Lesson 1: Growing and Shrinking

- Let’s calculate exponential change.

1.1: Bank Accounts

A bank account has a balance of \$120 on January 1. Describe a situation in which the account balance for each month (February 1, March 1, . . .) forms the following sequences. Write the first three terms of each sequence.

1. an arithmetic sequence

2. a geometric sequence

1.2: Shrinking a Passport Photo

The distance from Elena’s chin to the top of her head is 150 mm in an image. For a U.S. passport photo, this measurement needs to be between 25 mm and 35 mm.

PASSPORT PHOTO

1. Find the height of the image after it has been scaled by 80% the following number of times. Explain or show your reasoning.
   a. 3 times
   b. 6 times
2. How many times would the image need to be scaled by 80% for the image to be less than 35 mm?
3. How many times would the image need to be scaled by 80% to be less than 25 mm?

Are you ready for more?

Suppose you'd like to rescale the passport image that has been scaled down 7 times back to its original size. At what percentage should you set the scale on the image editor?

1.3: Pond in a Park

On May 12, a fast-growing species of algae is accidentally introduced to a pond in an urban park. The area of the pond that the algae covers doubles each day. If not controlled, the algae will cover the entire surface of the pond, depriving the fish in the pond of oxygen. At the rate it is growing, this will happen on May 24.

1. On which day is the pond halfway covered?
2. On May 18, Clare visits the park. A park caretaker mentions to her that the pond will be completely covered in less than a week. Clare thinks that the caretaker must be mistaken. Why might she find the caretaker's claim hard to believe?
3. What fraction of the area of the pond was covered by the algae initially, on May 12? Explain or show your reasoning.

Lesson 1 Summary

Sometimes quantities change by the same factor at regular intervals.

For example, a bacteria population might be 10,000 on the first day of measurement and then double each day after that point. This means that one day after the initial measurement, the population would be 20,000, two days after the measurement, it would be 40,000, and three days after, it would be 80,000.

The relationship can be modeled by an exponential function because the population changes by the same factor for each passing day. If n is the number of days since the bacteria population was first measured, then the population on day n is $10{,}000 \cdot 2^n$. The population is also a geometric sequence because each term is found by multiplying the previous term by 2.

days since population is measured	population
0	10,000
1	$10{,}000 \cdot 2$
2	$10{,}000 \cdot 2^2$
3	$10{,}000 \cdot 2^3$
n	$10{,}000 \cdot 2^n$