Lesson 10: Bases and Heights of Triangles

Goals

- Draw and label the height that corresponds to a given base of a triangle, making sure it is perpendicular to the base and the correct length.
- Evaluate (orally) the usefulness of different base-height pairs for finding the area of a given triangle.

Learning Targets

- I can identify pairs of base and corresponding height of any triangle.
- When given information about a base of a triangle, I can identify and draw a corresponding height.

Lesson Narrative

This lesson furthers students' ability to identify and work with a base and height in a triangle in two ways:

1. By learning to draw (not just to recognize) a segment to show the corresponding height for any given base, and
2. By learning to choose appropriate base-height pairs to enable area calculations.

Students have seen that the area of a triangle can be determined in multiple ways. Using the base and height measurements and the formula is a handy approach, but because there are three possible pairs of bases and heights, some care is needed in identifying the right combination of measurements. Some base-height pairs may be more practical or efficient to use than others, so it helps to be strategic in choosing a side to use as a base.

Alignments

Addressing

- 6.EE.A.2.c: Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole-number exponents, in the conventional order when there are no parentheses to specify a particular order (Order of Operations). For example, use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length $s = 1/2$.
- 6.G.A.1: Find the area of right triangles, other triangles, special quadrilaterals, and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Index cards

Required Preparation

From the geometry toolkit, each student especially needs an index card for the Hunting for Heights activity.

> **Student Learning Goals**
>
> Let's use different base-height pairs to find the area of a triangle.

10.1 An Area of 12

Warm Up: 10 minutes (there is a digital version of this activity)
So far, students have determined area given a triangle and some measurements. In this warm-up, students are invited to reverse the process. They are given an area measure and are asked to create several triangles with that area.

Expect students to gravitate toward right triangles first (or to halve rectangles that have factors of 12 as their side lengths). This is a natural and productive starting point. Prompting students to create non-right triangles encourages them to apply insights from their experiences with non-right parallelograms.

As students work alone and discuss with partners, notice the strategies they use to draw their triangles and to verify their areas. Identify a few students with different strategies to share later.

Addressing

- 6.G.A.1

Instructional Routines

- Think Pair Share

Launch

Arrange students in groups of 2. Give students 2–3 minutes of quiet think time and 2 minutes to share their drawings with their partner afterwards. Encourage students to refer to previous work as needed. Provide access to their geometry toolkits. Tell students to draw a different triangle with the same area if they finish their first one early.

During partner discussion, each partner should convince the other that the triangle drawn is indeed 12 square units.

Anticipated Misconceptions

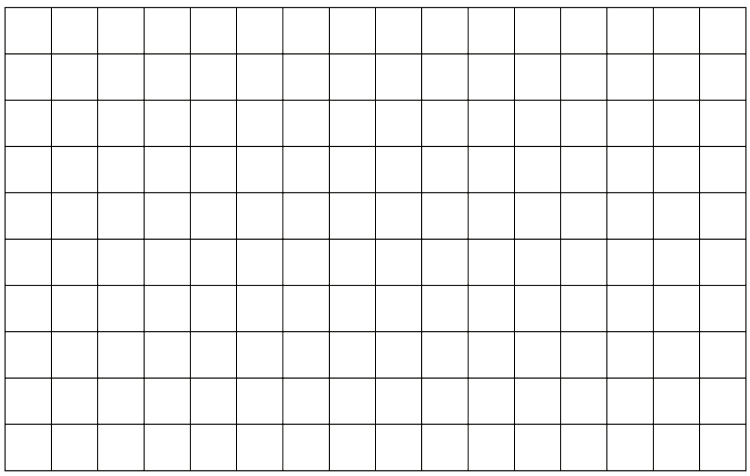
If students have trouble getting started, ask:

- “Can you draw a quadrilateral with an area of 12?”
- “Can you use what you know about parallelograms to help you?”
- “Can you use any of the area strategies—decomposing, rearranging, enclosing, subtracting—to arrive at an area of 12?”

Students who start by drawing rectangles and other parallelograms may use factors of 12, instead of factors of 24, for the base and height. If this happens, ask them what the area of the their quadrilateral is and how it relates to the triangle they are trying to draw.

Student Task Statement

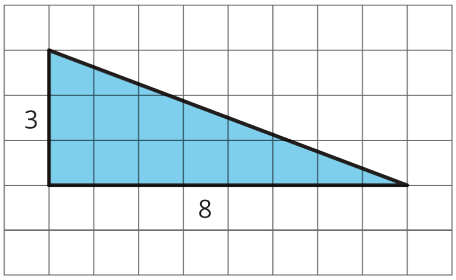
On the grid, draw a triangle with an area of 12 square units. Try to draw a non-right triangle. Be prepared to explain how you know the area of your triangle is 12 square units.



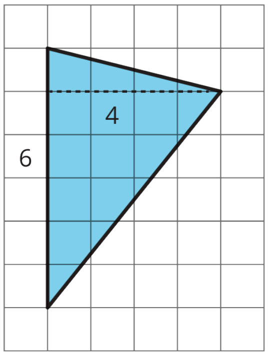
Student Response

Drawings and explanations vary. Sample responses:

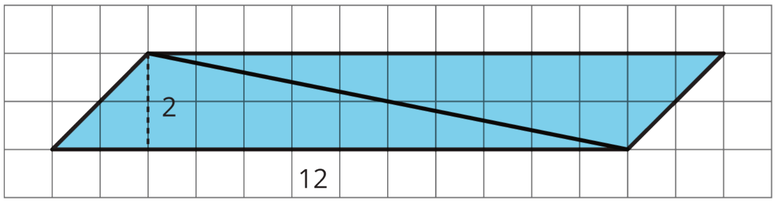
- This right triangle has a base of 8 units and a height of 3 units. The area is half of $3 \cdot 8$ or half of 24, which is 12.



