

## Lesson 19: End Behavior of Rational Functions

- Let's explore the end behavior of rational functions.

### 19.1: Different Divisions, Revisited

Complete all three representations of the polynomial division following the forms of the integer division.

$$\begin{array}{r}
 252 \\
 11 \overline{) 2775} \\
 \underline{2200} \\
 575 \\
 \underline{550} \\
 25 \\
 \underline{22} \\
 3
 \end{array}$$

$$2775 = 11(252) + 3$$

$$\frac{2775}{11} = 252 + \frac{3}{11}$$

$$\begin{array}{r}
 2x^2 \\
 x + 1 \overline{) 2x^3 + 7x^2 + 7x + 5}
 \end{array}$$

$$2x^3 + 7x^2 + 7x + 5 =$$

$$\frac{2x^3 + 7x^2 + 7x + 5}{x + 1} =$$

### 19.2: Combined Fuel Economy

In 2000, the Environmental Protection Agency (EPA) reported a combined fuel efficiency for cars that assumes 55% city driving and 45% highway driving. The expression for the combined fuel efficiency of a car that gets  $x$  mpg in the city and  $h$  mpg on the highway can be written as  $\frac{100xh}{55x+45h}$ .

- Several conventional cars have a fuel economy for highway driving that is about 10 mpg higher than for city driving. That is,  $h = x + 10$ . Write a function  $f$  that represents the combined fuel efficiency for cars like these in terms of  $x$ .
- Rewrite  $f$  in the form  $q(x) + \frac{r(x)}{b(x)}$  where  $q(x)$ ,  $r(x)$ , and  $b(x)$  are polynomials.

## 19.3: Exploring End Behavior

function	degree of num.	degree of den.	rewritten in the form of $q(x) + \frac{r(x)}{b(x)}$	end behavior
$g(x) = -\frac{5}{x+2}$				
$h(x) = \frac{7x-5}{x+2}$				
$j(x) = \frac{3x^2+7x-5}{x+2}$				
$k(x) = \frac{2x^3+3x^2+7x-5}{x+2}$				
$m(x) = \frac{x+2}{2x^3+3x^2+7x-5}$				

1. Complete the table to explore the end behavior for rational functions.

2. What do you notice about the end behavior of different types of rational functions?

### Are you ready for more?

1. Graph  $y = j(x)$  and the line it approaches.

2. Under what conditions would the end behavior of the graph of a rational function approach a line that is not horizontal?
  
3. Create a rational function that approaches the line  $y = 2x - 3$  as  $x$  gets larger and larger in either the positive or negative direction.

### Lesson 19 Summary

In earlier lessons, we saw rational functions whose end behavior could be described by a horizontal asymptote. For example, we can rewrite functions like  $d(x) = \frac{x+4}{x}$  as  $d(x) = 1 + \frac{4}{x}$  to see more clearly that as  $x$  gets larger and larger in either the positive or negative direction, the value of  $\frac{4}{x}$  gets closer and closer to 0, which means the value of  $d(x)$  gets closer to 1. We can use similar thinking to understand rational functions that do not have horizontal asymptotes.

For example, consider  $f(x) = \frac{x^2+4x+5}{x-3}$ . Using division, the expression can be rewritten as  $f(x) = x + 7 + \frac{26}{x-3}$ . As  $x$  gets larger and larger in either the positive or negative direction, the value of the term  $\frac{26}{x-3}$  gets closer and closer to 0, which means the value of  $d(x)$  gets closer to the value of  $x + 7$ . This means that the end behavior of  $f$  can be described by the line  $y = x + 7$ . Here is a graph of  $y = f(x)$ , the line  $y = x + 7$ , and the vertical asymptote of the function at  $x = 3$ :

