid lines to the left and half has many grid lines up—that is, 2 grid lines to the left and 1 grid line up from P.



When the grid has coordinates, it is easier to communicate the location of new points. In the figure, we have $A = (0, 0)$ and $B = (2, 1)$.What is the dilation of B with center A and scale factor 3? To communicate the answer, we can just say $(6, 3)$. It is three times as far to the right and 3 times as far up from A as B so it is the desired point.

4.4 A Dilated Image

Cool Down: 5 minutes

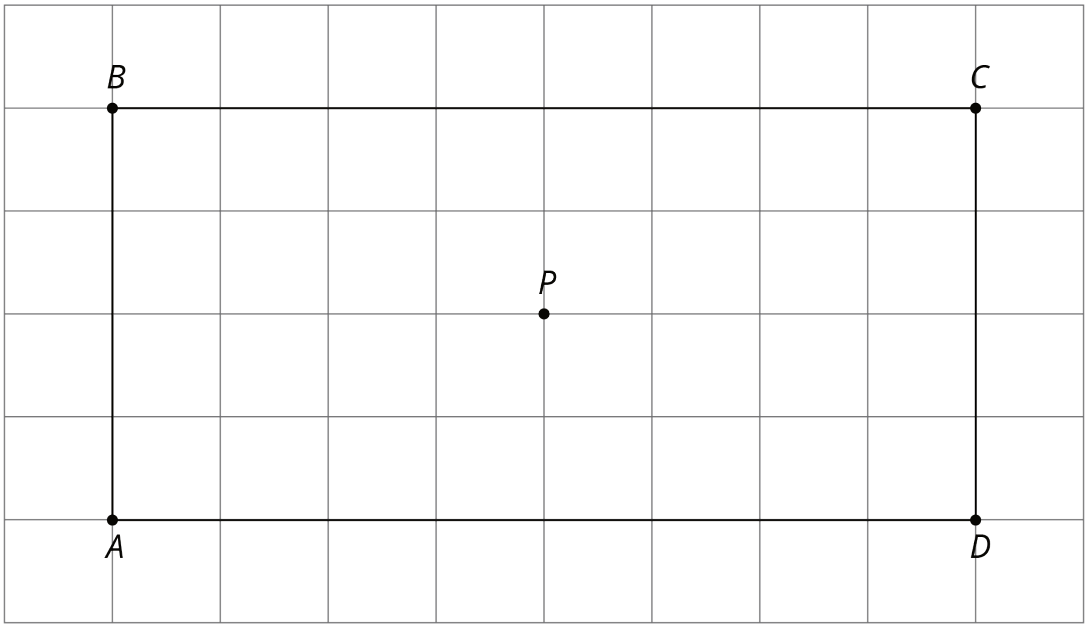
Students apply a dilation to a polygon where the center of dilation is on the interior of the figure. The polygon is on a grid without coordinates and the structure of the grid can be efficiently used to find the dilation.

Addressing

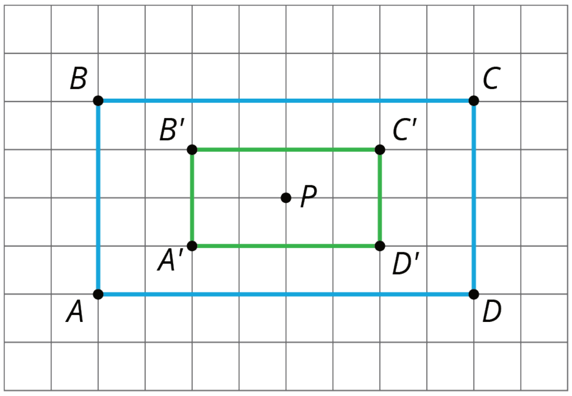
- 8.G.A

Student Task Statement

Draw the image of rectangle $ABCD$ under dilation using center P and scale factor $\frac{1}{2}$.