- This triangle has a side of 6 units. This can be the base. Draw a height segment that is perpendicular to the base and is 4 units long. The area of the triangle is $b \cdot h \div 2$, so it is $6 \cdot 4 \div 2$, which is 12.



- Draw a parallelogram with a base of 12 and a height of 2, and then draw a diagonal line to create two identical triangles. Each of the triangles has an area of 12 because it is half of a parallelogram with an area of 24.



Activity Synthesis

Invite a few students to share their drawings and ways of reasoning with the class. For each drawing shared, ask the creator for the base and height and record them for all to see. Ask the class:

- "Did anyone else draw an identical triangle?"
- "Did anyone draw a different triangle but with the same base and height measurements?"

To reinforce the relationship between base, height, and area, discuss:

- "Which might be a better way to draw a triangle: by starting with the base measurement or with the height? Why?"
- "Can you name other base-height pairs that would produce an area of 12 square units without drawing? How?"

10.2 Hunting for Heights

25 minutes

Students may be able to recognize a measurement that can be used for height when they see it, but identifying and drawing an appropriate segment is more challenging. This activity, and the demonstration needed to launch it, gives students a concrete strategy for identifying a height accurately. When students use a strategy of drawing an auxiliary line to solve problems, they are looking for and making use of structure (MP7). Explicit instruction, as in this activity, is often needed before students can be expected to use this strategy spontaneously.

Addressing

- 6.G.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Explain to students that they will try to draw a height that corresponds to each side of a triangle. Arrange students in groups of 2. Give each student an index card and 1–2 minutes to complete the first question. Remind them that there is more than one correct way to draw the corresponding height for a base. Ask them to pause after the first question. As students work, notice how students are using the index cards (if at all).

Afterwards, solicit a few quick comments on the exploration. Ask questions such as:

- "How did you know where to draw the segments?"
- "How did you draw them?"
- "Why were you given index cards? How might they help?"

Explain that you will now demonstrate a way to draw heights effectively. (If any students used the index card correctly, acknowledge that they were on the right track.)

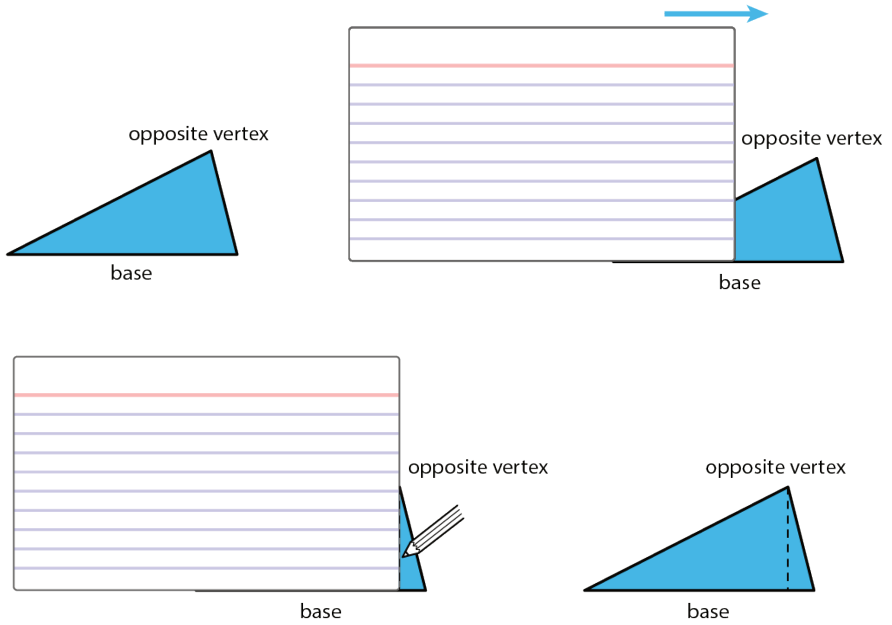
Remind students that any line we draw to show the height of a triangle must be drawn *perpendicular* to the base. Having a tool with a right angle and with straight edges can help us make sure the line we draw is both straight and perpendicular to the base. This is what the index card is for.

Ask: "How do we know where to stop this line we are drawing? How long should it be?"

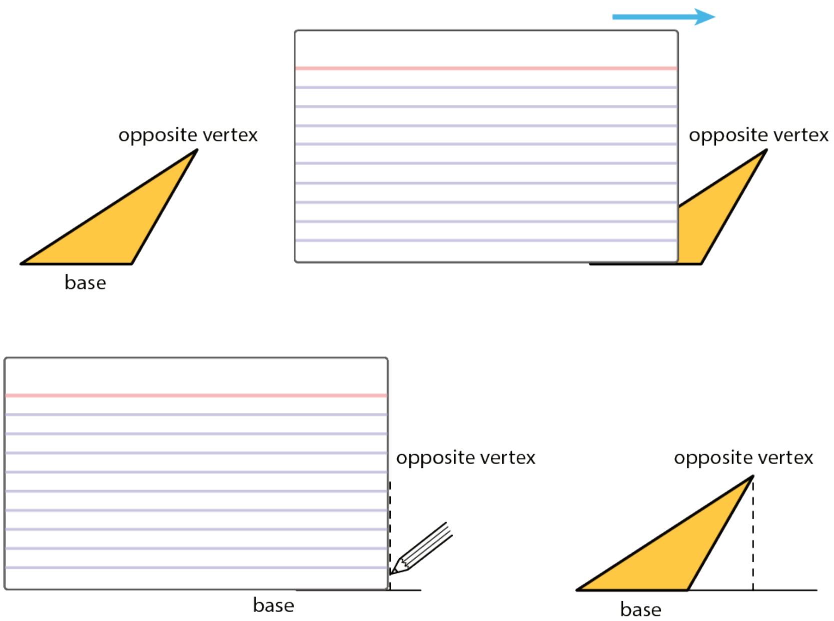
Explain that the easiest way is to draw the line so it would pass through the vertex opposite of the chosen base. Draw or display a triangle for all to see. Demonstrate the following.

