Lesson 12: Using Equations for Lines

Goals

- Create an equation of a line with positive slope on a coordinate grid using knowledge of similar triangles.

- Generalize (orally) a process for dilating a slope triangle ABC on a coordinate plane with center of dilation A and scale factor s.

- Justify (orally) that a point(x, y) is on a line by verifying that the values of x and y satisfy the equation of the line.

Learning Targets

- I can find an equation for a line and use that to decide which points are on that line.

Lesson Narrative

In the previous two lessons, students saw that all slope triangles for a line give the same slope value, and this value is called the slope of the line. They also began writing relationships satisfied by all points (x, y) on a line. In this lesson, they continue to write equations but with less scaffolding, that is no similar triangles are selected so students need to figure out what to do given a line and a few points on the line.

The properties of slope triangles that make the slope of a line meaningful have to do with dilations. In particular, dilations do not change the quotient of the vertical side length and horizontal side length of a slope triangle. Students return to dilations in this lesson, applied to a single slope triangle with varying scale factor. This gives a different way of seeing how the coordinates of points on a line vary.

Both techniques, using equations and studying all of the dilations of a single slope triangle, give expressions representing points on a line.

Alignments

Addressing

- 8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

- 8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Building Towards

- 8.EE.B.6: Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR3: Clarify, Critique, Correct

- MLR7: Compare and Connect

- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

> **Student Learning Goals**
>
> Let's write equations for lines.

12.1 Missing center

Warm Up: 5 minutes
Given a point, the image of the point under a dilation, and the scale factor of the dilation, students must identify the center of the dilation. This requires thinking about the meaning of dilations and the fact that the center of dilation, the point dilated, and the image, are collinear.
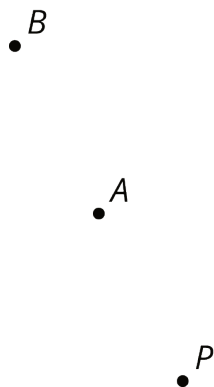
Addressing

- 8.G.A

Launch

Provide access to geometry toolkits. (In particular, a ruler or index card is needed.)

> **Student Task Statement**
>
> A dilation with scale factor 2 sends A to B. Where is the center of the dilation?

Student Response

The point, marked as P in the figure, is on the same line as A and B, the same distance from B to A, but on the other side of A.

Activity Synthesis

Ask students what they know about the center of dilation that helps to solve this problem. The key fact is that the center of dilation lies on the same line as A and B. The scale factor is 2 so if P is the center of dilation, then the length of segment PB is twice the length of segment PA.

12.2 Writing Relationships from Two Points

10 minutes

In the previous lesson, students found an equation satisfied by the points on a line using properties of slope triangles and a general point, labeled (x, y), on the line. In this activity, they pursue this work but the scaffold of the given slope triangles has been removed. Once students draw appropriate slope triangles, this is an opportunity to practice and consolidate learning. In addition, students use the equation (satisfied by points on the line) to check whether or not specific points lie on the line.

Note that the y-intercept was intentionally left off of this diagram, so that students who may have seen some of this material are discouraged from jumping straight to $y = mx + b$ and encouraged to engage with thinking about similar triangles.

There are many slope triangles that students can draw, but the one joining (5,3) and (7,7) is the most natural for calculating the slope and then (x, y) and either $(5, 3)$ or $(7, 7)$ can be used to find an equation. Monitor for students who make these choices (and use them to decide whether or not the given points lie on the line) and invite them to present during the discussion.

Building Towards

- 8.EE.B.6

Instructional Routines

- MLR3: Clarify, Critique, Correct

- Think Pair Share

Launch

Provide access to geometry toolkits (in particular, a straightedge is helpful). Give 2–3 minutes of quiet work time. Then ask them to share their responses and reasoning with a partner, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of using slope triangles to find the equation of a line to support connections between new situations and prior understandings. Consider using these prompts: "What does this demonstration have in common with the graph in this activity?" or "How can you apply your reasoning with slope triangles to this graph?"
Supports accessibility for: Conceptual processing; Visual-spatial processing