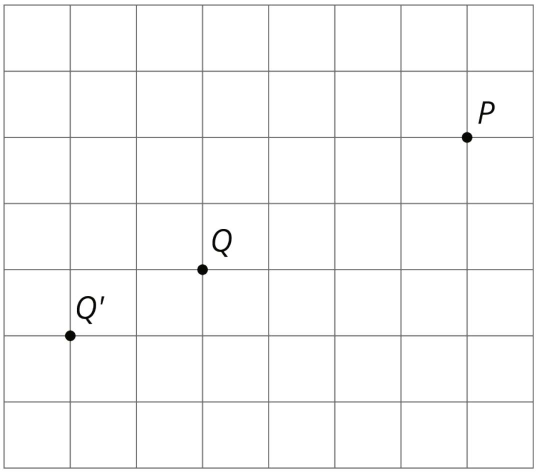
Student Response



Student Lesson Summary

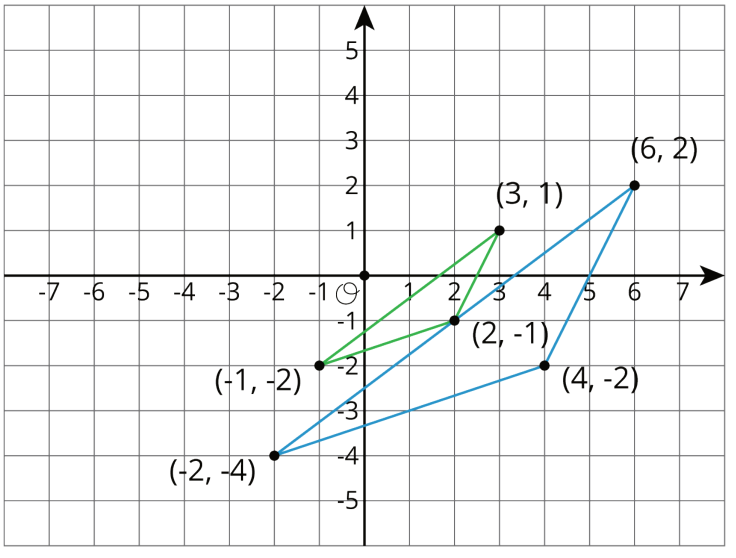
Square grids can be useful for showing dilations. The grid is helpful especially when the center of dilation and the point(s) being dilated lie at grid points. Rather than using a ruler to measure the distance between the points, we can count grid units.

For example, suppose we want to dilate point Q with center of dilation P and scale factor $\frac{3}{2}$. Since Q is 4 grid squares to the left and 2 grid squares down from P, the dilation will be 6 grid squares to the left and 3 grid squares down from P (can you see why?). The dilated image is marked as Q' in the picture.



Sometimes the square grid comes with coordinates. The coordinate grid gives us a convenient way to *name* points, and sometimes the coordinates of the image can be found with just arithmetic.

For example, to make a dilation with center $(0, 0)$ and scale factor 2 of the triangle with coordinates $(\text{-}1, \text{-}2)$, $(3, 1)$, and $(2, \text{-}1)$, we can just double the coordinates to get $(\text{-}2, \text{-}4)$, $(6, 2)$, and $(4, \text{-}2)$.



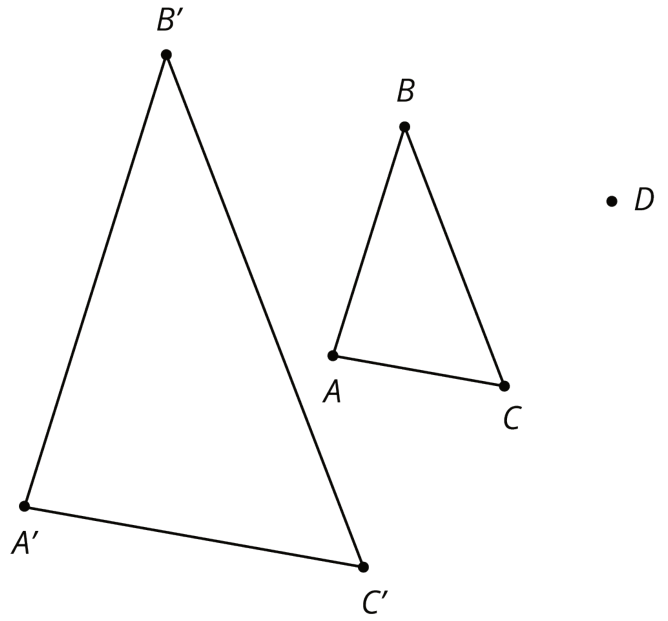
Lesson 4 Practice Problems

Problem 1

Statement

Triangle ABC is dilated using D as the center of dilation with scale factor 2.