- Choose one side of the triangle as the base. Identify the opposite vertex.
- Line up one edge of the index card with that base.

- Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
- Use that edge to draw a line segment from that vertex to the base. The measure of that segment is the height.



Ask: What if the opposite vertex is not directly over the base? Explain that sometimes we need to extend the line of the base and demonstrate the process.



Demonstrate the process with another example in which the card needs to slide from right to left (e.g., by rotating the obtuse triangle above clockwise). Left-handed students may find this particularly helpful.

Prompt students to use this method to check the heights they drew in the first question, revise the drawings if they were incorrect, and share their revisions with their partners. Circulate and support students as they draw. Those who finish verifying the heights in the first question can move on to complete the rest of the activity with their partners.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Support access to tools and assistive technologies. Monifor for students who may need an additional demonstration, or assistance lining up one edge of their index card with the chosen base. Consider displaying the images from the Launch as students complete this task.
Supports accessibility for: Visual-spatial processing; Fine-motor skills

Access for English Language Learners

Representing, Speaking, Listening: MLR2 Collect and Display. While students work, circulate and collect examples of student drawings of line segments showing the base and height. To do this, take digital pictures, or sketch students' drawings onto a display. Look for examples that show the height inside and outside of the triangle, as well as bases that are horizontal or vertical. Then, in the whole-class discussion, display the various examples and ask students to compare the diagrams. For example, ask students, "Do any two diagrams have similar methods for determining the base or height?" Listen for and amplify the mathematical language students use to support their reasoning.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

Some students may use the index card simply as a straightedge and therefore draw heights that are not perpendicular to the given base. Remind them that a height needs to be perpendicular or at a right angle to the base.

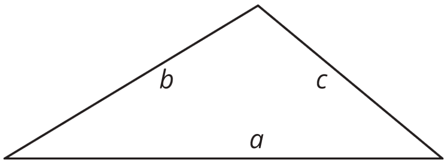
Students may mistakenly think that a base must be a horizontal side of a triangle (or one closest to being horizontal) and a height must be drawn inside of the triangle. Point to some examples from earlier work to remind students that neither is true. Remind them to align their index card to the side labeled "base."

Some students may find it awkward to draw height segments when the base is not horizontal. Encourage students to rotate their paper as needed to make drawing easier.

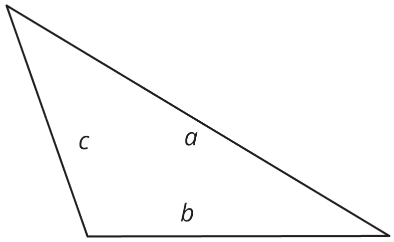
Student Task Statement

1. Here are three copies of the same triangle. The triangle is rotated so that the side chosen as the base is at the bottom and is horizontal. Draw a height that corresponds to each base. Use an index card to help you.

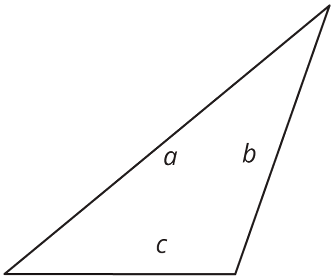
Side a as the base:



Side b as the base:



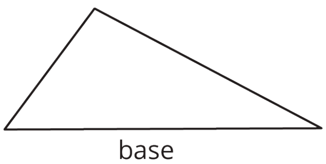
Side c as the base:



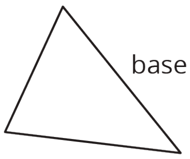
Pause for your teacher’s instructions before moving to the next question.

