Lesson 6: Similarity

Goals

- Comprehend that the phrase “similar figures” (in written and spoken language) means there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other.
- Justify (orally) the similarity of two figures using a sequence of transformations that takes one figure to the other.

Learning Targets

- I can apply a sequence of transformations to one figure to get a similar figure.
- I can use a sequence of transformations to explain why two figures are similar.

Lesson Narrative

In the previous unit, students saw that two figures are congruent when there is a sequence of translations, rotations, and reflections that takes one figure to another. Now dilations, studied in previous lessons, are added to the possible set of “moves” taking one shape to another. Two figures are **similar** if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to the other. When two figures are similar, there are always many different sequences that show that they are similar. One method is to apply a dilation to one figure so that the corresponding figures are congruent. Then a sequence of rigid motions will finish taking one shape to the other. Alternatively, we could translate one pair of corresponding vertices together, apply rotations and reflections to adjust the orientations, and then conclude with a dilation so that they match.

In future lessons, students will learn shortcuts for some polygons (including all triangles), but in this lesson they focus on the definition of similarity in terms of transformations. They will see that two dilations with the same scale factor but different centers differ by a translation. They will also study how transformations from polygon A to polygon B can be reversed to take polygon B to polygon A.

Alignments

Building On

- 7.NS.A: Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
- 8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

Addressing

- 8.G.A.2: Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
- 8.G.A.4: Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time
- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

If you decide to do the optional “Methods for Translations and Dilations” activity, print and cut out 1 set of cards for every 2 students.

> **Student Learning Goals**
>
> Let’s explore similar figures.

6.1 Equivalent Expressions

Warm Up: 5 minutes
This warm-up prompts students to use what they know about operations and properties of operations to create related expressions. While many warm-ups encourage students to work mentally and verbally, students will write their responses to this prompt. Since many different responses are possible, the task is accessible to all students and provides an opportunity to hear how each student reasons about the operations. Some different ideas that may emerge are:

- commutative property
- distributive property
- inverse operations
- adjusting factors (for example, doubling and halving)

Examples of each are given in the student response section. Students are not expected to use these terms, but highlight the terms if students do use them.

Building On

- 7.NS.A

Launch

Arrange students in groups of 2. Tell students they are writing a list of several expressions equivalent to $10(2+3)-8\cdot 3$. Give students 2 minutes of quiet think time followed by 1 minute to discuss their responses with a partner.

> ### Student Task Statement
>
> Use what you know about operations and their properties to write three expressions equivalent to the expression shown.
>
> $$10(2+3)-8\cdot 3$$

Student Response

Answers vary. Possible responses:

- commutative property: $10(3+2)-8\cdot 3$ or $\text{-}8\cdot 3+10(2+3)$
- distributive property: $10\cdot 2+10\cdot 3-8\cdot 3$
- inverse operations: $10(2+3)+\text{-}8\cdot 3$
- associative property: $10(2+3)-16\cdot 1.5$

Activity Synthesis

Much of the discussion takes place between partners. Ask students to share any expressions that they aren't sure about, but try to resolve these and move on quickly.

6.2 Similarity Transformations (Part 1)

20 minutes (there is a digital version of this activity)
In this activity, students learn that two figures are *similar* when there is a sequence of translations, reflections, rotations and dilations that takes one figure to the other. Students practice discovering these sequences for two pairs of figures.

When two shapes are similar but not congruent, the sequence of steps showing the similarity usually has a single dilation and then the rest of the steps are rigid transformations. The dilation can come at any time. It does not matter which figure you start with. An important thing for students to notice in this activity is that there is more than one sequence of transformations that show two figures are similar. Monitor for students who insert a dilation at different places in the sequence. Also monitor for how students find the scale factor for the hexagons.

Building On

- 8.G.A.2

Addressing

- 8.G.A.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR1: Stronger and Clearer Each Time