The image is triangle $A'B'C'$. Clare says the two triangles are congruent, because their angle measures are the same. Do you agree? Explain how you know.



Solution

No. The triangles are not congruent because their side lengths are different.

Problem 2

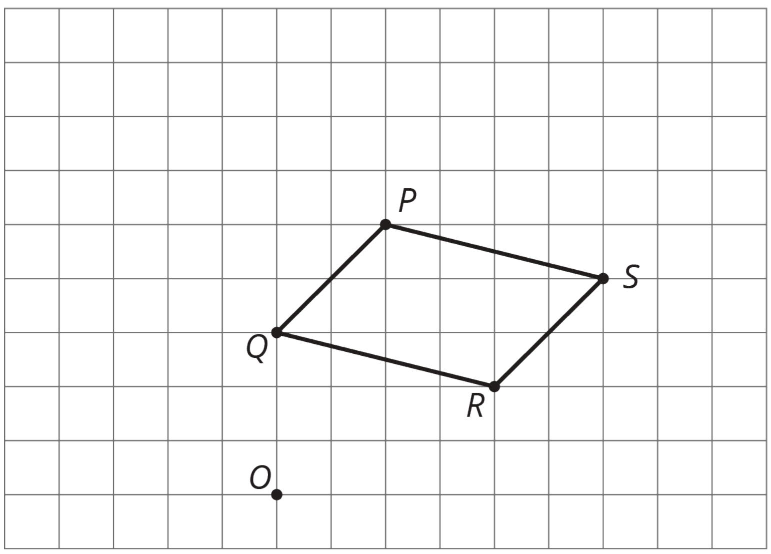
Statement

On graph paper, sketch the image of quadrilateral PQRS under the following dilations:

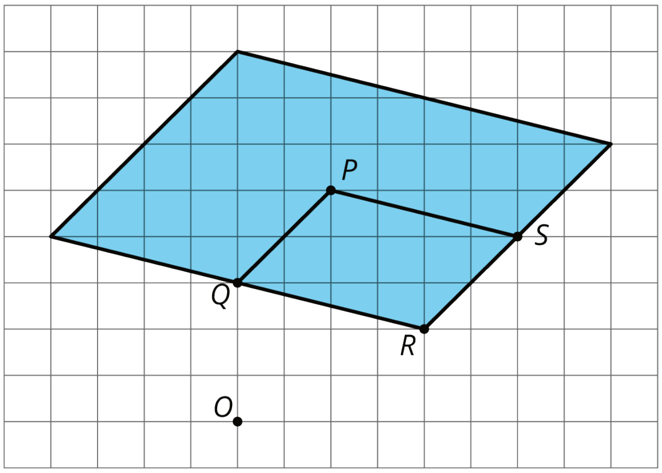
a. The dilation centered at R with scale factor 2.

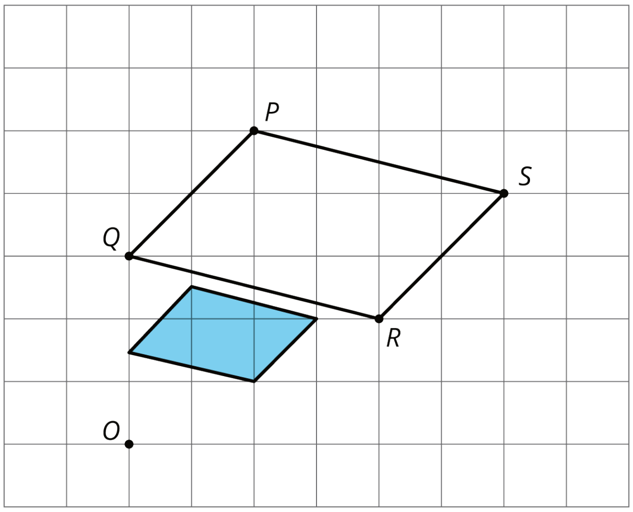
b. The dilation centered at O with scale factor $\frac{1}{2}$.

