Lesson 3: What a Point in a Scatter Plot Means

Goals

- Coordinate (orally and in writing) data in a table and points on a scatter plot.
- Describe (orally) the trend of the data, and use the trend to predict unknown values.
- Interpret (orally and in writing) a point on a scatter plot in context.

Learning Targets

- I can describe the meaning of a point in a scatter plot in context.

Lesson Narrative

In this lesson, students continue their investigation of scatter plots. They interpret points in a scatter plot in terms of a context, and add points to a scatter plot given information about an individual in the population. They compare individuals represented by different points and informally discuss trends in the data.

There are two levels of analysis needed to successfully make sense of scatter plots: what is happening for a particular individual, and what is happening at a global level for the entire population. The ability to move between these two zoom levels develops over time. In this lesson, students spend a lot of time looking at the details of a scatter plot, naming the quantities represented in a scatter plot, and focusing on the meaning of individual points (MP6).

Alignments

Building On

- 5.G.A.2: Represent real world and mathematical problems by graphing points in the first quadrant of the coordinate plane, and interpret coordinate values of points in the context of the situation.

Addressing

- 8.SP.A.1: Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
- 8.SP.A.3: Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. For example, in a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR5: Co-Craft Questions

- Think Pair Share

Student Learning Goals

Let's investigate points in scatter plots.

3.1 The Giant Panda

Warm Up: 5 minutes
The purpose of this warm-up is for students to interpret the meaning of a single point in a scatter plot by looking at the point's coordinates and the graph's axis labels (MP2).

Building On

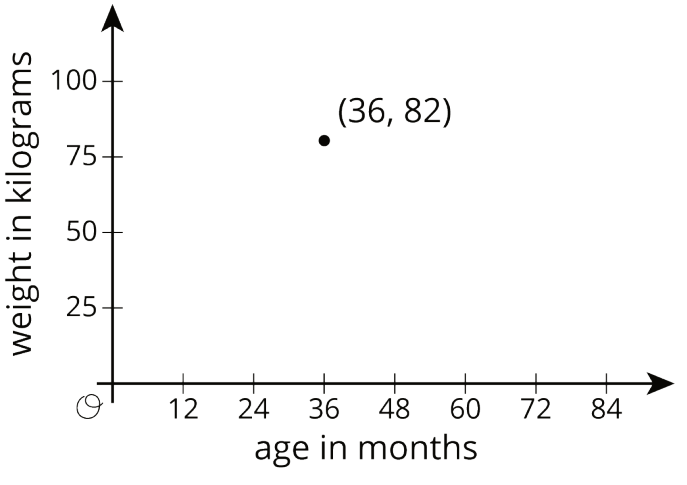
- 5.G.A.2

Launch

Display the graph for all to see. Give students 1 minute of quiet think time followed by a whole-class discussion.

Student Task Statement

A giant panda lives in a zoo. What does the point on the graph tell you about the panda?



Student Response

At 36 months of age, the panda weighed 82 kilograms.

Activity Synthesis

Ask students to share their responses. Record and display their responses for all to see. If all of the students responds, “At 36 months of age, the panda weighed 82 kilograms,” ask them the following questions to clarify where they found that information in the graph:

- "What did the first coordinate of the point tell you?" (The first coordinate shows how far to go on the horizontal axis.)
- "What did the second coordinate of the point tell you?" (The second coordinate shows how far to go on the vertical axis.)
- "How did you know the meaning of each of the numbers?" (You can read the meaning of the numbers by looking at the labels on the axes.)
- "What specific information did each axis give you?" (The labels on the axes indicate attribute, age or weight, and the units in which each attribute is measured, months or kilograms.)

3.2 Weight and Fuel Efficiency

15 minutes (there is a digital version of this activity)
The analysis of scatter plots continues with data about the mass of automobiles and their fuel efficiency. Again, students connect points in the scatter plot with rows of data in a table, but now there is a third column that gives a name to each pair.

After picking out information about points in the scatter plot and table, students make their first foray into thinking about how scatter plots can help them make predictions. Specifically, when looking at two particular points in the scatter plot, students are asked if the results are surprising given the overall trend of the data. In later lessons, students develop more sophisticated tools for answering questions like this (specifically, by fitting lines to the data), so a heuristic discussion is all that is expected at this point.

