## Lesson 3: Measuring Dilations

* Let’s dilate polygons.

### 3.1: Dilating Out

Dilate triangle $FGH$ using center $C$ and a scale factor of 3.



### 3.2: All the Scale Factors

Here is a center of dilation and a triangle.



1. Measure the sides of triangle $EFG$ (to the nearest mm).
2. Your teacher will assign you a scale factor. Predict the relative lengths of the original figure and the image after you dilate by your scale factor.
3. Dilate triangle $EFG$ using center $C$ and your scale factor.
4. How does your prediction compare to the image you drew?
5. Use tracing paper to copy point $C$, triangle $EFG$, and your dilation. Label your tracing paper with your scale factor.
6. Align your tracing paper with your partner’s. What do you notice?

#### Are you ready for more?



1. Dilate triangle $FEG$ using center $C$ and scale factors:
	1. $\frac{1}{2}$
	2. 2
2. What scale factors would cause some part of triangle $E^{′}F^{′}G^{′}$’ to intersect some part of triangle $EFG$?

### 3.3: What Stays the Same?

1. Dilate quadrilateral $ABCD$ using center $P$ and your scale factor.
* 
1. Complete the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| * Ratio
 | * $\frac{PA^{′}}{PA}$
 | * $\frac{PB^{′}}{PB}$
 | * $\frac{PC^{′}}{PC}$
 | * $\frac{PD^{′}}{PD}$
 |
| * Value
 |  |  |  |  |

1. What do you notice? Can you prove your conjecture?
2. Complete the table.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| * Ratio
 | * $\frac{B^{′}A^{′}}{BA}$
 | * $\frac{C^{′}B^{′}}{CB}$
 | * $\frac{D^{′}C^{′}}{DC}$
 | * $\frac{A^{′}D^{′}}{AD}$
 |
| * Value
 |  |  |  |  |

1. What do you notice? Does the same reasoning you just used also prove this conjecture?

### Lesson 3 Summary

We know a *dilation* with center $P$ and positive *scale factor* $k$ takes a point $A$ along the ray $PA$ to another point whose distance is $k$ times farther away from $P$ than $A$ is.

The triangle $A^{′}B^{′}C^{′}$ is a dilation of the triangle $ABC$ with center $P$ and with scale factor 2. So $A^{′}$ is 2 times farther away from $P$ than $A$ is, $B^{′}$ is 2 times farther away from $P$ than $B$ is, and $C^{′}$ is 2 times farther away from $P$ than $C$ is.

Because of the way dilations are defined, all of these quotients give the scale factor: $\frac{PA^{′}}{PA}=\frac{PB^{′}}{PB}=\frac{PC^{′}}{PC}=2$.



If triangle $ABC$ is dilated from point $P$ with scale factor $\frac{1}{3}$, then each vertex in $A^{″}B^{″}C^{″}$ is on the ray from P through the corresponding vertex of $ABC$, and the distance from $P$ to each vertex in $A^{″}B^{″}C^{″}$ is one-third as far as the distance from $P$ to the corresponding vertex in $ABC$.

$\frac{PA^{″}}{PA}=\frac{PB^{″}}{PB}=\frac{PC^{″}}{PC}=\frac{1}{3}$



The dilation of a line segment is longer or shorter according to the same ratio given by the scale factor. In other words, If segment $AB$ is dilated from point $P$ with scale factor $k$, then the length of segment $AB$ is multiplied by $k$ to get the corresponding length of $A^{′}B^{′}$.

$\frac{A^{″}B^{″}}{AB}=\frac{B^{″}C^{″}}{BC}=\frac{A^{″}C^{″}}{AC}=k$.

Corresponding side lengths of the original figure and dilated image are all in the same proportion, and related by the same scale factor $k$.



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