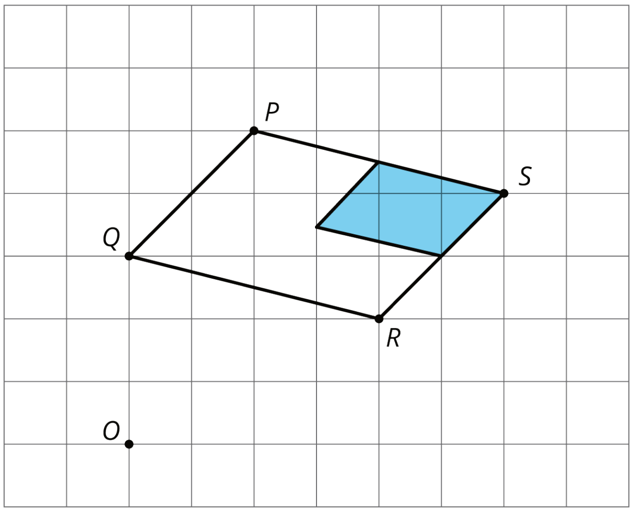
c. The dilation centered at S with scale factor $\frac{1}{2}$.



Solution



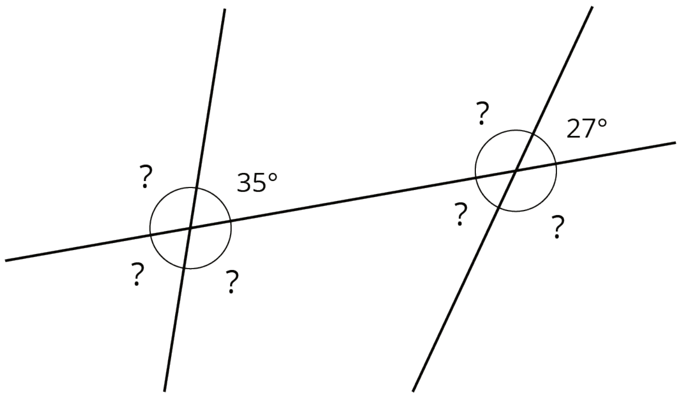




Problem 3

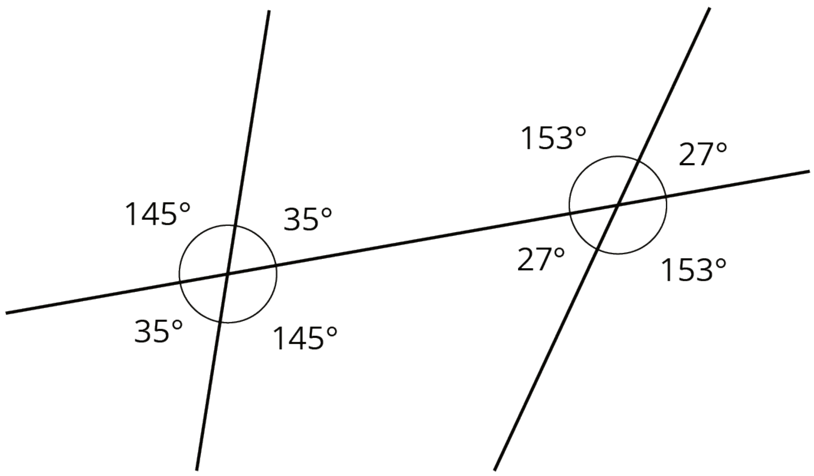
Statement

The diagram shows three lines with some marked angle measures.



Find the missing angle measures marked with question marks.

Solution

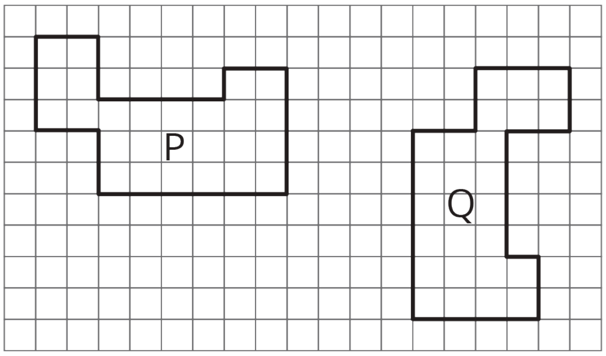


(From Unit 1, Lesson 14.)

Problem 4

Statement

Describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.



Solution

Answers vary. Sample response: P is rotated 90 degrees clockwise and translated until the corresponding vertices match up.

(From Unit 1, Lesson 4.)

Problem 5

Statement

Point B has coordinates $(-2, -5)$. After a translation 4 units down, a reflection across the y-axis, and a translation 6 units up, what are the coordinates of the image?

Solution

$(2, -3)$

(From Unit 1, Lesson 6.)