Student Task Statement

Here is a line.
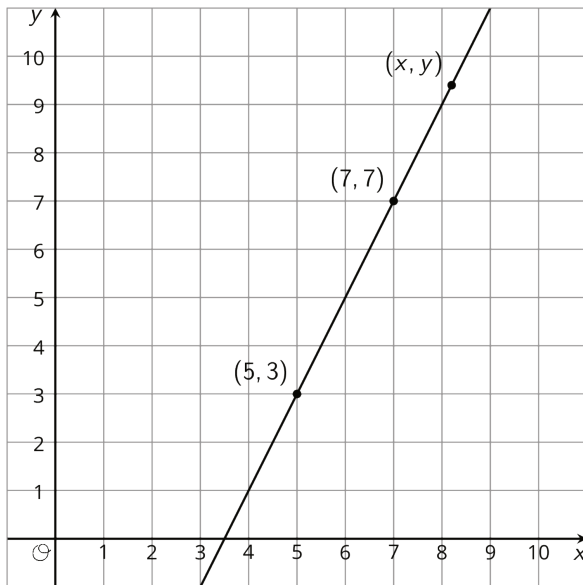
1. Using what you know about similar triangles, find an equation for the line in the diagram.

2. What is the slope of this line? Does it appear in your equation?

3. Is $(9, 11)$ also on the line? How do you know?

4. Is $(100, 193)$ also on the line?

Student Response

1. $\frac{y-7}{x-7} = 2$ or equivalent.

2. 2. Answers vary based on equation.

3. Yes. Possible strategies:
   - For every 1 to the right, you go 2 up. I started at $(7, 7)$ and counted until I got to $(9, 11)$.
   - When y is 11 and x is 9, it makes my equation true.

4. Yes. When y is 193 and x is 100, it satisfies my equation.

Are You Ready for More?

There are many different ways to write down an equation for a line like the one in the problem. Does $\frac{y-3}{x-6} = 2$ represent the line? What about $\frac{y-6}{x-4} = 5$? What about $\frac{y+5}{x-1} = 2$? Explain your reasoning.

Student Response

No. The equation $\frac{y-3}{x-6} = 2$ represents a line of slope 2 containing the point $(6, 3)$. Since our line does not contain the point $(6, 3)$, this is not our line. The equation $\frac{y-6}{x-4} = 5$ represents a line of slope 5 containing the point $(4, 6)$. Since our line has slope 2, this is not our line. The equation $\frac{y+5}{x-1} = 2$ represents a line of slope 2 containing the point $(1, -5)$. Since our line contains the point $(1, -5)$ and has slope 2, this is another possible equation for our line.

Activity Synthesis

Invite selected students to show how they arrived at their equation. Also, ask them how the equation helps to determine whether or not the points in the last two questions lie on the line.

Emphasize that the graph, as shown, is not helpful for checking whether or not $(100, 193)$ is on the line, but the equation is true for these numbers, so this point must be on the line.

Highlight that using (x, y) and $(5, 3)$ for a slope triangle gives an equation such as $\frac{y-3}{x-5} = 2$ while using (x, y) and $(7, 7)$ gives the equation $\frac{y-7}{x-7} = 2$. These equations look different, but they both work to check whether or not a point (x, y) is on the line. Using algebra to show that these two equations are equivalent is not necessary (or appropriate in grade 8), but students can see from the picture that either equation can be used to test whether or not a point (x, y) is on the line. If students have not done so already when they share their solutions, draw and label the two slope triangles that correspond to these two equations.

12.3 Dilations and Slope Triangles

15 minutes
This activity investigates the coordinates of points on a line from the point of view of dilations. At the beginning of this unit, students experimented with dilations and made numerous important discoveries including

- Dilations change distances between points by a scale factor s.

- Dilations preserve angles.

- Dilations take lines to lines.

Having developed the key ideas of *similar triangles* and *slope,* this activity returns to dilations, applying them systematically to a single slope triangle. All of these dilations of the triangle are similar, their long sides all lie on the same line, and the coordinates of the points on that line have a structure intimately linked with the dilations used to produce them.

For the third question, monitor for students who

- look for and express a pattern for the coordinates of the points from earlier questions (MP8); a scale factor of 1 gives $C = (2, 2)$, a scale factor of 2 gives $C = (4, 3)$, a scale factor of 2.5 gives $C = (5, 3.5)$, so the x-coordinate appears to be twice the scale factor while the y-coordinate appears to be one more than the scale factor.

- use the structure of $\triangle ABC$ and the definition of dilations.

