Lesson 8: Rewriting Quadratic Expressions in Factored Form (Part 3)

- Let’s look closely at some special kinds of factors.

8.1: Math Talk: Products of Large-ish Numbers

Find each product mentally.

$9 \cdot 11$

$19 \cdot 21$

$99 \cdot 101$

$109 \cdot 101$

8.2: Can Products Be Written as Differences?

1. Clare claims that $(10+3)(10-3)$ is equivalent to $10^2 - 3^2$ and $(20+1)(20-1)$ is equivalent to $20^2 - 1^2$. Do you agree? Show your reasoning.

2. a. Use your observations from the first question and evaluate $(100+5)(100-5)$. Show your reasoning.

   b. Check your answer by computing $105 \cdot 95$.

3. Is $(x+4)(x-4)$ equivalent to $x^2 - 4^2$? Support your answer:

   With a diagram:

	x	4
x		
-4		

   Without a diagram:

4. Is $(x+4)^2$ equivalent to $x^2 + 4^2$? Support your answer, either with or without a diagram.

Are you ready for more?

1. Explain how your work in the previous questions can help you mentally evaluate $22 \cdot 18$ and $45 \cdot 35$.

2. Here is a shortcut that can be used to mentally square any two-digit number. Let's take 83^2, for example.

   - 83 is $80 + 3$.
   - Compute 80^2 and 3^2, which give 6,400 and 9. Add these values to get 6,409.
   - Compute $80 \cdot 3$, which is 240. Double it to get 480.
   - Add 6,409 and 480 to get 6,889.

   Try using this method to find the squares of some other two-digit numbers. (With some practice, it is possible to get really fast at this!) Then, explain why this method works.

8.3: What If There is No Linear Term?

Each row has a pair of equivalent expressions.

Complete the table.

If you get stuck, consider drawing a diagram. (Heads up: one of them is impossible.)

factored form	standard form
$(x-10)(x+10)$	
$(2x+1)(2x-1)$	
$(4-x)(4+x)$	
	x^2-81
	$49-y^2$
	$9z^2-16$
	$25t^2-81$
$(c+\frac{2}{5})(c-\frac{2}{5})$	
	$\frac{49}{16}-d^2$
$(x+5)(x+5)$	
	x^2-6
	x^2+100

Lesson 8 Summary

Sometimes expressions in standard form don't have a linear term. Can they still be written in factored form?

Let's take $x^2 - 9$ as an example. To help us write it in factored form, we can think of it as having a linear term with a coefficient of 0: $x^2 + 0x - 9$. (The expression $x^2 - 0x - 9$ is equivalent to $x^2 - 9$ because 0 times any number is 0, so $0x$ is 0.)

We know that we need to find two numbers that multiply to make -9 and add up to 0. The numbers 3 and -3 meet both requirements, so the factored form is $(x + 3)(x - 3)$.

To check that this expression is indeed equivalent to $x^2 - 9$, we can expand the factored expression by applying the distributive property: $(x + 3)(x - 3) = x^2 - 3x + 3x + (\text{-}9)$. Adding $\text{-}3x$ and $3x$ gives 0, so the expanded expression is $x^2 - 9$.

In general, a quadratic expression that is a difference of two squares and has the form:

$a^2 - b^2$	can be rewritten as:	$(a + b)(a - b)$

Here is a more complicated example: $49 - 16y^2$. This expression can be written $7^2 - (4y)^2$, so an equivalent expression in factored form is $(7 + 4y)(7 - 4y)$.

What about $x^2 + 9$? Can it be written in factored form?

Let's think about this expression as $x^2 + 0x + 9$. Can we find two numbers that multiply to make 9 but add up to 0? Here are factors of 9 and their sums:

- 9 and 1, sum: 10
- -9 and -1, sum: -10
- 3 and 3, sum: 6
- -3 and -3, sum: -6

For two numbers to add up to 0, they need to be opposites (a negative and a positive), but a pair of opposites cannot multiply to make positive 9, because multiplying a negative number and a positive number always gives a negative product.

Because there are no numbers that multiply to make 9 and also add up to 0, it is not possible to write $x^2 + 9$ in factored form using the kinds of numbers that we know about.