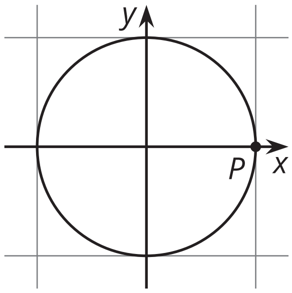
Lesson 9: Introduction to Trigonometric Functions

- Let’s graph cosine and sine.

9.1: An Angle and a Circle

Suppose there is a point P on the unit circle at $(1, 0)$.

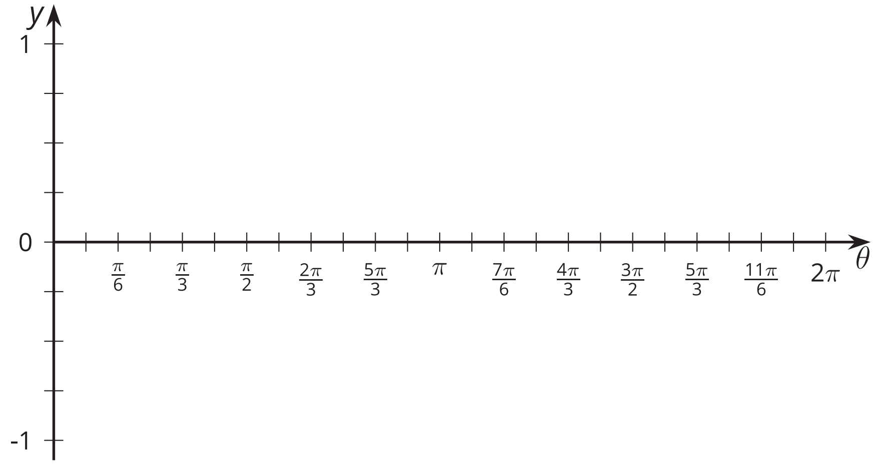


1. Describe how the x-coordinate of P changes as it rotates once counterclockwise around the circle.

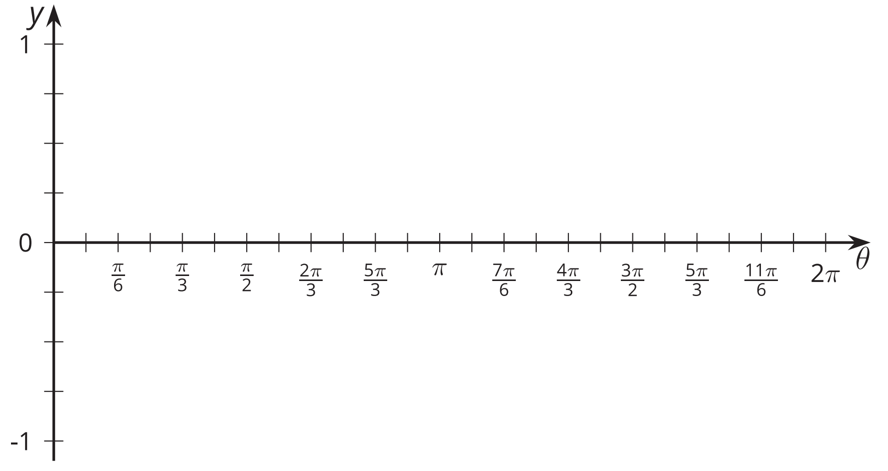
2. Describe how the y-coordinate of P changes as it rotates once counterclockwise around the circle.

9.2: Do the Wave

1. For each tick mark on the horizontal axis, plot the value of $y = \cos(\theta)$, where θ is the measure of an angle in radians. Use the class display of the unit circle, the unit circle from an earlier lesson, or technology to estimate the value of $\cos(\theta)$.



2. For each tick mark on the horizontal axis, plot the value of $y = \sin(\theta)$. Use the class display of the unit circle, the unit circle from an earlier lesson, or technology to estimate the value of $\sin(\theta)$.



3. What do you notice about the two graphs?

4. Explain why any angle measure between 0 and 2π gives a point on each graph.

5. Could these graphs represent functions? Explain your reasoning.

9.3: Graphs of Cosine and Sine

1. Looking at the graphs of $y = \cos(\theta)$ and $y = \sin(\theta)$, at what values of θ do $\cos(\theta) = \sin(\theta)$? Where on the unit circle do these points correspond to?

2. For each of these equations, first predict what the graph looks like, and then check your prediction using technology.
   a. $y = \cos(\theta) + \sin(\theta)$

   b. $y = \cos^2(\theta)$

   c. $y = \sin^2(\theta)$

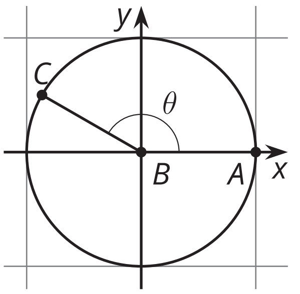
   d. $y = \cos^2(\theta) + \sin^2(\theta)$

Are you ready for more?

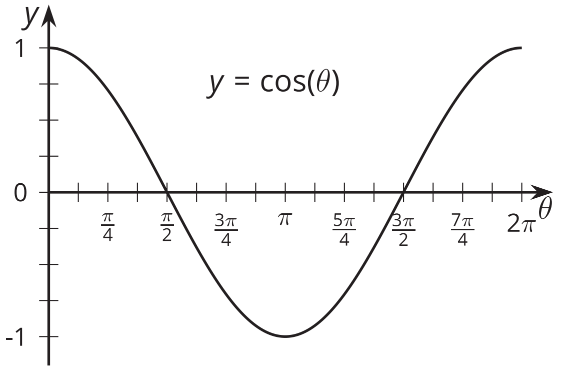
For the equation given, predict what the graph looks like, and then check your prediction using technology: $y = \theta + \cos(\theta)$.

Lesson 9 Summary

Using the unit circle, we can make sense of $\cos(\theta)$ and $\sin(\theta)$ for any angle measure θ between 0 and 2π radians. For an angle θ starting at the positive x-axis, there is a point C where the terminal ray of the angle intersects the unit circle. The coordinates of that point are $(\cos(\theta), \sin(\theta))$.

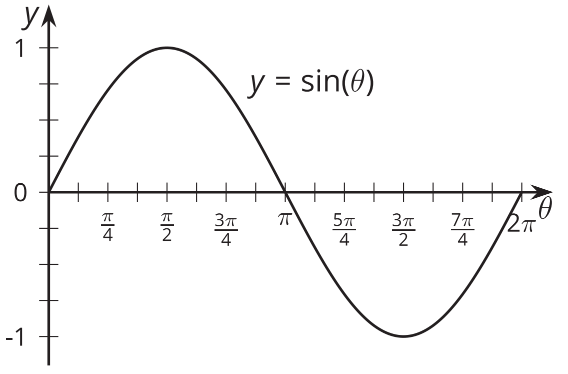


But what if we wanted to think about just the horizontal position of point C as θ goes from 0 to 2π? The horizontal location is defined by the x-coordinate, which is $\cos(\theta)$. If we graph $y = \cos(\theta)$, we get:



This graph is 1 when θ is 0 because the x-coordinate of the point at 0 radians on the unit circle is $(1, 0)$. The graph then decreases to -1 (the smallest x-value on the unit circle) before increasing back to 1.

We can do the same for the y-coordinate of a point on the unit circle by graphing $y = \sin(\theta)$:



This graph is 0 when θ is 0, increases to 1 (the greatest y-value on the unit circle), then decreases to -1 before returning to 0.