For the final question, students can either

- reason through from scratch, looking at the x-coordinate or y-coordinate for example or

- use the coordinates they find for a scale factor of s in the third question.

Monitor for both approaches and invite students to share during the discussion.

Addressing

- 8.G.A.3

Building Towards

- 8.EE.B.6

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect

- MLR7: Compare and Connect

Launch

Access to geometry toolkits. Give 2–3 minutes of quiet work time, then ask students to share their reasoning with a partner. Follow with a whole-class discussion.


Access for Students with Disabilities

Representation: Develop Language and Symbols. Maintain a display of important terms and vocabulary. During the launch, take time to review terms students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of: dilation and scale factor.
*Supports accessibility for: Conceptual processing; Language*

Student Task Statement
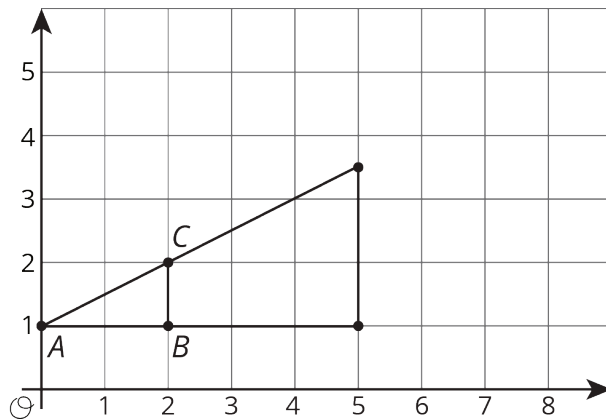
Here is triangle ABC.
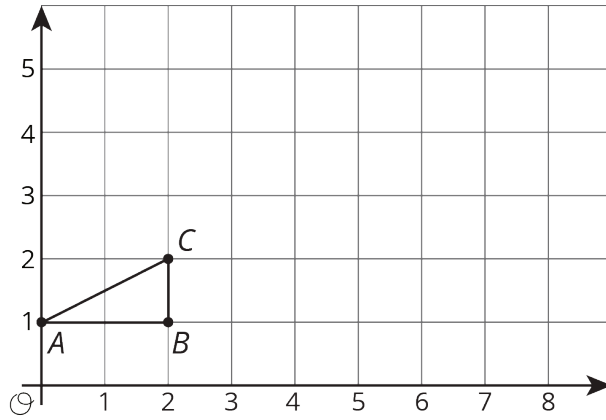
1. Draw the dilation of triangle ABC with center $(0, 1)$ and scale factor 2.

2. Draw the dilation of triangle ABC with center $(0, 1)$ and scale factor 2.5.

3. Where is C mapped by the dilation with center $(0, 1)$ and scale factor s?

4. For which scale factor does the dilation with center $(0, 1)$ send C to $(9, 5.5)$? Explain how you know.

Student Response
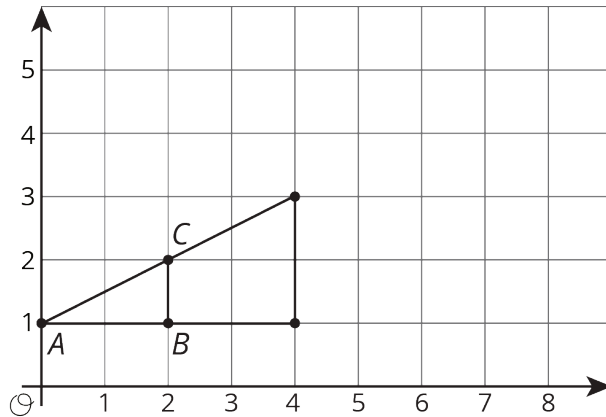
1.

2.

3. $(2s, s + 1)$. To get from (0,1) to (2,2), we move to the right by 2 units and up 1 unit. Applying a scale factor of s will multiply each of these side lengths by s. So C will go to the point $2s$ units to the right and s units up from $(0, 1)$. That is the point $(2s, s + 1)$.

4. $s = 4.5$. We want $(2s, s + 1)$ to be $(9, 5.5)$. Solve either the equation $2s = 9$ or $s + 1 = 5.5$ to get $s = 4.5$.

Activity Synthesis

First, focus on the third question, inviting selected students to present, in this sequence: first those who identified a pattern from the first two questions and then those who studied the impact of a dilation with scale factor s on $\triangle ABC$. Point out that the argument looking at where C is taken by the dilation with scale factor s and center A explains *why* the x coordinate doubles and the y coordinate is 1 more than the scale factor.