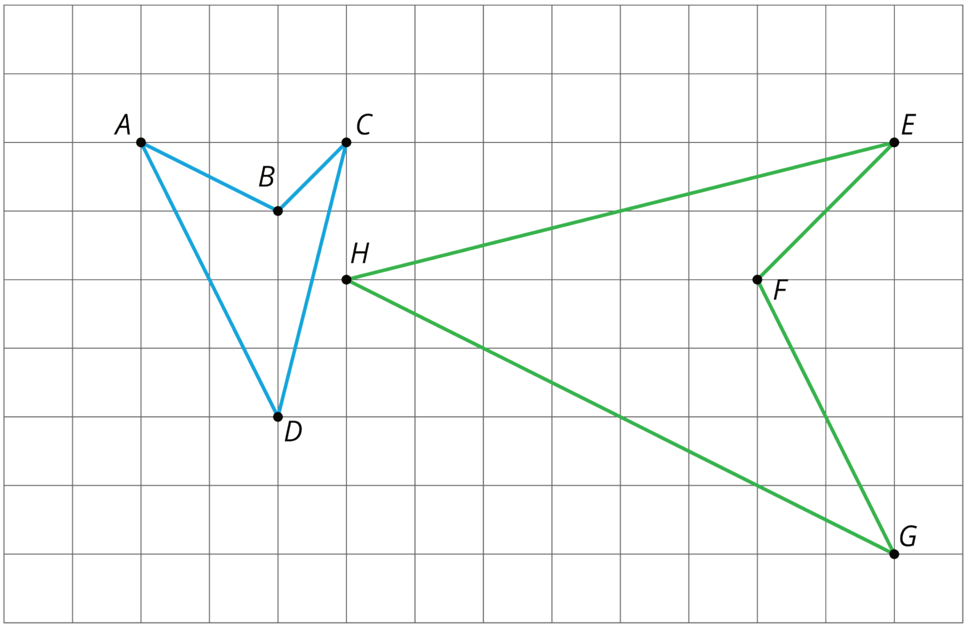
Launch

Before beginning this task, define what it means for two figures to be *similar*.

Tell students: We have talked about how one figure can be a scaled copy of another. This relationship goes in both directions. For example, if triangle DEF is a scaled copy of triangle ABC with scale factor of 2 then triangle ABC is a scaled copy of triangle DEF with scale factor $\frac{1}{2}$. We have learned that the transformation that creates scaled copies is called a dilation.

We say that triangles ABC and DEF are **similar**. The previous unit explored how translations, rotations, and reflections define congruent figures. The inclusion of dilations can change the size of the figure as well as its location and orientation.

We will start our investigation of similar figures by identifying sequences of translations, rotations, reflections, and dilations that show two figures are similar. Demonstrate using this example.



There are many methods to make this work. Explain at least two. First, identify the corresponding parts. Then come up with a plan to take one figure to the other. Ensure students understand, through demonstration with this example, that the work of showing two figures are similar requires communicating the details of each transformation in the sequence with enough precision. Some sample methods:

1. Method 1 ($ABCD$ to $GFEH$: Dilate, Translate, Rotate, Reflect)
    a. Dilate using D as the center with scale factor 2.

    b. Translate D to H

    c. Rotate using H as the center clockwise by 90 degrees

    d. Reflect using the line that contains H and F.

2. Method 2 ($ABCD$ to $GFEH$: Reflect, Translate, Rotate, Dilate)
    a. Reflect using the line that contains D and B.

    b. Translate D to H.

    c. Rotate using H as the center clockwise by 90 degrees

    d. Dilate using H as the center with a scale factor of 2.

3. Method 3 ($ABCD$ to $GFEH$: Translate, Rotate, Reflect, Dilate)
    a. Translate B to F.

    b. Rotate using F as the center clockwise by 90 degrees.

    c. Reflect using the line that contains F and H.

    d. Dilate using F as the center with scale factor 2.

These arguments can also be applied to figures that are not on a grid: the grid helps to identify directions and distances of translation, 90 degree angles of rotation, and horizontal and vertical lines of reflection.

If using the print version, provide access to geometry toolkits. If using the digital version, remind students of the meaning and functionality of each transformation tool, as necessary.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following term and maintain the display for reference throughout the unit: similar. On this display, include the step-by-step instructions of at least 2 of the 3 given methods for creating similar polygons using transformations.
Supports accessibility for: Memory; Language

