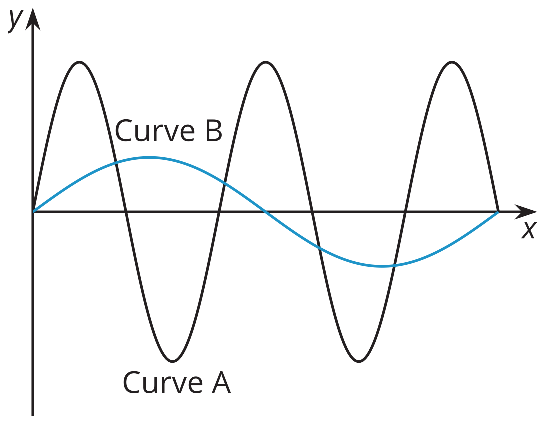
Lesson 15: Features of Trigonometric Graphs (Part 1)

- Let’s compare graphs and equations of trigonometric functions.

15.1: Notice and Wonder: Musical Notes

Here are pictures of sound waves for two different musical notes:



What do you notice? What do you wonder?

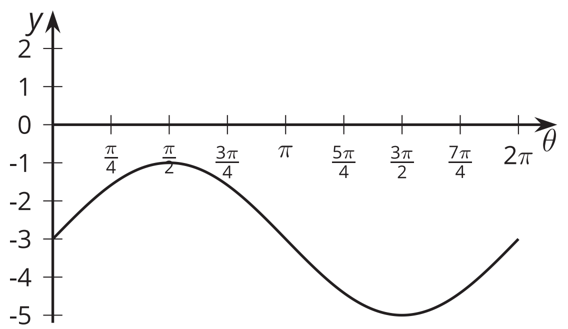
15.2: Equations and Graphs

Match each equation with its graph. More than 1 equation can match the same graph.

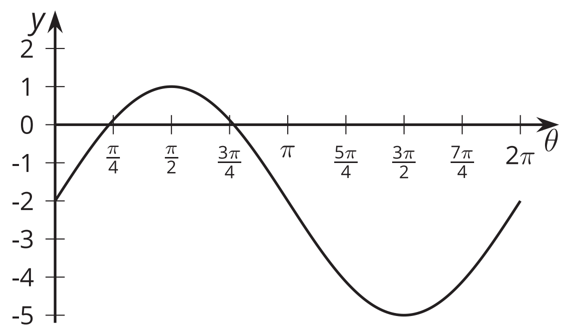
Equations:

1. $y = \text{-}\cos(\theta)$
2. $y = 2\sin(\theta) - 3$
3. $y = \cos\left(\theta + \frac{\pi}{2}\right)$
4. $y = 3\sin(\theta) - 2$
5. $y = \sin(\theta - \frac{\pi}{2})$
6. $y = \sin(\theta + \pi)$

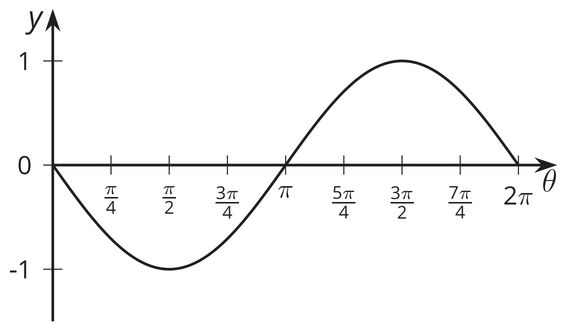
A



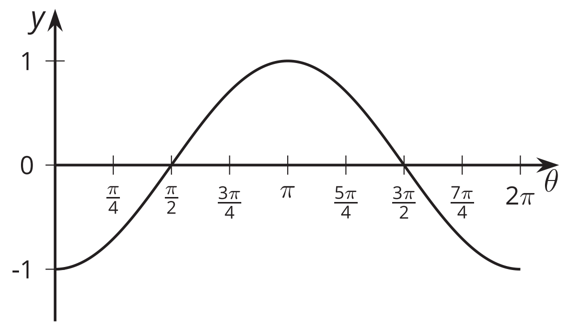
B



C

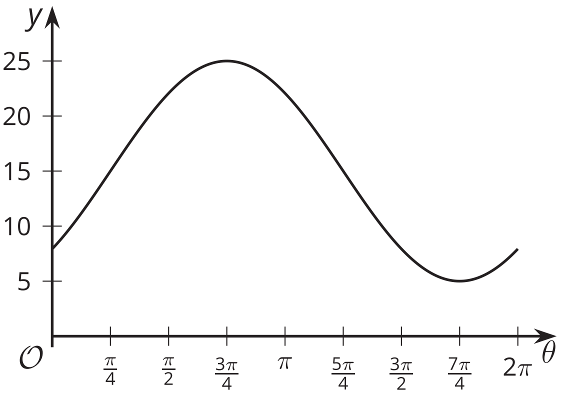


D



Are you ready for more?

1. Find an equation for this graph using the sine function.
2. Find another equation for the same graph using a cosine function.