2. Draw a line segment to show the height for the chosen base in each triangle.

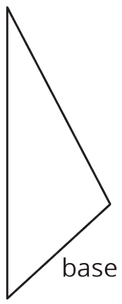
A



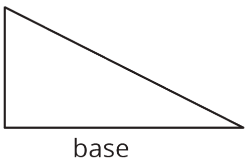
B



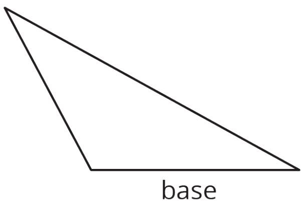
C



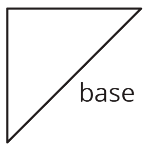
D



E

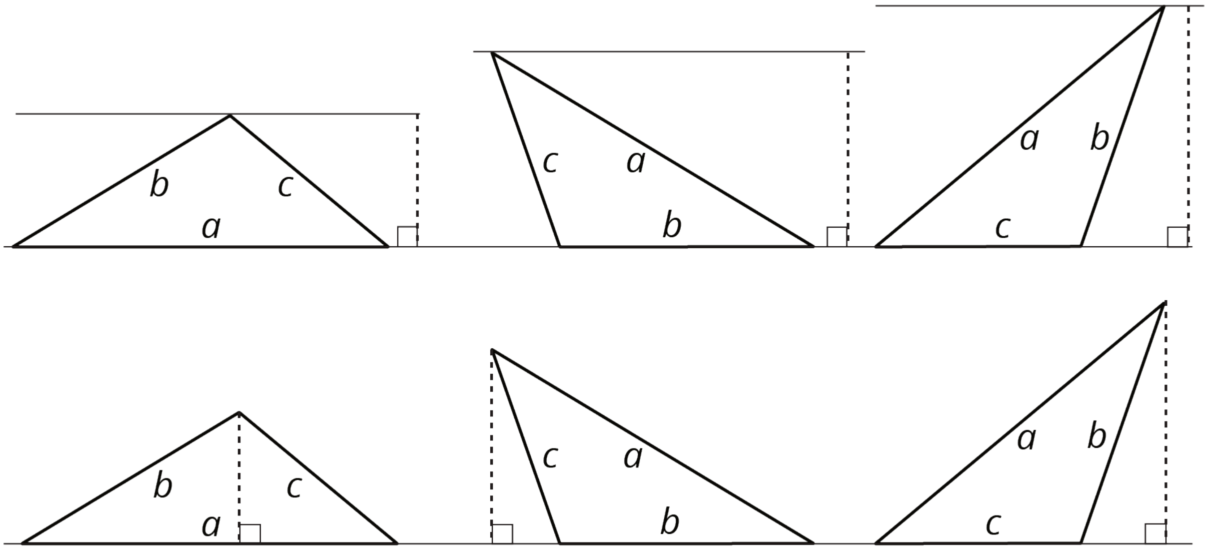


F

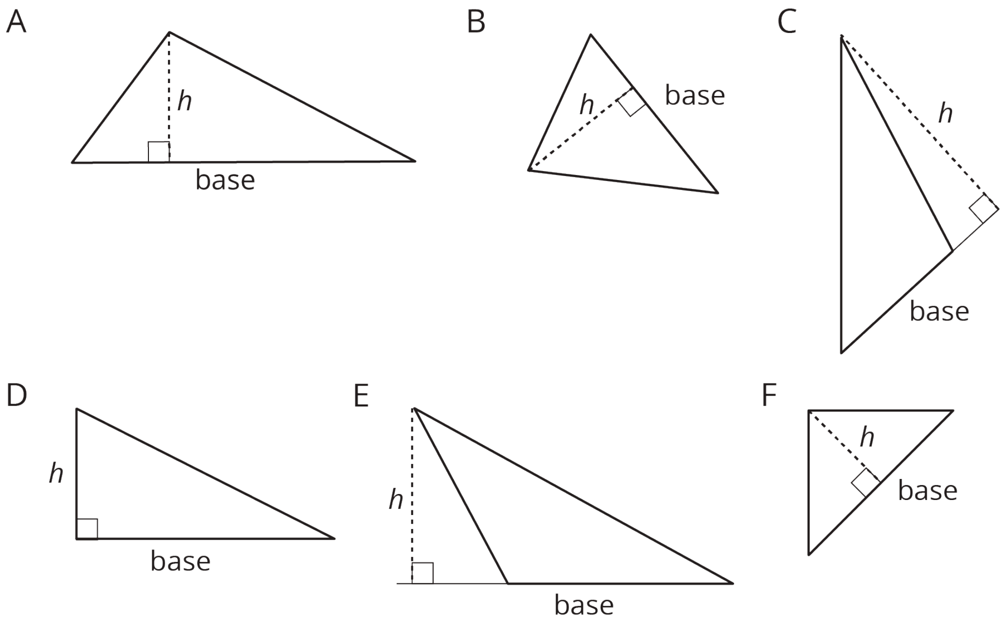


Student Response

1. Drawings vary. Sample drawings:



2.



Activity Synthesis

If time permits, consider selecting one student to share the height drawing for each triangle, or display the solutions in the Student Response for all to see. To help students reflect on their work, discuss questions such as:

- “For which triangles was it easy to find the corresponding height for the given base?”
- “For which triangles was it harder?”
- “How was the process of finding the height of triangle D different from that of the others?” (The height of a right triangle is already drawn: it is the other segment framing the right angle.)

- “When might we need to extend the line of the base, or draw a height line outside of the triangle?” (When dealing with obtuse triangles, or when the opposite vertex is not directly over the base.)

10.3 Some Bases Are Better Than Others

Optional: 15 minutes
This activity allows students to practice identifying the base and height of triangles and using them to find areas.

Because there are no directions on which base or height to use, and because not all sides would enable them to calculate area easily, students need to think structurally and choose strategically. All triangles in the problems have either a vertical or a horizontal side. Choosing such a side as the base makes it easier to identify the corresponding height.

In some cases, students may opt to use a combination of area-reasoning strategies rather than finding the base and height of the shaded triangles and applying the formula. For instance, they may enclose a shaded triangle with a rectangle and subtract the areas of extra triangles (with or without using the formula on those extra triangles). Notice students who use such strategies so they could share later.

Addressing

- 6.EE.A.2.c
- 6.G.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in groups of 2. Explain that they will now practice locating or drawing heights and using them to find area of triangles. Give students 8–10 minutes of quiet think time and time to share their responses with a partner afterwards. Provide access to their geometry toolkits (especially index cards).

If time is limited, consider asking students find the area of two or three triangles instead of all four.

Anticipated Misconceptions

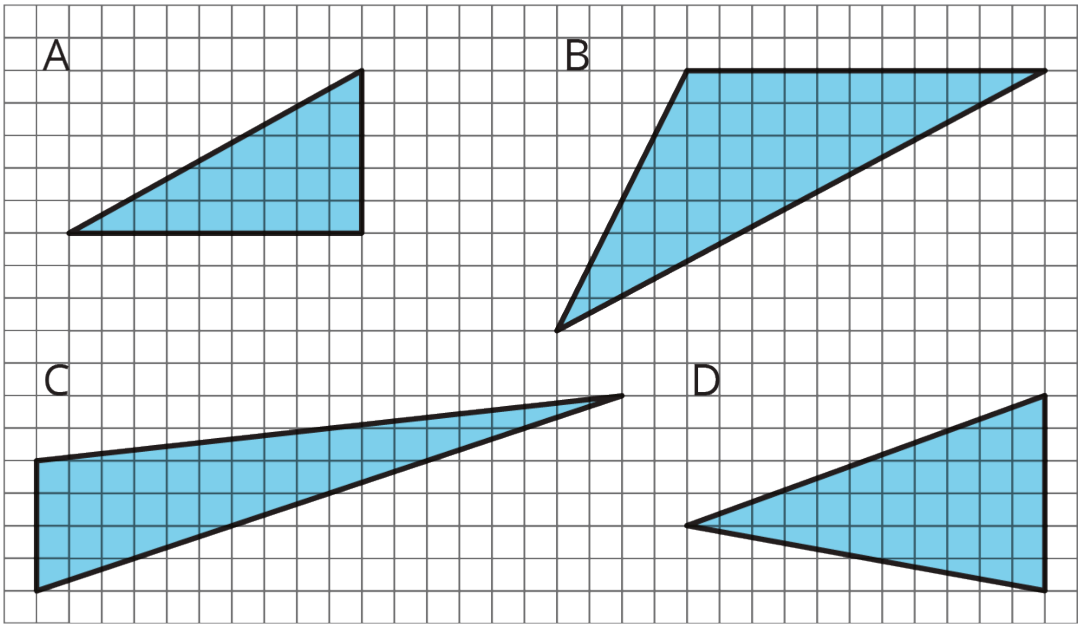
Students may think that a vertical side of a triangle is the height regardless of the segment used as the base. If this happens, have them use an index card as a straightedge to check if the two segments they are using as base and height are perpendicular.