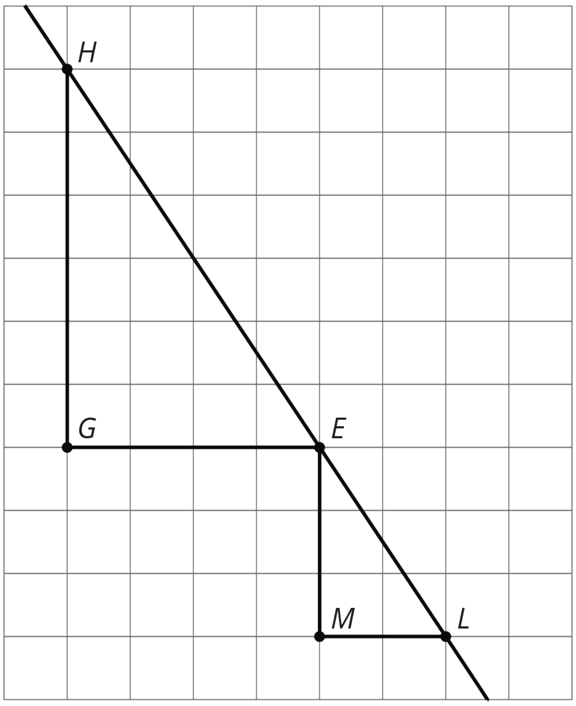
Anticipated Misconceptions

If students do not recall the three types of rigid transformations, refer them to the classroom display that provides an example of a rotation, a reflection, and a translation.

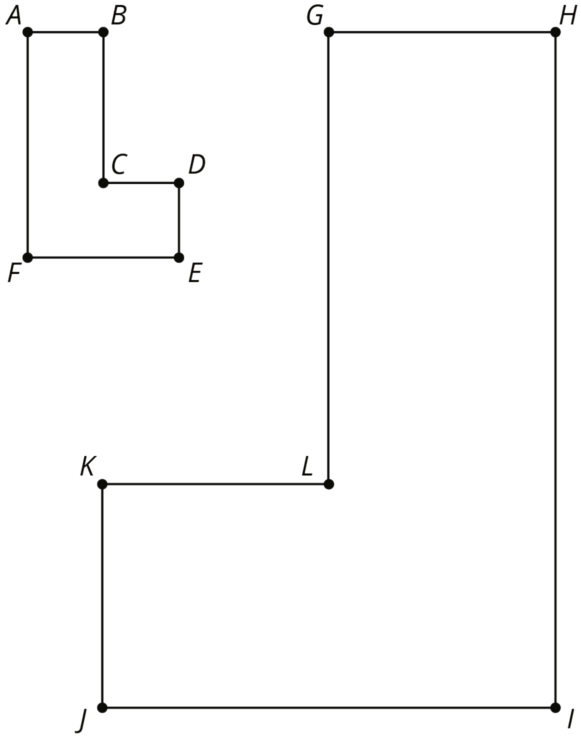
For the second problem, students may need encouragement to experiment moving the shapes (using tracing paper for example). If they get stuck finding the scale factor, tell them that they can approximate by measuring sides of the two figures.

Student Task Statement

1. Triangle EGH and triangle LME are **similar**. Find a sequence of translations, rotations, reflections, and dilations that shows this.



2. Hexagon $ABCDEF$ and hexagon $HGLKJI$ are similar. Find a sequence of translations, rotations, reflections, and dilations that shows this.



Student Response

1. Answers vary. Sample response:
   a. Begin with triangle LME

b. Translate L to E

c. Dilate using E as the center with scale factor 2

2. Answers vary. Sample response:
    a. Begin with figure $ABCDEF$

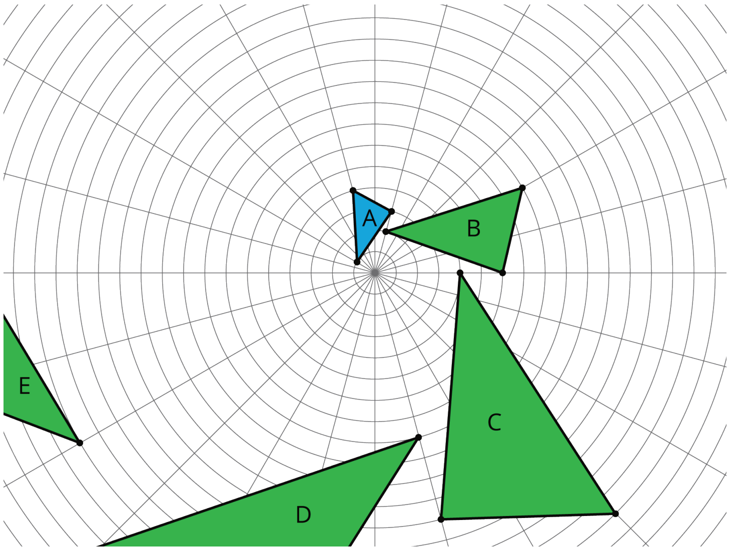
    b. Reflect using the line that contains A and F

    c. Translate F to I

    d. Dilate using I as the center with scale factor 3

Are You Ready for More?

