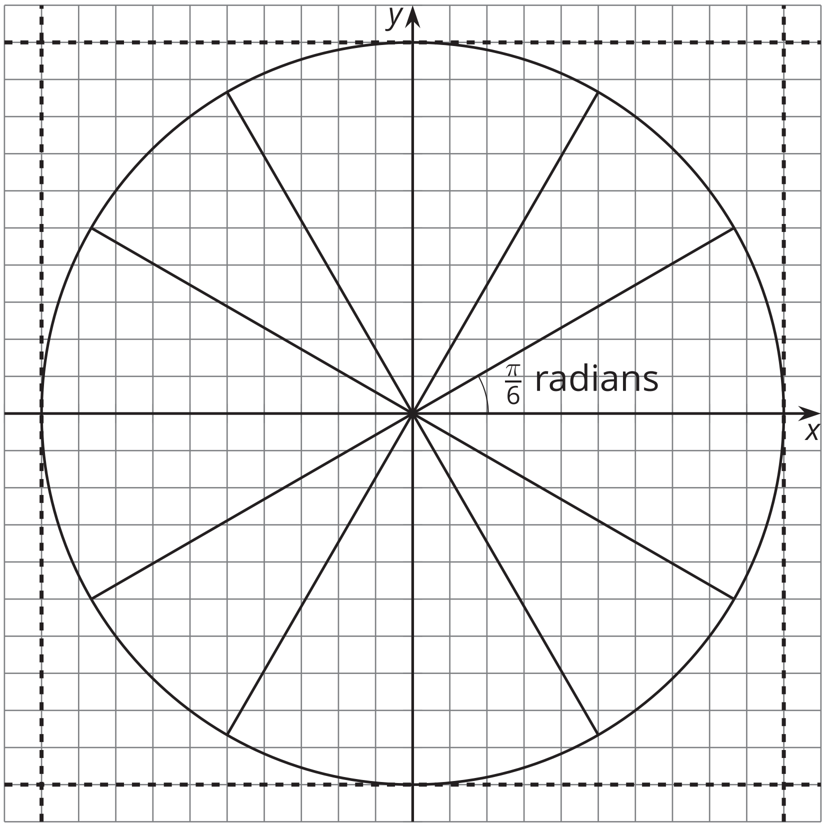
Lesson 4: The Unit Circle (Part 2)

- Let’s look at angles and points on the unit circle.

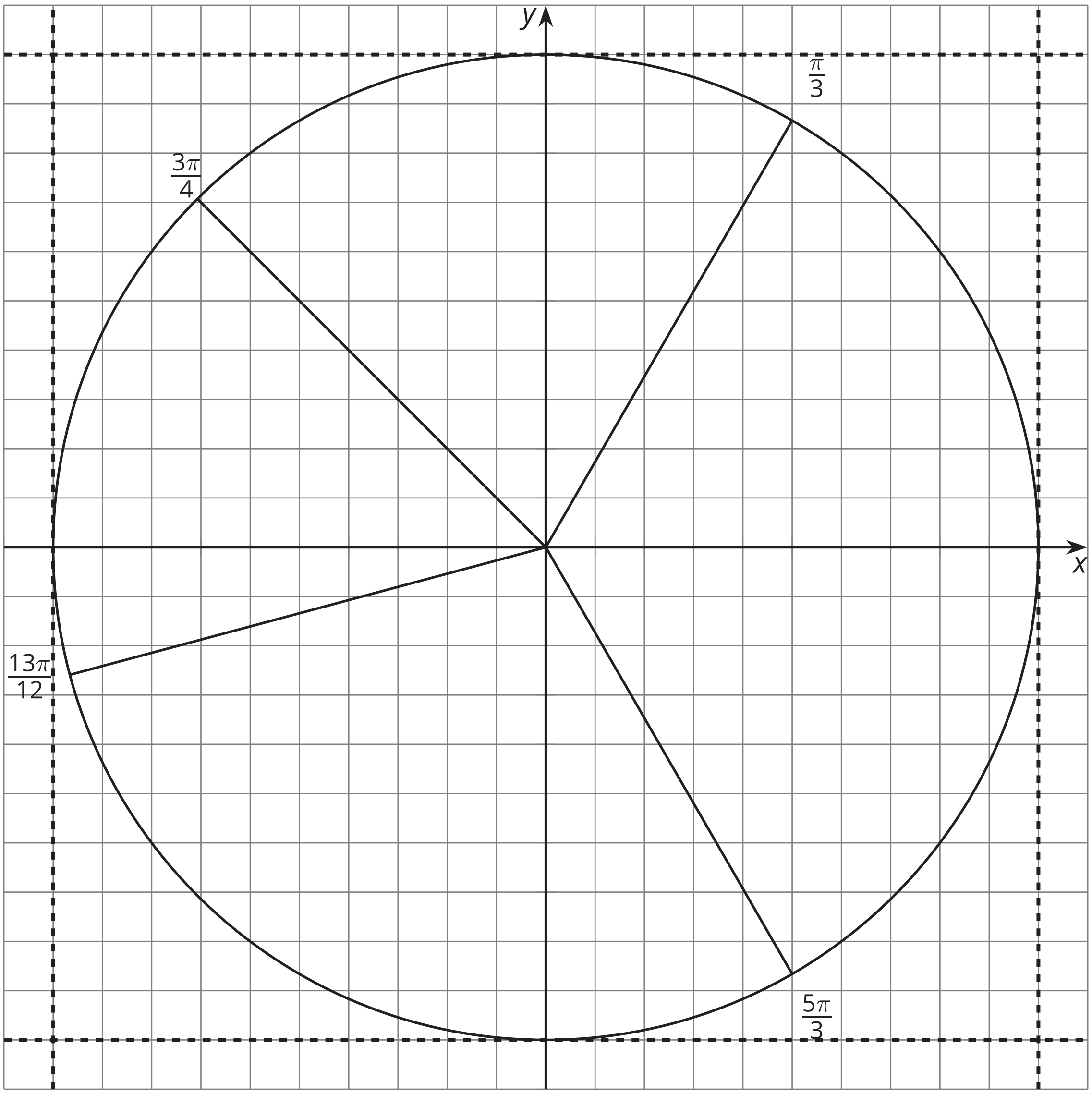
4.1: Notice and Wonder: Angles Around the Unit Circle

