## Lesson 12: Reasoning about Exponential Graphs (Part 1)

Let’s study and compare equations and graphs of exponential functions.

### 12.1: Spending Gift Money

Jada received a gift of $180. In the first week, she spent a third of the gift money. She continues spending a third of what is left each week thereafter. Which equation best represents the amount of gift money $g$, in dollars, she has after $t$ weeks? Be prepared to explain your reasoning.

1. $g=180−\frac{1}{3}t$
2. $g=180⋅\left(\frac{1}{3}\right)^{t}$
3. $g=\frac{1}{3}⋅180^{t}$
4. $g=180⋅\left(\frac{2}{3}\right)^{t}$

### 12.2: Equations and Their Graphs

1. Each of the following functions $f$, $g$$,$ $h$, and $j$ represents the amount of money in a bank account, in dollars, as a function of time $x$, in years. They are each written in form $m\left(x\right)=a⋅b^{x}$.
$f\left(x\right)=50⋅2^{x}$
$g\left(x\right)=50⋅3^{x}$
$h\left(x\right)=50⋅\left(\frac{3}{2}\right)^{x}$
$j\left(x\right)=50⋅\left(0.5\right)^{x}$
	1. Use graphing technology to graph each function on the same coordinate plane.
	2. Explain how changing the value of $b$ changes the graph.
2. Here are equations defining functions $p$, $q$, and $r$. They are also written in the form $m\left(x\right)=a⋅b^{x}$.
$p\left(x\right)=10⋅4^{x}$
$q\left(x\right)=40⋅4^{x}$
$r\left(x\right)=100⋅4^{x}$
	1. Use graphing technology to graph each function and check your prediction.
	2. Explain how changing the value of $a$ changes the graph.

#### Are you ready for more?

As before, consider bank accounts whose balances are given by the following functions:

$f\left(x\right)=10⋅3^{x}    g\left(x\right)=3^{x+2}    h\left(x\right)=\frac{1}{2}⋅3^{x+3}$

Which function would you choose? Does your choice depend on $x$?

### 12.3: Graphs Representing Exponential Decay

$m\left(x\right)=200⋅\left(\frac{1}{4}\right)^{x}$
$n\left(x\right)=200⋅\left(\frac{1}{2}\right)^{x}$
$p\left(x\right)=200⋅\left(\frac{3}{4}\right)^{x}$
$q\left(x\right)=200⋅\left(\frac{7}{8}\right)^{x}$



1. Match each equation with a graph. Be prepared to explain your reasoning.
2. Functions $f$ and $g$ are defined by these two equations: $f\left(x\right)=1,​000⋅\left(\frac{1}{10}\right)^{x}$ and $g\left(x\right)=1,​000⋅\left(\frac{9}{10}\right)^{x}$.
	1. Which function is decaying more quickly? Explain your reasoning.
	2. Use graphing technology to verify your response.

### Lesson 12 Summary

An exponential function can give us information about a graph that represents it.

For example, suppose the function $q$ represents a bacteria population $t$ hours after it is first measured and $q\left(t\right)=5,​000⋅\left(1.5\right)^{t}$. The number 5,000 is the bacteria population measured, when $t$ is 0. The number 1.5 indicates that the bacteria population increases by a factor of 1.5 each hour.

A graph can help us see how the starting population (5,000) and growth factor (1.5) influence the population. Suppose functions $p$ and $r$ represent two other bacteria populations and are given by $p\left(t\right)=5,​000⋅2^{t}$ and  $r\left(t\right)=5,​000⋅\left(1.2\right)^{t}$. Here are the graphs of $p$, $q$, and $r$.



All three graphs start at $5,​000$ but the graph of $r$ grows more slowly than the graph of $q$ while the graph of $p$ grows more quickly. This makes sense because a population that doubles every hour is growing more quickly than one that increases by a factor of 1.5 each hour, and both grow more quickly than a population that increases by a factor of 1.2 each hour.



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