The same sequence of transformations takes Triangle A to Triangle B, takes Triangle B to Triangle C, and so on. Describe a sequence of transformations with this property.



Student Response

Answers vary. Sample response: Dilate, from the center of the circular grid, with scale factor 2, then rotate clockwise 75 degrees.

Activity Synthesis

Select students to give a variety of solutions for the different problems. Point out that there are multiple ways to do each pair and any valid sequence is allowed. Ensure that students communicate each transformation in the sequence in sufficient detail.

Access for English Language Learners

Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time. After students have determined the sequence of transformations that shows the polygons are similar, ask students to write a detailed sequence of the transformations on their paper. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language (e.g., “How did you know to translate point L to point E?”, and “How did you know to dilate the polygon by a scale factor of 3?”, etc.). Students can borrow ideas and language from each partner to refine and clarify their original explanation. This will help students refine their own explanation and learn about other ways to show polygons are similar using a sequence of transformations.
Design Principles(s): Optimize output (for explanation); Maximize meta-awareness

6.3 Similarity Transformations (Part 2)

10 minutes
This activity helps students visualize what happens to figures under different kinds of transformations. Students practice identifying which transformations might be used in the sequence of translations, rotations, reflections, and dilations in order to show figures are similar. By recognizing patterns in the image results after using certain transformations (MP8), students may be able to apply this to finding transformations for other problems. Students should pay special attention to see the connection between the orientations of the original figure and the resulting images after transformation.

Encourage students to draw rough sketches though it is also ok to use patty paper, for example, to execute the rigid motions. For the dilations, however, it could become time consuming to choose an explicit scale factor and measure carefully. Make sure, after students have worked on the first problem, to show some examples of sketches that are not exact but capture the main features of the figure.

Addressing

- 8.G.A.2
- 8.G.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

The figure in this task is intended to resemble a hand with all of the fingers together and the thumb sticking out. Encourage students to “sketch” the resulting images for this task (or make tracing paper available, indicating that the images do not need to be exact). They do not need to make the side lengths, angles, etc. perfect, but it should be clear where the corresponding parts of the image are and whether it is larger or smaller than the original. Other activities in this unit will ask students to be precise in their use of transformations, but the goal of this activity is to get an idea of how the different transformations affect a figure’s image.

Select a few good examples of student work (including some that have been sketched free hand) to share with the class after problem 1, in order to clearly communicate the expectation for level of precision.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. To support development of organizational skills, check in with students within the first 2–3 minutes of work time. Check to make sure students are not using exact or precise measurements, as this activity only requires a sketch. After the first problem, invite a few students to think aloud and share their sketches to guide the rest of the individual work time.
Supports accessibility for: Memory; Organization

Access for English Language Learners

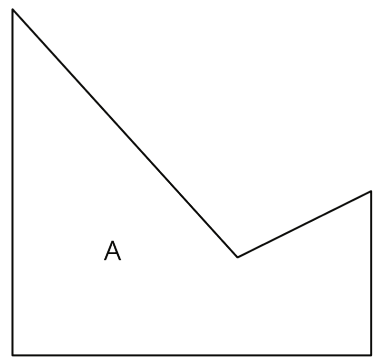
Speaking, Listening: MLR7 Compare and Connect. Ask students to prepare a visual display of their figures that are similar to Figure A. As students investigate each other’s work, ask students to share what transformations are especially clear in the display of similar figures. Listen for and amplify any comments about what might make the transformations clearer in the display. Then encourage students to make connections between the words “translation,” “rotation,” “reflection,” and “dilation” and how they affect the figure. Listen for and amplify language students use to describe what happens to figures under different kinds of transformations. This will foster students’ meta-awareness and support constructive conversations as they compare images of the same figure and make connections between transformations and their effects on figures.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

Anticipated Misconceptions

If students choose an exact scale factor, or measure the exact angle sizes, explain that precise measurements are not needed in this task. At this point, they are just sketching similar figures.

