## Unit 1 Lesson 8: The $n^{th}$ Term

### 1 Which One Doesn’t Belong: Repeated Operations (Warm up)

#### Student Task Statement

Which one doesn’t belong?

A. $5+2+2+2+2+2+2$

B. $5+6⋅2$

C. $5⋅2^{6}$

D. $5⋅2⋅2⋅2⋅2⋅2⋅2$

### 2 More Paper Slicing

#### Student Task Statement

1. Clare takes a piece of paper with length 8 inches and width 10 inches and cuts it in half. Then she cuts it in half again, and again. . .
	1. Instead of writing a recursive definition, Clare writes $C(n)=80⋅\left(\frac{1}{2}\right)^{n}$, where $C$ is the area, in square inches, of the paper after $n$ cuts. Explain where the different terms in her expression came from.
	2. Approximately what is the area of the paper after 10 cuts?
2. Kiran takes a piece of paper with length 8 inches and width 10 inches and cuts away one inch of the width. Then he does it again, and again. . .
	1. Complete the table for the area of Kiran’s paper $K(n)$, in square inches, after $n$ cuts.

|  |  |
| --- | --- |
| * + $n$
 | * + $K(n)$
 |
| * + 0
 | * + 80
 |
| * + 1
 |  |
| * + 2
 | * + $80−8−8=80−8(2)=64$
 |
| * + 3
 |  |
| * + 4
 |  |
| * + 5
 |  |

* 1. Kiran says the area after 6 cuts, in square inches, is $80−8⋅6$. Explain where the different terms in his expression came from.
	2. Write a definition for $K(n)$ that is not recursive.
1. Which is larger, $K(6)$ or $C(6)$?

### 3 A Sierpinski Triangle

#### Student Task Statement

A Sierpinski triangle can be created by starting with an equilateral triangle, breaking the triangle into 4 congruent equilateral triangles, and then removing the middle triangle. Starting from a single black equilateral triangle:



1. Let $S$ be the number of black triangles in Step $n$. Define $S(n)$ recursively.
2. Andre and Lin are asked to write an equation for $S$ that isn't recursive. Andre writes $S(n)=3^{n}$ for $n\geq 0$ while Lin writes $S(n)=3^{n−1}$ for $n\geq 1$. Whose equation do you think is correct? Explain or show your reasoning.



© CC BY 2019 by Illustrative Mathematics®