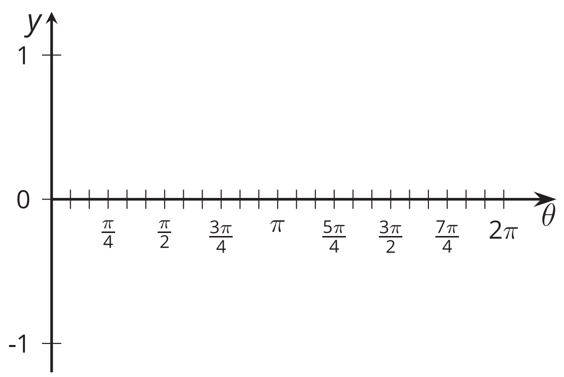


15.3: Double the Sine

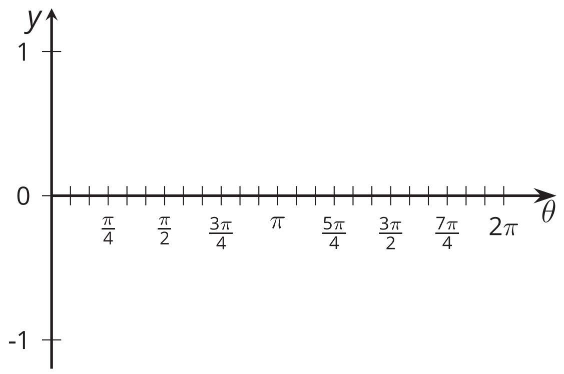
1. Complete the table of values for the expression $\sin(2\theta)$

θ	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π	$\frac{5\pi}{4}$	$\frac{3\pi}{2}$	$\frac{7\pi}{4}$	2π
$\sin(2\theta)$											

2. Plot the values and sketch a graph of the equation $y = \sin(2\theta)$. How does the graph of $y = \sin(2\theta)$ compare to the graph of $y = \sin(\theta)$?

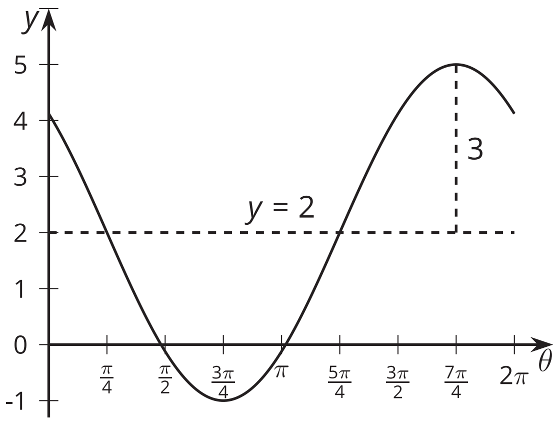


3. Predict what the graph of $y = \cos(4\theta)$ will look like and make a sketch. Explain your reasoning.

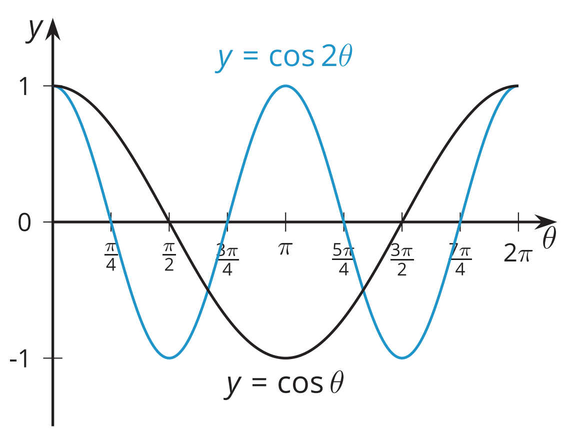


Lesson 15 Summary

We can find the amplitude and midline of a trigonometric function using the graph or from an equation. For example, let's look at the function given by the equation $y = 3\cos\left(\theta + \frac{\pi}{4}\right) + 2$. We can see that the midline of this function is 2 because of the vertical translation up by 2. This means the horizontal line $y = 2$ goes through the middle of the graph. The amplitude of the function is 3. This means the maximum value it takes is 5, 3 more than the midline value, and the minimum value it takes is -1, 3 less than the midline value. The horizontal translation is $\frac{\pi}{4}$ to the left, so instead of having, for example, a minimum at π, the minimum is at $\frac{3\pi}{4}$. Here is what the graph looks like:



Another type of transformation is one that affects the period and that is when a horizontal scale factor is used. For example, let's look at the equation $y = \cos(2\theta)$ where the variable θ is multiplied by a number. Here, 2 is the scale factor affecting θ. When $\theta = 0$, we have $2\theta = 0$ so the graph of this cosine equation starts at $(0, 1)$, just like the graph of $y = \cos(\theta)$. When $x = \pi$, we have $2\theta = 2\pi$ so the graph of $y = \cos(2\theta)$ goes through two full periods in the same horizontal span it takes $y = \cos(\theta)$ to complete one full period, as shown in their graphs.



Notice that the graph of $y = \cos(2\theta)$ has the same general shape as the graph of $y = \cos(\theta)$ (same midline and amplitude) but the waves are compressed together. And what if we wanted to give the graph of cosine a stretched appearance? Then we could use a horizontal scale factor between 0 and 1. For example, the graph of $y = \cos(\frac{\theta}{6})$ has a period of 12π.