Student Task Statement

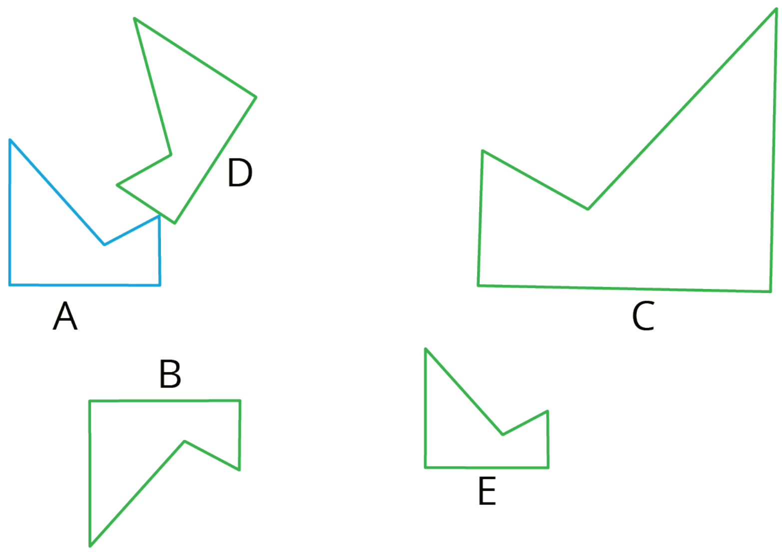
Sketch figures similar to Figure A that use only the transformations listed to show similarity.



1. A translation and a reflection. Label your sketch Figure B.
   Pause here so your teacher can review your work.
2. A reflection and a dilation with scale factor greater than 1. Label your sketch Figure C.
3. A rotation and a reflection. Label your sketch Figure D.
4. A dilation with scale factor less than 1 and a translation. Label your sketch Figure E.

Student Response

Answers vary. Sample responses:



Activity Synthesis

Select students to display their answers for each of the questions. Ask students to notice things the answers have in common so that they can make connections to the types of transformations that might be useful in showing that two figures are similar.

Assuming that the rotations are not through an angle that is a multiple of 360° and that the translations have a non-zero horizontal or vertical part, point out that:

- Dilations will create larger or smaller copies depending on the scale factor as seen in previous lessons.
- Translations will slide the figure in some direction.
- Rotations will “tilt” or “turn” the figure.
- Reflections will change the handedness so that the resulting image will look like the back of a right hand instead of the back of a left hand as in the original image.

6.4 Methods for Translations and Dilations

Optional: 10 minutes

The purpose of this task is for students to practice showing that two shapes are similar using only a few pre-determined rigid motions and dilations. Some students will start with triangle ABC and take this to triangle DEF while other start with DEF and take this to ABC. While there is flexibility in either direction, one way of getting from DEF to ABC is to “undo” the moves that take ABC to DEF.

Monitor for students who use different centers of dilation in their sequence, particularly as the first step in the sequence. Invite these students to share, highlighting the fact that two dilations with the same scale factor but different centers differ by a translation. Also monitor for students whose sequences of rigid motions and dilations are the same but in the opposite order, one set taking ABC to DEF and the opposite taking DEF back to ABC. Select these students to share this important observation during the discussion.

Addressing

- 8.G.A.4

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Again, remind students that two figures are similar if there is a sequence of translations, rotations, reflections, and dilations that takes one figure to another. Tell them that they need to find at least one way to show that triangle ABC and triangle DEF are similar using only the transformations they are given on their cards.

Arrange students in groups of 2. Give each group one complete set of cards.

Access for Students with Disabilities

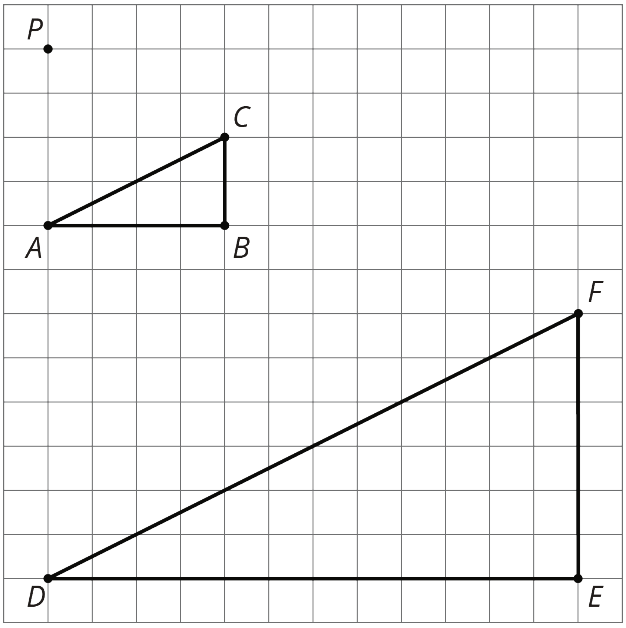
Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, "One thing that is the same is....”; "One thing that is different is....”; and "Another strategy to get the same result is....”
Supports accessibility for: Language; Social-emotional skills

Anticipated Misconceptions

Students might think that is it necessary to perform transformations in the same order or that one particular point needs to be the center. If partners choose the same methods, prompt them to try it another way that will have the same end result.

