## Lesson 17: Lines in Triangles

* Let’s investigate more special segments in triangles.

### 17.1: Folding Altitudes

Draw a triangle on tracing paper. Fold the altitude from each vertex.

### 17.2: Altitude Attributes

Triangle $ABC$ is graphed.



1. Find the slope of each side of the triangle.
2. Find the slope of each altitude of the triangle.
3. Sketch the altitudes. Label the point of intersection $H$.
4. Write equations for all 3 altitudes.
5. Use the equations to find the coordinates of $H$ and verify algebraically that the altitudes all intersect at $H$.

#### Are you ready for more?

Any triangle $ABC$ can be translated, rotated, and dilated so that the image $A^{′}$ lies on the origin, $B^{′}$ lies on the point $\left(1,0\right)$, and $C^{′}$ has position $\left(a,b\right)$. Use this as a starting point to prove that the altitudes of all triangles all meet at the same point.

### 17.3: Percolating on Perpendicular Bisectors

Triangle $ABC$ is graphed.



1. Find the midpoint of each side of the triangle.
2. Sketch the perpendicular bisectors, using an index card to help draw 90 degree angles. Label the intersection point $P$.
3. Write equations for all 3 perpendicular bisectors.
4. Use the equations to find the coordinates of $P$ and verify algebraically that the perpendicular bisectors all intersect at $P$.

### 17.4: Perks of Perpendicular Bisectors

Consider triangle $ABC$ from an earlier activity.

1. What is the distance from $A$ to $P$, the intersection point of the perpendicular bisectors of the triangle’s sides? Round to the nearest tenth.
2. Write the equation of a circle with center $P$ and radius $AP$.
3. Construct the circle. What do you notice?
4. Verify your hypothesis algebraically.

### 17.5: Amazing Points

Consider triangle $ABC$ from earlier activities.



1. Plot point $H$, the intersection point of the altitudes.
2. Plot point $P$, the intersection point of the perpendicular bisectors.
3. Find the point where the 3 medians of the triangle intersect. Plot this point and label it $J$.
4. What seems to be true about points $H,P,$ and $J$? Prove that your observation is true.

### 17.6: Tiling the (Coordinate) Plane

A tessellation covers the entire plane with shapes that do not overlap or leave gaps.

1. Tile the plane with congruent rectangles:
	1. Draw the rectangles on your grid.
	2. Write the equations for lines that outline 1 rectangle.
2. Tile the plane with congruent right triangles:
	1. Draw the right triangles on your grid.
	2. Write the equations for lines that outline 1 right triangle.
3. Tile the plane with any other shapes:
	1. Draw the shapes on your grid.
	2. Write the equations for lines that outline 1 of the shapes.

### Lesson 17 Summary

The 3 medians of a triangle always intersect in 1 point. We can use coordinate geometry to show that the altitudes of a triangle intersect in 1 point, too. The 3 altitudes of triangle $ABC$ are shown here. They appear to intersect at the point $\left(4,6\right)$. By finding their equations, we can prove this is true.



The slopes of sides $AB,BC,$ and $AC$ are 0, $-\frac{2}{3}$, and 2. The altitude from $C$ is the vertical line $x=4$. The slope of the altitude from $A$ is $\frac{3}{2}$. Since the altitude goes through $\left(0,0\right),$ its equation is $y=\frac{3}{2}x$. The slope of the altitude from $B$ is $-\frac{1}{2}$. Following this slope over to the $y$-axis we can see that the $y$-intercept is 8. So the equation for this altitude is $y=-\frac{1}{2}x+8$.

We can now verify that $\left(4,6\right)$ lies on all 3 altitudes by showing that the point satisfies the 3 equations. By substitution we see that each equation is true when $x=4$ and $y=6$.



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