Some students may not immediately see that choosing a side that is either vertical or horizontal would enable them to find the corresponding height very easily. They may choose a non-vertical or non-horizontal side and not take advantage of the grid. Ask if a different side might make it easier to determine the base-height lengths without having to measure.

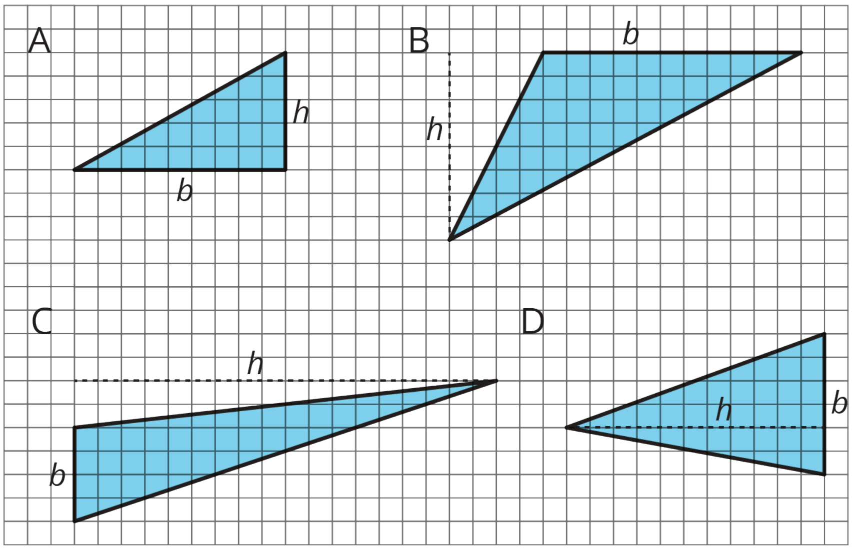
Student Task Statement

For each triangle, identify and label a base and height. If needed, draw a line segment to show the height.

Then, find the area of the triangle. Show your reasoning. (The side length of each square on the grid is 1 unit.)



Student Response



Triangle A: $b = 9$ and $h = 5$, $9 \cdot 5 \div 2 = 22.5$, area: 22.5 square units

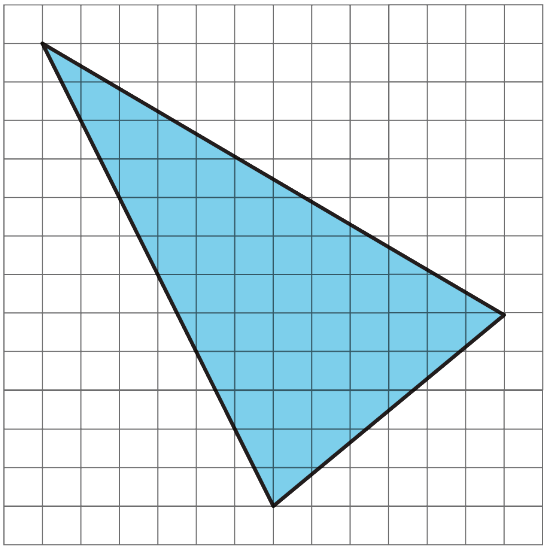
Triangle B: $b = 11$ and $h = 8$, $11 \cdot 8 \div 2 = 44$, area: 44 square units

Triangle C: $b = 4$ and $h = 18$, $4 \cdot 18 \div 2 = 36$, area: 36 square units.

Triangle D: $b = 6$ and $h = 11$, $6 \cdot 11 \div 2 = 33$, area: 33 square units.

Are You Ready for More?

Find the area of this triangle. Show your reasoning.



Student Response

51, since we can enclose the given triangle in a square that has an area of 144 ($12 \cdot 12 = 144$), then subtract away the area from right triangles in each corner.

Activity Synthesis

Focus the whole-class discussion how students went about identifying bases and heights. Discuss:

- “Which side did you choose as the base for triangle B? C? Why?”
- “Aside from choosing a vertical or horizontal side as the base, is there another way to find the area of the shaded triangles without using their bases and heights?” (Invite a couple of students who use the enclose-and-subtract method to find the area of B, C, or D to share.)
- “Which strategy do you prefer or do you think is more efficient?”
- “Can you think of an example where it might be preferable to find the base and height of the triangle of interest?” (Students may point to any of the triangles in the task.)
- “Can you think of an example where it might be preferable to enclose the triangle of interest and subtract other areas?” (Students may point to the triangle shown in "Are you ready for more?", where none of the shaded triangle's sides are horizontal or vertical.)

Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support whole-class discussion. Provide sentence frames to help students produce statements that describe the strategies they use to identify bases and heights, and to find area. For example, “For triangle ___, I chose side ___ as the base because . . .” or “The next time I need to find the area of a triangle, the strategy I will use is _____, because . . . .”
Design Principle(s): Support sense-making

Lesson Synthesis

In this lesson, we looked closely at the heights of a triangle. We located or drew a height for any side of a triangle. We also considered which pair of base and height to use to find area.

- "What must we remember about the relationship between a base of a triangle and its corresponding height?" (The height must be perpendicular to the base.)
- "What tools might help us draw a height segment? What is it about an index card or a ruler that helps us?" (A tool with straight edges and a right angle can help us draw perpendicular segments.)

When we have a base and a corresponding height, we can find the area quite simply, but for every triangle there are multiple base-height pairs.

- "Does it matter which side we choose as the base? How do we decide?" (For the base, we need a side with a known length. For the height, we need a segment that is perpendicular to that base and whose length we can determine.)

10.4 Stretched Sideways

Cool Down: 5 minutes

Addressing

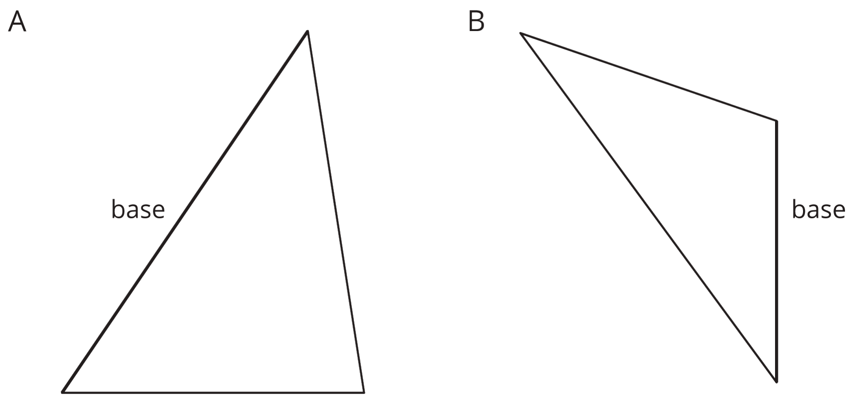
- 6.G.A.1

Launch

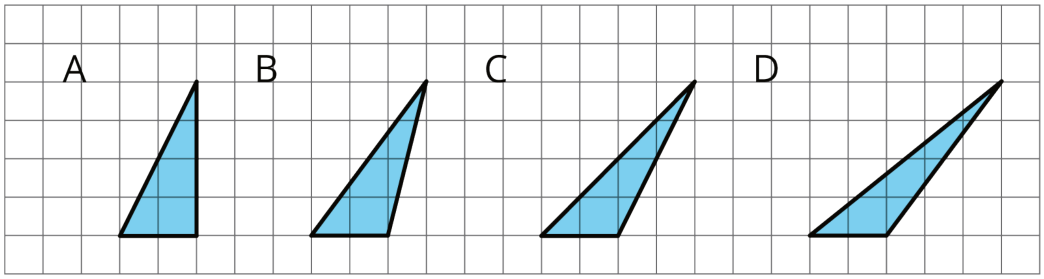
Provide access to geometry toolkits.

Student Task Statement

1. For each triangle, draw a height segment that corresponds to the given base, and label it h. Use an index card if needed.

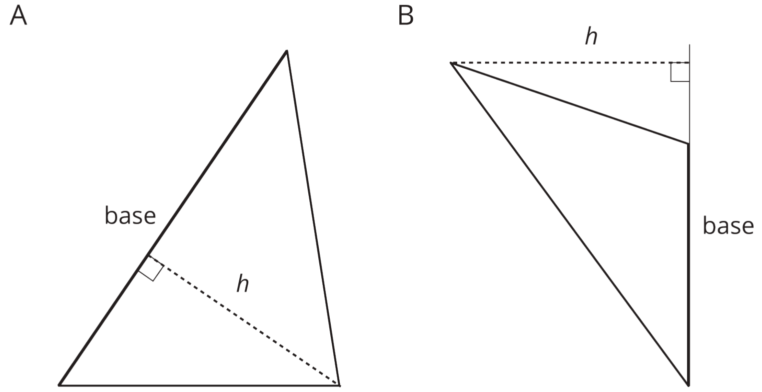


2. Which triangle has the greatest area? The least area? Explain your reasoning.



Student Response

1. Answers vary. There are many possible locations for a height segment. The segments shown are the most straightforward.



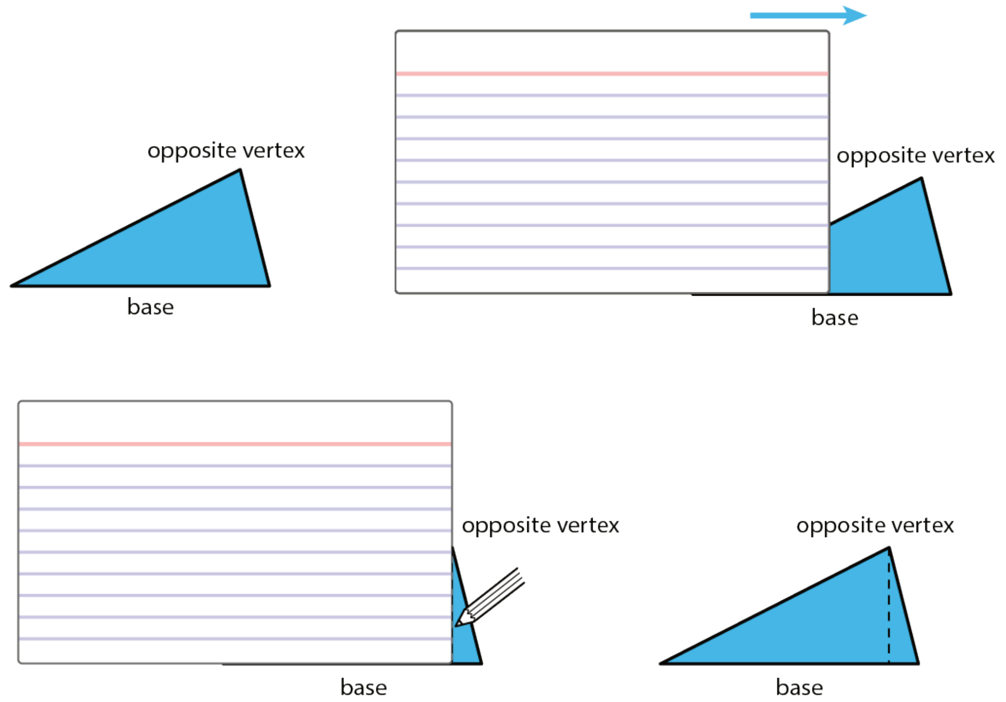
2. All of the triangles have the same area: 4 square units. They all have a base of 2 units and a height of 4 units.