Student Task Statement

Your teacher will give you a set of five cards and your partner a different set of five cards. Using only the cards you were given, find at least one way to show that triangle ABC and triangle DEF are similar. Compare your method with your partner's method. What is the same about your methods? What is different?



Student Response

Partner A can:

1. Dilate triangle ABC using center point P and scale factor 3.

2. Dilate using center point A and scale factor 3 followed by translating from A to D.
3. Translate from A to D followed by dilating using center D and scale factor 3.

Partner B can:

1. Dilate triangle DEF using center point P and scale factor $\frac{1}{3}$.
2. Dilate using center point D and scale factor $\frac{1}{3}$ followed by translating from D to A.
3. Translate from D to A followed by dilating using center A and scale factor $\frac{1}{3}$.

Activity Synthesis

Invite selected students to share, highlighting methods of moving ABC to DEF and DEF to ABC which are “opposite” of one another, for example dilations with center P and reciprocal scale factors.

As students share their responses, highlight these points:

- The scale factors for the dilations are reciprocals regardless of when the dilations are done in the sequence.
- If used, the translations are inverses of each other (eg “Translate A to D” instead of “Translate D to A”).
- Dilations with different centers but the same scale factor produce congruent figures that differ by a translation.
- The order in which transformations are applied can influence the result.

One important conclusion (the third bullet point) is that when you are showing that two figures are similar, you can pick any point as the center of dilation if you know the scale factor, because you can always adjust the position using a translation.

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their sequence of transformations to show that triangle ABC and triangle DEF are similar, present an incorrect sequence of transformations. For example, "Translate from A to D. Then dilate using center P and scale factor 3." Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss in partners, listen for students who clarify the meaning of the center of dilation. Prompt students to share their critiques and corrected explanations with the class. Listen for and amplify the language students use to describe what happens when P is the center of dilation and explain why D should be the center of dilation. This routine will engage students in meta-awareness as they clarify how the center of dilation affects the dilated figure.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

Lesson Synthesis

Review the definition of similar figures and any important insights that arose during the lesson. Insights to highlight:

Two figures are similar if there is a sequence of translations, rotations, reflections, and dilations that maps one to the other. Scaled copies of figures, studied in grade 7, are all examples of similar figures. In this lesson, we found transformations that showed that two figures were similar.

We saw that there is more than one sequence of transformations that shows two figures are similar. One way to think about it is that you need to use a dilation to make corresponding side lengths the same size. The figures will be congruent after this step. Once you do that, you just need to find a sequence of rigid transformations that align the congruent figures. You can also do it the other way, by bringing the figures into alignment and then dilating one to match up with the other.

Add the term *similar* along with a definition and example to your classroom display such as a word wall or anchor chart.