Addressing

- 8.SP.A.1

Instructional Routines

- MLR5: Co-Craft Questions

Launch

Explain a bit about fuel efficiency so that students understand this measurement. “Fuel efficiency” is a measure of the average distance a car will travel using a certain amount of gas. Commonly this is measured in "miles per gallon." For example, a car that has a fuel efficiency of 25 miles per gallon (mpg) should be able to drive approximately 25 miles while using up a gallon of gas. Many factors can influence fuel efficiency, including the way an engine is engineered (to produce more power, an engine may use more gasoline), driving conditions (more frequent stopping and starting or hills will

use fuel less efficiently), and what accessories are being used (air conditioning requires energy that is not used for actually driving the car).

Before beginning the activity, ask "What would a scatter plot including these two variables look like?" to encourage students to predict how a car's mass might influence fuel efficiency.

If using the digital version of the activity, after completing the launch discussion, students will use an applet to analyze a scatter plot, identify points, and draw conclusions about the data. The digital activity mirrors the analog activity.

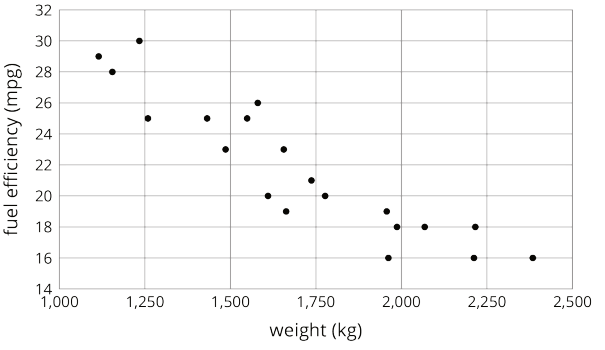
Access for English Language Learners

Writing, Conversing: MLR5 Co-Craft Questions. Display only the table without revealing the graph or questions that follow. Ask students to work with a partner to write possible questions that could be answered by the data in the table. Invite 2–3 groups to share their questions with the class. Look to see whether/how the weights and fuel efficiencies of the vehicles shows up in students' questions. Next, reveal the questions of the activity. This routine helps students consider the context of this problem and to increase awareness about language used to talk about weights and fuel efficiencies.
Design Principle(s): Maximize meta-awareness; Support sense-making

Student Task Statement

The table and scatter plot show weights and fuel efficiencies of 18 cars.

car	weight (kg)	fuel efficiency
A	1,549	25
B	1,610	20
C	1,737	21
D	1,777	20
E	1,486	23
F	1,962	16
G	2,384	16
H	1,957	19
I	2,212	16
J	1,115	29
K	2,068	18
L	1,663	19
M	2,216	18
N	1,432	25
O	1,987	18
P	1,580	26
Q	1,234	30
R	1,656	23



1. Which point in the scatter plot represents Car L's measurements?
2. What is the fuel efficiency of the car with the greatest weight?
3. What is the weight of the car with the greatest fuel efficiency?
4. Car S weighs 1,912 kilograms and gets 16 miles per gallon. On the scatter plot, plot a point that represents Car S's measurements.
5. Cars N and O, shown in the scatter plot, are made by the same company. Compare their weights and fuel efficiencies. Does anything surprise you about these cars?
6. A different company makes Cars F and G. Compare their weights and fuel efficiencies. Does anything surprise you about these cars?