Student Lesson Summary

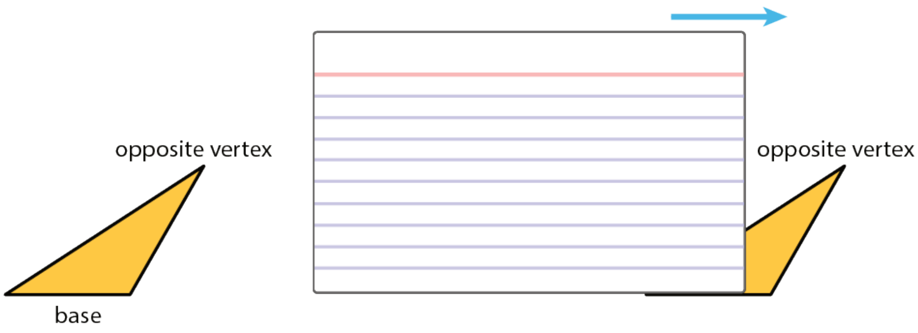
A height of a triangle is a perpendicular segment between the side chosen as the base and the opposite vertex. We can use tools with right angles to help us draw height segments.

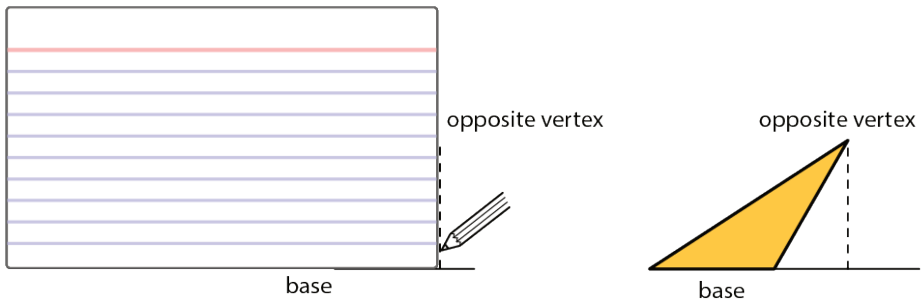
An index card (or any stiff paper with a right angle) is a handy tool for drawing a line that is perpendicular to another line.

1. Choose a side of a triangle as the base. Identify its opposite vertex.
2. Line up one edge of the index card with that base.
3. Slide the card along the base until a perpendicular edge of the card meets the opposite vertex.
4. Use the card edge to draw a line from the vertex to the base. That segment represents the height.



Sometimes we may need to extend the line of the base to identify the height, such as when finding the height of an obtuse triangle, or whenever the opposite vertex is not directly over the base. In these cases, the height segment is typically drawn *outside* of the triangle.

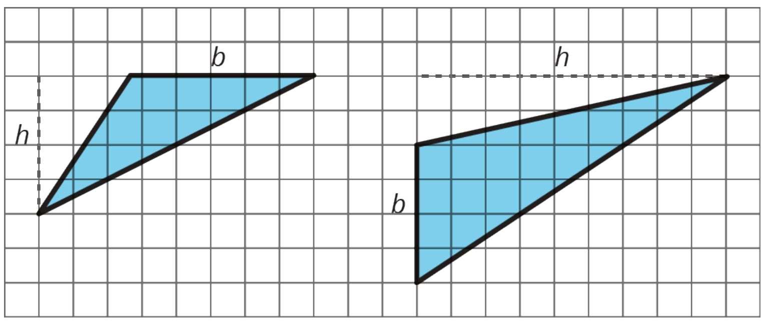




Even though any side of a triangle can be a base, some base-height pairs can be more easily determined than others, so it helps to choose strategically.

For example, when dealing with a right triangle, it often makes sense to use the two sides that make the right angle as the base and the height because one side is already perpendicular to the other.

If a triangle is on a grid and has a horizontal or a vertical side, you can use that side as a base and use the grid to find the height, as in these examples:



Glossary

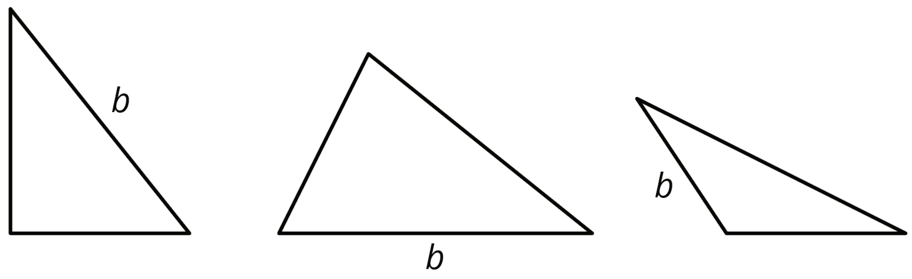
- edge
- vertex

Lesson 10 Practice Problems

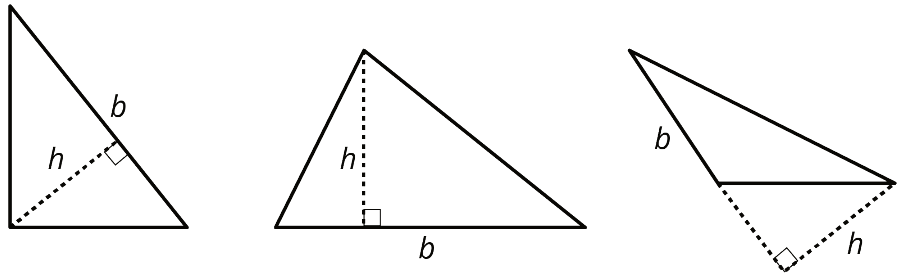
Problem 1

Statement

For each triangle, a base is labeled b. Draw a line segment that shows its corresponding height. Use an index card to help you draw a straight line.