6.5 Showing Similarity

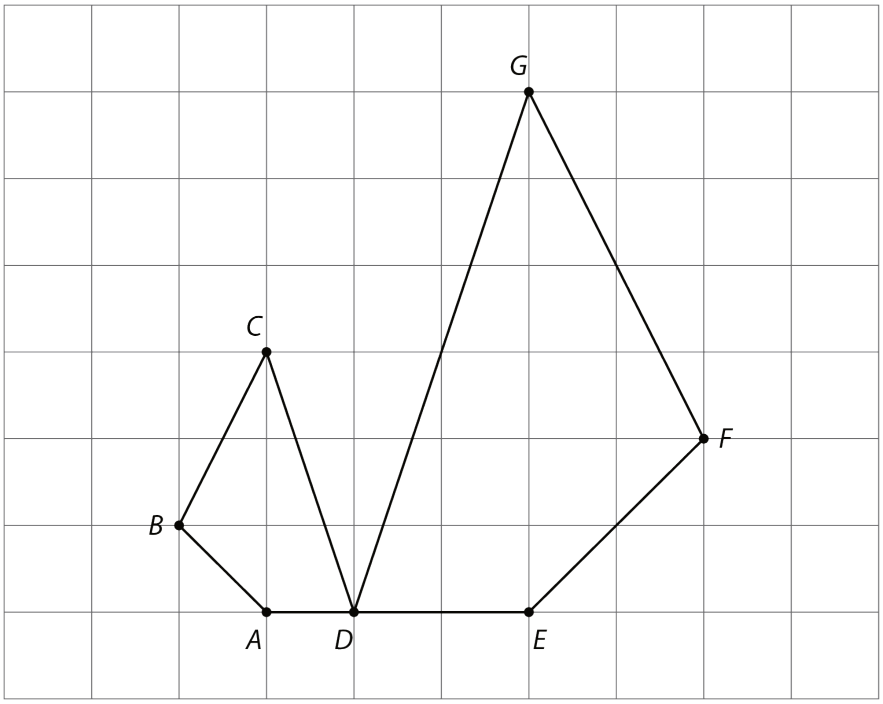
Cool Down: 5 minutes
Students analyze and correct a proposed sequence of rigid transformations and dilations to show that two figures are similar.

Addressing

- 8.G.A.4

Student Task Statement

Elena gives the following sequence of transformations to show that the two figures are similar by transforming $ABCD$ into $EFGD$.



1. Dilate using center D and scale factor 2.
2. Reflect using the line AE.

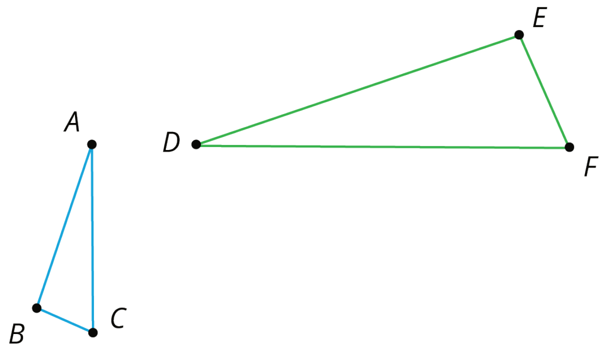
Is Elena's method correct? If not, explain how you could fix it.

Student Response

The figures are similar, but the transformations do not take $ABCD$ to $EFGD$. After dilating $ABCD$ using D as the center with a scale factor of 2, Elena can reflect over the vertical line through D rather than the horizontal line.

Student Lesson Summary

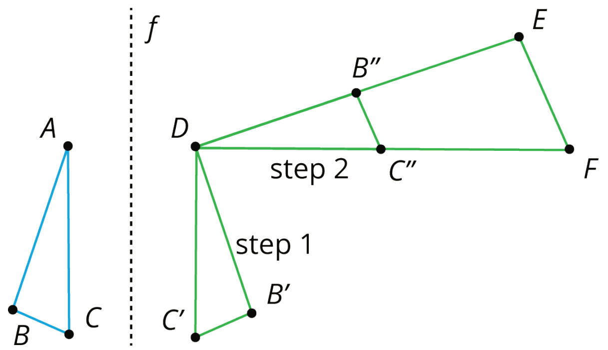
Let's show that triangle ABC is similar to triangle DEF:



Two figures are **similar** if one figure can be transformed into the other by a sequence of translations, rotations, reflections, and dilations. There are many correct sequences of transformations, but we only need to describe one to show that two figures are similar.

One way to get from ABC to DEF follows these steps:

- step 1: reflect across line f
- step 2: rotate 90° counterclockwise around D
- step 3: dilate with center D and scale factor 2