Student Response

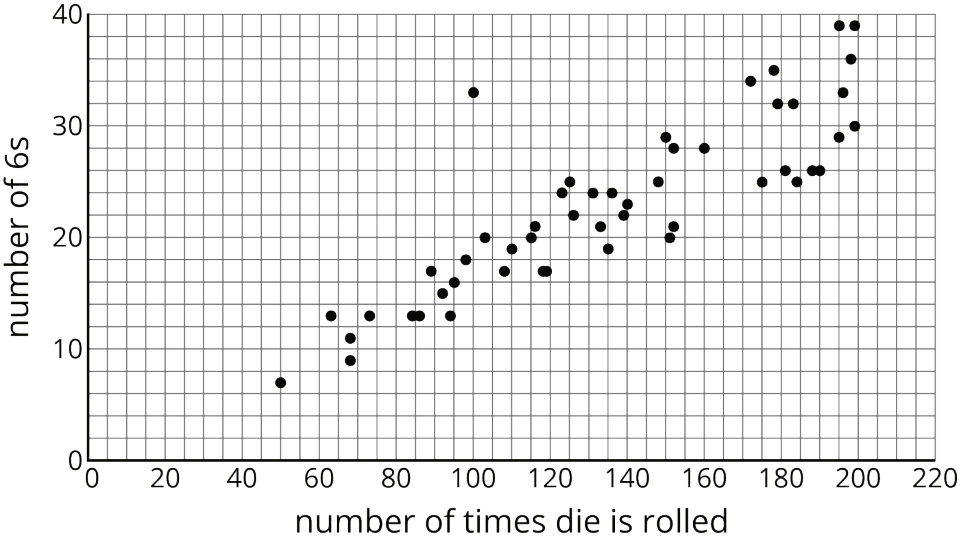
1. The point with coordinates $(1{,}663, 19)$
2. 16 mpg
3. 1,234 kg
4. The point with coordinates $(1{,}912, 16)$

5. Car N has a lower weight and a higher fuel efficiency than car O. Answers vary. Sample response: Nothing is surprising about these cars because they fit in with the trend of the data.

6. Their weights are quite different, but they still have the same fuel efficiency. Answers vary. Sample response: It is surprising that cars with such different weights made by the same company have the same fuel efficiency since generally the heavier the car, the less fuel efficient it is.

Are You Ready for More?

After a board game competition, the tournament director collects 50 dice from the games played and rolls each one until he gets bored and tries a different one. The scatter plot shows the number of times he rolled each die and the number of 6s that resulted during those rolls.



Select a point in the scatter plot and give its approximate coordinates, then tell the story of that point in the context of the problem.

Student Response

Answers vary. Sample response: The point at $(100, 33)$ lies apart from the rest of the data. Someone may have brought a die from home that is weighted to roll more 6s, expecting to cheat and win the tournament.

Activity Synthesis

The purpose of the discussion is for students to compare pairs of data in both representations and to begin thinking about trends in the data. Ask students to share their thinking about the last two questions. Draw out the fact that cars N and O seem to fit the overall trend that an increase in mass typically corresponds to a decrease in fuel efficiency but cars F and G don't fit that trend when compared with each other. Mention that we will see many examples of trends that don't necessarily apply to individuals in a population.

Consider asking some of the following questions to conclude the discussion:

- “How did you find the automobile with the greatest weight?” (I looked for the point farthest to the right.)
- “How did you find the automobile with the greatest fuel efficiency?” (I looked for the point that is highest.)
- “The data in this table is from automobiles that run solely on gasoline. Where do you think a hybrid car would appear on the graph?” (A hybrid car would use less gas for the same weight, so it's point would be below the other points with a similar weight.)
- “Did the scatter plot match your prediction?”

3.3 Coat Sales

15 minutes
In this activity, students continue to identify points on a scatter plot as representatives of a single month when two variables are measured. Students are also asked to explain the meaning of some abstract points in the context of the problem and identify when it does not make sense to extrapolate information from the graph based on the context (MP2).

Monitor for students who understand that 60 degrees Celsius is unreasonable for an average monthly temperature in most parts of the world, as well as those who identify the trend to predict negative sales which does not make sense in this context either.

Addressing

- 8.SP.A.1
- 8.SP.A.3

Instructional Routines

- MLR1: Stronger and Clearer Each Time
- Think Pair Share

Launch

Arrange students in groups of 2. Allow students 5 minutes quiet work time followed by 5 minutes partner discussion and whole-class discussion.

Ask students to predict what a scatter plot might look like when the temperature is along the x-axis and the sales of coats at a store is along the y-axis.