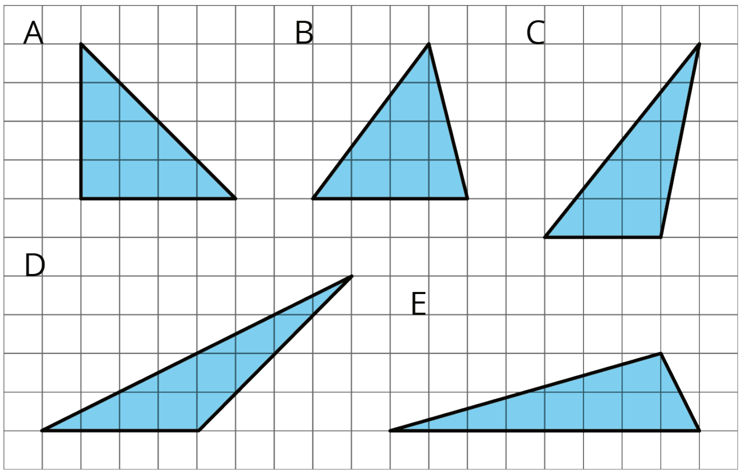
Solution



Problem 2

Statement

Select all triangles that have an area of 8 square units. Explain how you know.



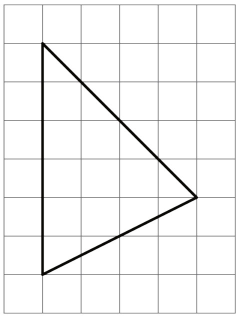
Solution

A, B, D, and E. Triangles A, B, and D all have a horizontal base of 4 units and a height of 4 units. $\frac{4 \cdot 4}{2} = 8$, so the area of each is 8 square units. Triangle C has a horizontal base of 3 units and a height of 5 units, so its area is 7.5 square units. Triangle E has a horizontal base of 8 units and a height of 2 units, so its area is 8 square units, since $\frac{8 \cdot 2}{2} = 8$.

Problem 3

Statement

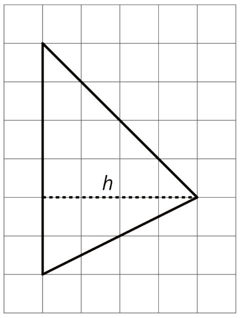
Find the area of the triangle. Show your reasoning.



If you get stuck, carefully consider which side of the triangle to use as the base.

Solution

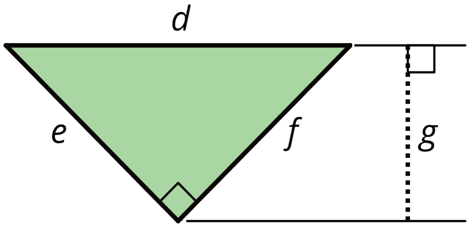
12 square units. Explanations vary. Sample response: The vertical side is 6 units long, and this side can be used as the base. The corresponding height, shown in the diagram, is 4 units. So the area is 12 square units. Another method is to surround the triangle with a rectangle then subtract the parts that are not in the triangle.



Problem 4

Statement

Can side d be the base for this triangle? If so, which length would be the corresponding height? If not, explain why not.



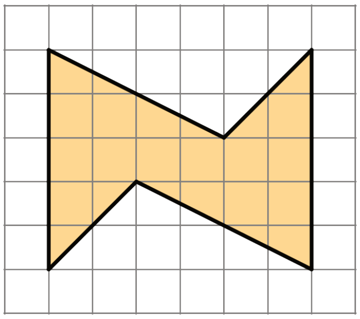
Solution

Yes, side d can be the base, because it is a side of the triangle. The corresponding height is g.

Problem 5

Statement

Find the area of this shape. Show your reasoning.



Solution

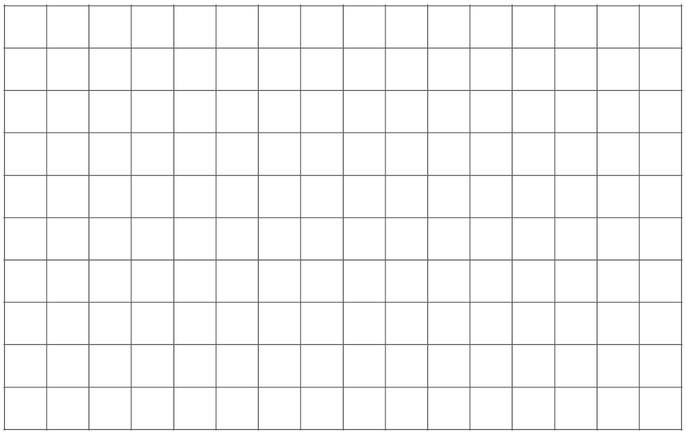
18 square units. Reasoning varies.

(From Unit 1, Lesson 3.)

Problem 6

Statement

On the grid, sketch two different parallelograms that have equal area. Label a base and height of each and explain how you know the areas are the same.



Solution

Answers vary.

(From Unit 1, Lesson 6.)