What do you notice? What do you wonder?



4.2: Angles Everywhere

Here is a circle of radius 1 with some radii drawn.



1. Draw and label angles, with the positive x-axis as the starting ray for each angle, measuring $\frac{\pi}{12}, \frac{\pi}{6}, \frac{\pi}{4} \dots, 2\pi$ in the counterclockwise direction. Four of these angles, one in each quadrant, have been drawn for you. There should be a total of 24 angles labeled when you are finished, including those that line up with the axes. Be prepared to share any strategies you used to make the angles.

2. Label the points, where the rays meet the unit circle, for which you know the exact coordinate values.

4.3: Angle Coordinates Galore

Your teacher will assign you a section of the unit circle.

1. Find and label the coordinates of the points assigned to you where the angles intersect the circle.

2. Compare and share your values with your group.

3. What relationships or patterns do you notice in the coordinates? Be prepared to share what you notice with the class.

Are you ready for more?

Other than $(1, 0)$, $(0, 1)$, $(\text{-}1, 0)$, and $(0, \text{-}1)$, the coordinates we used in this activity involved approximations. The point $(0.8, 0.6)$, however, lies exactly on our unit circle.

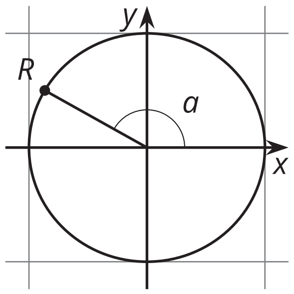
1. Explain why this must be true.

2. Find all other points on the unit circle that also lie exactly at the intersection of two grid lines.

3. What are the approximate angle measures needed to intersect at $(0.8, 0.6)$ and each of these new points?

Lesson 4 Summary

Given any point in a quadrant on the unit circle and its associated angle, like R shown here, we can make some statements about other points that must also be on the unit circle.



For example, if the coordinates of R are $(\text{-}0.87, 0.5)$ and a is $\frac{5\pi}{6}$ radians, then there is a point S in quadrant 1 with coordinates $(0.87, 0.5)$. Since R is $\frac{\pi}{6}$ radians from a half circle, the angle associated with point S must be $\frac{\pi}{6}$ radians. Similarly, there is a point T at $(\text{-}0.87, \text{-}0.5)$ with an angle $\frac{\pi}{6}$ radians greater than a half circle. This means point T is at angle $\frac{7\pi}{6}$ radians, since $\pi + \frac{\pi}{6} = \frac{7\pi}{6}$.

What is the matching point to R in quadrant 4? (A point at $(0.87, \text{-}0.5)$ and angle $\frac{11\pi}{6}$ radians.)

In future lessons, we'll learn about how to find the coordinates of point R ourselves using its angle a and what we know about right triangles.