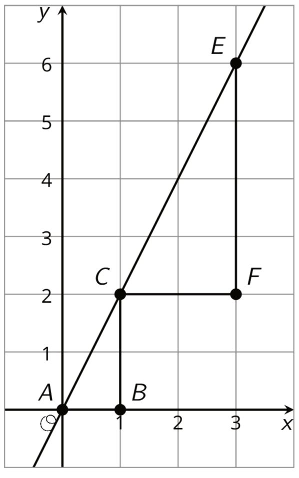


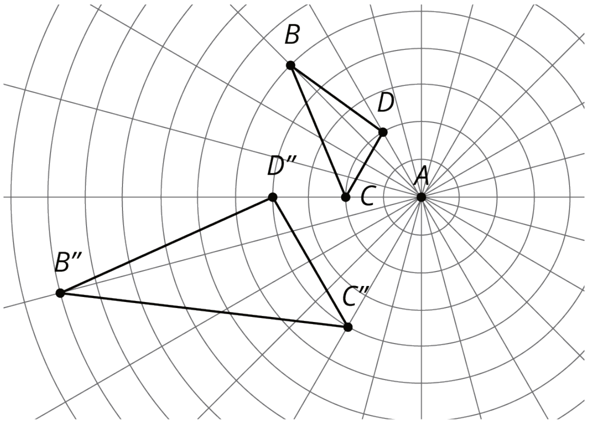
Another way would be to dilate triangle ABC by a scale factor of 2 with center of dilation A, then translate A to D, then reflect over a vertical line through D, and finally rotate it so it matches up with triangle DEF. What steps would you choose to show the two triangles are similar?

Glossary

- similar

Lesson 6 Practice Problems
Problem 1
Statement
Each diagram has a pair of figures, one larger than the other. For each pair, show that the two figures are similar by identifying a sequence of translations, rotations, reflections, and dilations that takes the smaller figure to the larger one.

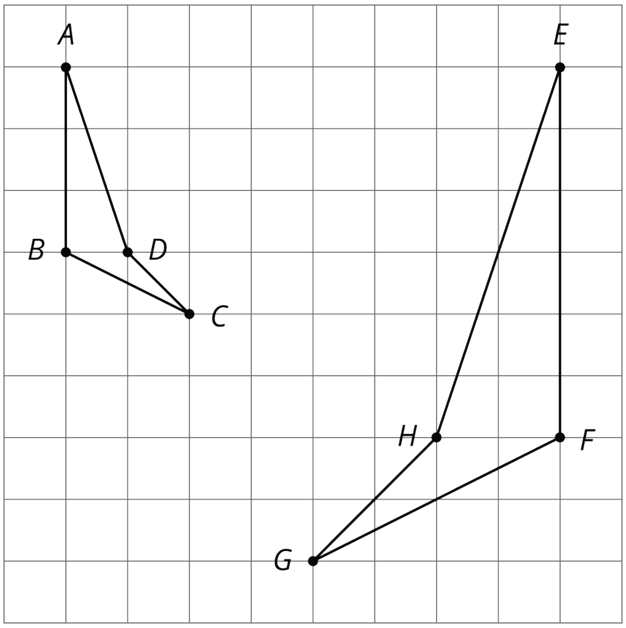




Solution
- Translate A to C, and then dilate with center C by a factor of 2.
- Rotate 60° counter-clockwise with center A, and then dilate using a scale factor of 2 centered at A.

Problem 2
Statement
Here are two similar polygons.

Measure the side lengths and angles of each polygon. What do you notice?



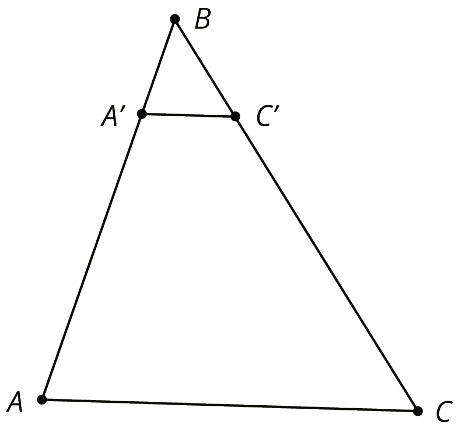
Solution

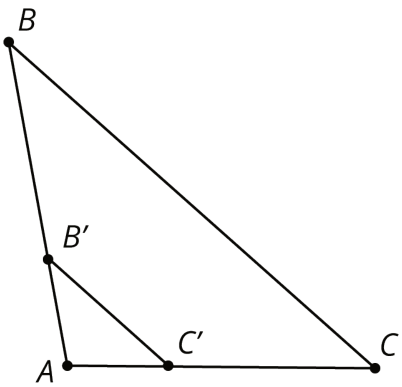
Answers vary. Sample response: Corresponding side lengths in the larger polygon are double the side lengths of the smaller polygon, while corresponding angles all have the same measure.

Problem 3

Statement

Each figure shows a pair of similar triangles, one contained in the other. For each pair, describe a point and a scale factor to use for a dilation moving the larger triangle to the smaller one. Use a measurement tool to find the scale factor.





Solution

Center of dilation: B, scale factor: $\frac{1}{4}$; center of dilation: A, scale factor: $\frac{1}{3}$