Next, invite selected students to share their answers to the last question.

In previous activities, students have used similar triangles to show that the points (x, y) on the line containing the long side of triangle ABC satisfy the relationship $\frac{y-1}{x} = \frac{1}{2}$. In this activity, they find that points (x, y) on this line are of the form $(2s, s + 1)$ where s is a (positive) real number. Make sure that students understand that the equation $\frac{y-1}{x} = \frac{1}{2}$ is true if we take $y - 1 = s + 1$ and $x = 2s$. Also, emphasize the key role that dilations play in these arguments.

The big take away from this lesson is that the structure of the coordinates of points on a line can be derived from properties of dilations.

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. When discussing where C is mapped, some students may need support to generalize that C will be mapped to $(2s, s + 1)$ after a dilation with center $(0, 1)$ and scale factor s and how these distances relate to the visual image. Ask students to write a short explanation or create a visual display of how they determined where C is mapped. Provide students time to examine the different approaches or representations of their classmates. Ask students what worked well in different approaches. Next, ask students to explain how the lengths in their explanations relate to the generalized form. Ask students where they see $2s$ and $s + 1$ represented in the slope triangles. Be sure to demonstrate asking questions that students can ask each other, rather than asking questions to test understanding.
Design Principle(s): Maximize meta-awareness, Cultivate conversation

Lesson Synthesis

The coordinates of points on a line have a nice structure that is useful for checking whether or not a given point is on a line. Here is a line with a couple of labeled points.
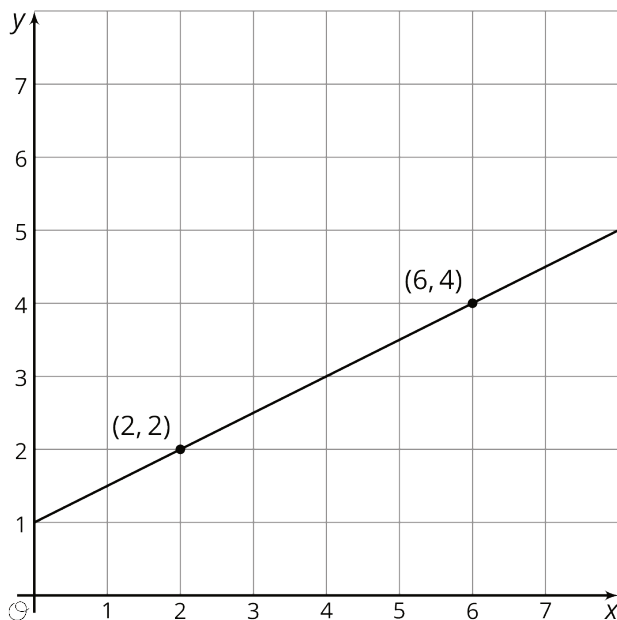
- What is the slope of this line? It's $\frac{1}{2}$ because a slope triangle (draw in the slope triangle) for the two labeled points has horizontal side length 4 and vertical side length 2.

- What is an equation for the line? One example is $\frac{y-2}{x-2} = \frac{1}{2}$. Label a general point on the line (x, y), and draw in a slope triangle to show this relationship.

