## Lesson 12: Tangent

* Let’s learn more about tangent.

### 12.1: Notice and Wonder: An Unusual Function

What do you notice? What do you wonder?

| $θ$ | $cos\left(θ\right)$ | $sin\left(θ\right)$ | $tan\left(θ\right)$ |
| --- | --- | --- | --- |
| $-\frac{π}{2}$ | 0 | -1 |   |
| $-\frac{π}{3}$ | 0.5 | -0.87 |   |
| $-\frac{π}{6}$ | 0.87 | -0.5 |   |
| 0 | 1 | 0 |   |
| $\frac{π}{6}$ | 0.87 | 0.5 |   |
| $\frac{π}{3}$ | 0.5 | 0.87 |   |
| $\frac{π}{2}$ | 0 | 1 |   |

### 12.2: A Tangent Ratio

1. Complete the table. For each positive angle in the table, add the corresponding point and the segment between it and the origin to the unit circle.
* 

| * $θ$
 | * $cos\left(θ\right)$
 | * $sin\left(θ\right)$
 | * $tan\left(θ\right)$
 |
| --- | --- | --- | --- |
| * $-\frac{π}{2}$
 | * 0
 | * -1
 | *
 |
| * $-\frac{π}{3}$
 | * 0.5
 | * -0.87
 | *
 |
| * $-\frac{π}{6}$
 | * 0.87
 | * -0.5
 | *
 |
| * 0
 | * 1
 | * 0
 | *
 |
| * $\frac{π}{6}$
 | * 0.87
 | * 0.5
 | *
 |
| * $\frac{π}{3}$
 | * 0.5
 | * 0.87
 | *
 |
| * $\frac{π}{2}$
 | * 0
 | * 1
 | *
 |
| * $\frac{2π}{3}$
 | *
 | *
 | *
 |
| * $\frac{5π}{6}$
 | *
 | *
 | *
 |
| * $π$
 | *
 | *
 | *
 |
| * $\frac{7π}{6}$
 | *
 | *
 | *
 |
| * $\frac{4π}{3}$
 | *
 | *
 | *
 |
| * $\frac{3π}{2}$
 | *
 | *
 | *
 |
| * $\frac{5π}{3}$
 | *
 | *
 | *
 |
| * $\frac{11π}{6}$
 | *
 | *
 | *
 |
| * $2π$
 | *
 | *
 | *
 |

1. How are the values of $tan\left(θ\right)$ like the values of $cos\left(θ\right)$ and $sin\left(θ\right)$? How are they different?

#### Are you ready for more?

1. Where does the line $x=1$ intersect the line that passes through the origin and the point corresponding to the angle $\frac{π}{6}$?
2. Where does the line $x=1$ intersect the line that passes through the origin and the point corresponding to the angle $θ$?
3. Where do you think the name “tangent” of an angle comes from?
* 
*

### 12.3: The Tangent Function

Before we graph $y=tan\left(θ\right)$, let’s figure out some things that must be true.

1. Explain why the graph of $tan\left(θ\right)$ has a vertical asymptote at $x=\frac{π}{2}$.
2. Does the graph of $tan\left(θ\right)$ have other vertical asymptotes? Explain how you know.
3. For which values of $θ$ is $tan\left(θ\right)$ zero? For which values of $θ$ is $tan\left(θ\right)$ one? Explain how you know.
4. Is the graph of $tan\left(θ\right)$ periodic? Explain how you know.

### Lesson 12 Summary

The tangent of an angle $θ$, $tan\left(θ\right)$, is the quotient of the sine and cosine: $tan\left(θ\right)=\frac{sin\left(θ\right)}{cos\left(θ\right)}$. Here is a graph of $y=tan\left(θ\right)$.



We can see from the graph that $tan\left(θ\right)=0$ when $θ$ is $-2π,-π,0,π,or 2π$. This makes sense because the sine is 0 for these values of $θ$. Since sine and cosine are never 0 at the same $θ$, we can say that tangent has a value of 0 whenever sine has a value of 0.

We can also see the asymptotes of tangent $-\frac{3π}{2},-\frac{π}{2},\frac{π}{2},and \frac{3π}{2}$. Let’s look more closely at what happens when $θ=\frac{π}{2}$. We have $sin\frac{π}{2}=1$ and $cos\frac{π}{2}=0$. This means $tan\left(\frac{π}{2}\right)=\frac{1}{0}$, which is not defined. Whenever $cos\left(θ\right)=0$, the tangent is not defined and has a vertical asymptote.

Like the sine and cosine functions, the tangent function is periodic. This makes sense because it is defined using sine and cosine. The period of tangent is only $π$ while the period of sine and cosine is $2π$.



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