How can we find out whether or not the point $(72, 37)$ is on this line? The points on the line satisfy the equation $\frac{y-2}{x-2} = \frac{1}{2}$. Since $\frac{37-2}{72-2} = \frac{1}{2}$, the point $(72, 37)$ is on the line!
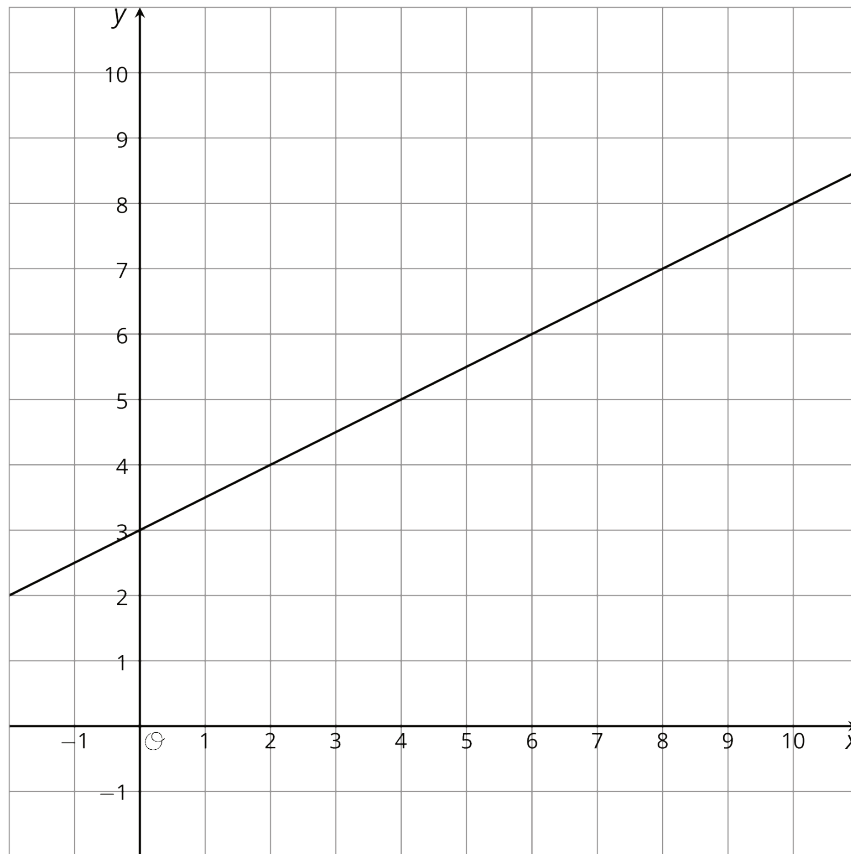
12.4 Is the Point on the Line?

Cool Down: 5 minutes
Students determine whether or not a point with given coordinates lies on a line. The line is presented graphically with no scaffolding and the coordinates of the point are too large to check whether or not it lies on the line by examining the graph. Students are expected to write an equation satisfied by points on the line and apply this.

Addressing

- 8.EE.B.6

Student Task Statement

Is the point (20,13) on this line? Explain your reasoning.

Student Response

Yes. The slope of the line is $\frac{1}{2}$: moving to the right 2 units and up 1 unit gives a point on the line. Since $(0, 3)$ is on the line, moving right 20 units and up 10 units gives another point on the line. This is the point $(20, 13)$.

Student Lesson Summary

We can use what we know about slope to decide if a point lies on a line. Here is a line with a few points labeled.
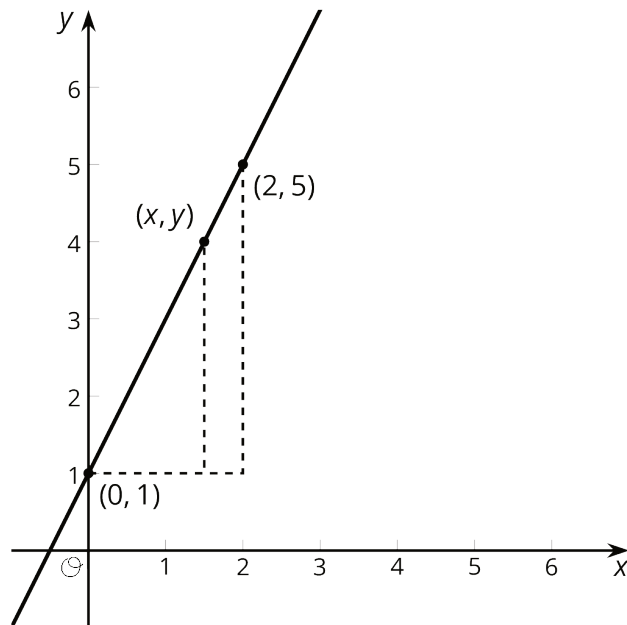
The slope triangle with vertices $(0, 1)$ and $(2, 5)$ gives a slope of $\frac{5-1}{2-0} = 2$. The slope triangle with vertices $(0, 1)$ and (x, y) gives a slope of $\frac{y-1}{x}$. Since these slopes are the same, $\frac{y-1}{x} = 2$ is an equation for the line. So, if we want to check whether or not the point $(11, 23)$ lies on this line, we can check that $\frac{23-1}{11} = 2$. Since $(11, 23)$ is a solution to the equation, it is on the line!

Lesson 12 Practice Problems
Problem 1
Statement
Select **all** the points that are on the line through $(0, 5)$ and $(2, 8)$.

A. $(4, 11)$
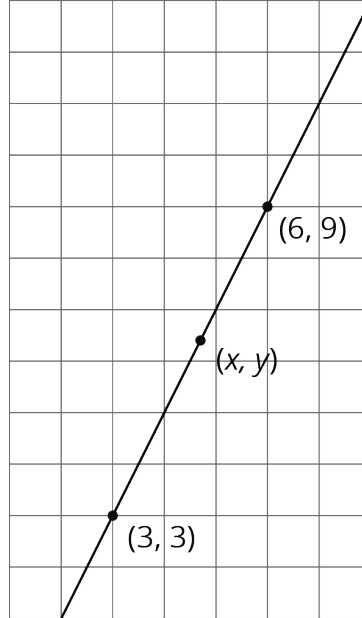
B. $(5, 10)$

C. $(6, 14)$

D. $(30, 50)$

E. $(40, 60)$

Solution
["A", "C", "D"]

Problem 2

Statement

All three points displayed are on the line. Find an equation relating x and y.
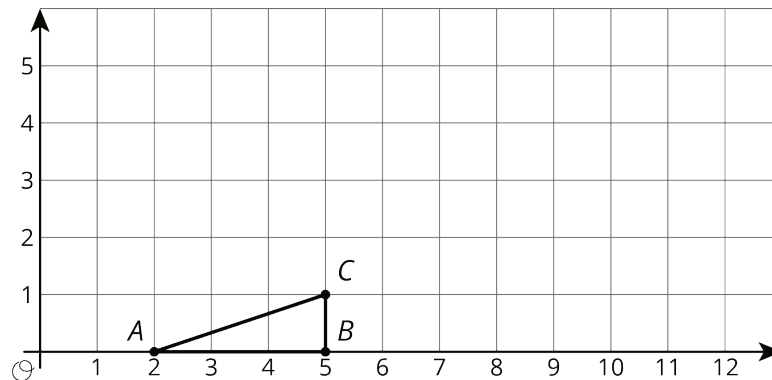

Solution

Answers vary. Sample response: $\frac{y-3}{x-3} = 2$ (or $y = 2x - 3$)

Problem 3

Statement

Here is triangle ABC.

a. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor 2.

b. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor 3.

c. Draw the dilation of triangle ABC with center $(2, 0)$ and scale factor $\frac{1}{2}$.
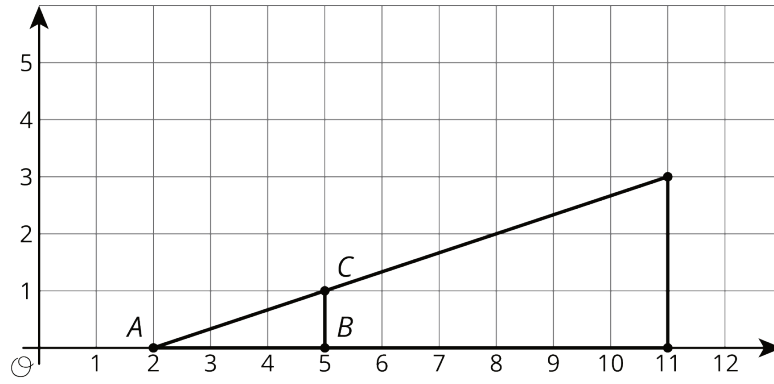
d. What are the coordinates of the image of point C when triangle ABC is dilated with center $(2, 0)$ and scale factor s?

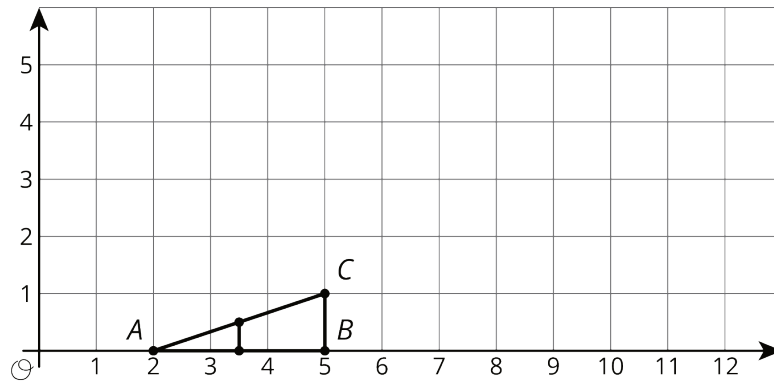e. Write an equation for the line containing all possible images of point C.
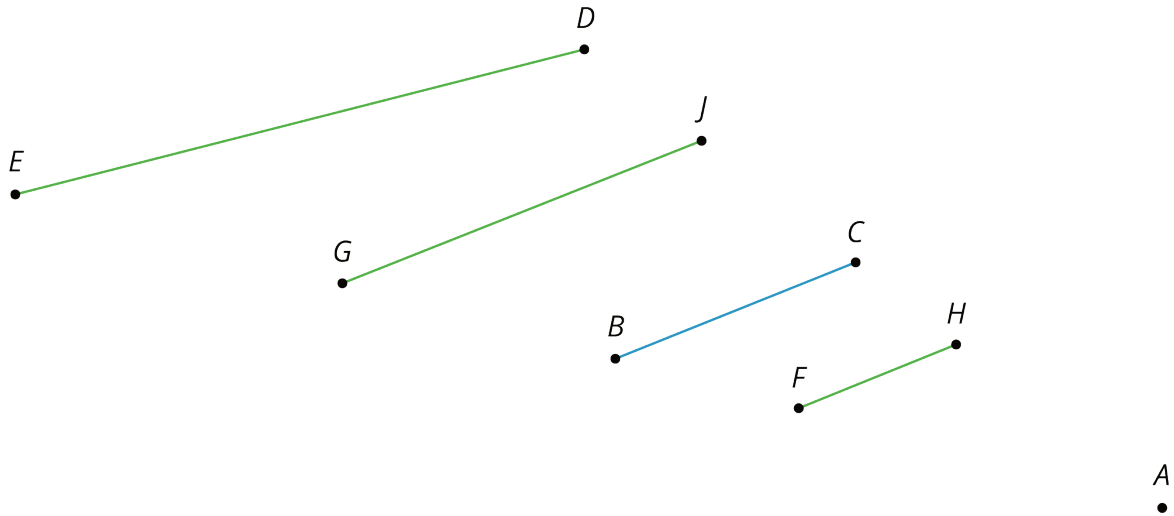
Solution

a.
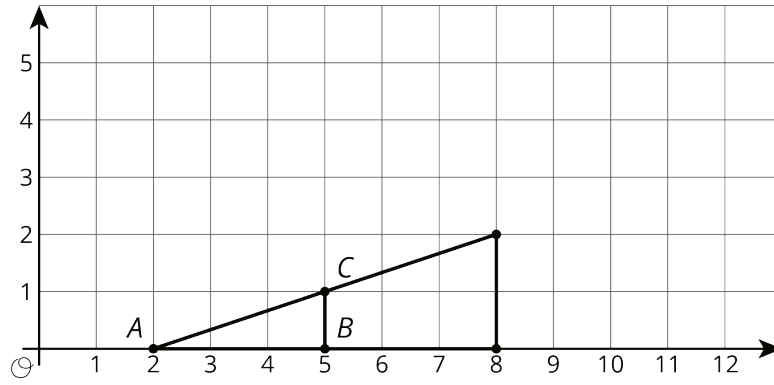
b.

c.

d. $(2 + 3s, s)$

e. $\frac{y}{x-2} = \frac{1}{3}$ (or equivalent)

Problem 4

Statement

Here are some line segments.

a. Which segment is a dilation of $\overline{BC}$ using A as the center of dilation and a scale factor of $\frac{2}{3}$?

b. Which segment is a dilation of $\overline{BC}$ using A as the center of dilation and a scale factor of $\frac{3}{2}$?

c. Which segment is not a dilation of $\overline{BC}$, and how do you know?

Solution

a. Segment $\overline{FH}$ (A scale factor of $\frac{2}{3}$ produces a parallel line segment with shorter length.)

b. Segment $\overline{GJ}$ (A scale factor of $\frac{3}{2}$ will produce a parallel line segment with longer length.)

c. Segment $\overline{DE}$ (Dilations take lines to parallel lines, and $\overline{DE}$ is not parallel to $\overline{FH}$.)

(From Unit 2, Lesson 4.)