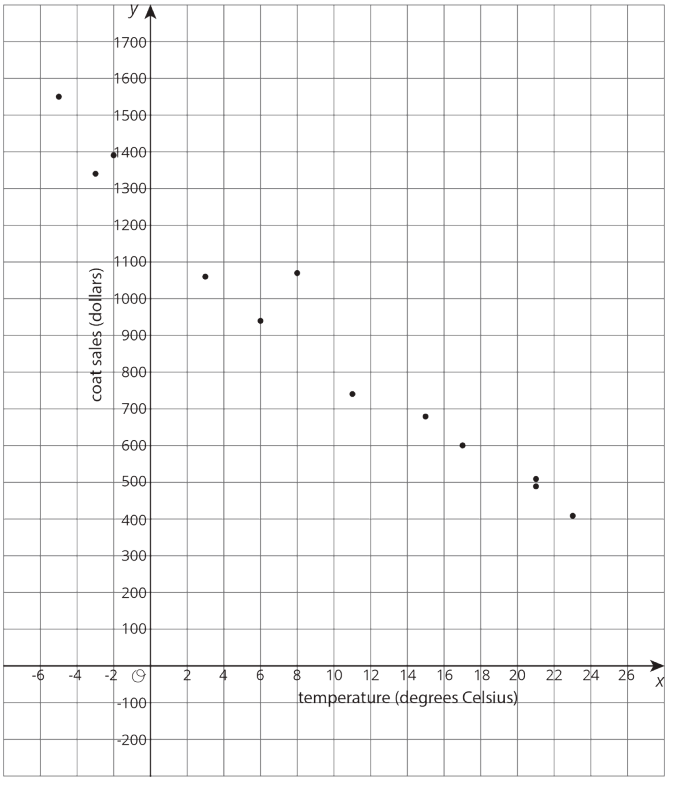
Access for Students with Disabilities

Representation: Internalize Comprehension. Demonstrate and encourage students to use color coding and annotations to highlight connections between representations in a problem. For example, ask students to use the different colors to represent and annotate temperature and coat sales in the graph and table.
Supports accessibility for: Visual-spatial processing

Student Task Statement

A clothing store keeps track of the average monthly temperature in degrees Celsius and coat sales in dollars.

temperature (degrees Celsius)	coat sales (dollars)
-5	1,550
-3	1,340
3	1,060
8	1,070
15	680
21	490
23	410
21	510
17	600
11	740
6	940
-2	1,390



1. What does the point $(15, 680)$ represent?

2. For the month with the lowest average temperature, estimate the total amount made from coat sales. Explain how you used the table to find this information.
3. For the month with the smallest coat sales, estimate the average monthly temperature. Explain how you used the scatter plot to find this information.
4. If there were a point at $(0, A)$ what would it represent? Use the scatter plot to estimate a value for A.
5. What would a point at $(B, 0)$ represent? Use the scatter plot to estimate a value for B.
6. Would it make sense to use this trend to estimate the value of sales when the average monthly temperature is 60 degrees Celsius? Explain your reasoning.

Student Response

1. When the average monthly temperature is 15 degree Celsius, the store sold $680 of coats.
2. $1,550. In the table, I found the lowest temperature and wrote the value for sales in the same row.
3. 23 degrees Celsius. In the graph, I found the lowest point and estimated the x coordinate.
4. When the temperature averaged 0 degrees Celsius for the month, the store sold A dollars of coats. Estimates vary. Correct answers should range between $1,200 and $1,300.
5. When the store sold no coats for the month, the average monthly temperature is B degrees Celsius. Estimates vary. Correct answers should be approximately 30 degrees Celsius.
6. It does not make sense. 60 degrees Celsius (140 Fahrenheit) is hotter than has been recorded on Earth. Also, based on the trend, there would be negative dollars sold which does not really make sense.

Activity Synthesis

The purpose of the discussion is for students to understand that each point in the graph represents a single month in which two measurements are made. Select students to share their responses, including those identified for the last problem.

Consider asking some of the following questions:

- "When discussing functions, what is a point of the form $(0, A)$ called? What about a point of the form $(B, 0)$?" (The y-intercept. The x-intercept.)
- "In a scatter plot is there a y-intercept?" (It depends on the context. For some situations, it may not make sense for the variable represented along the x-axis to be zero. For example if the variable is the height of a building. On other situations, there may be more than one point along the y-axis. For example, in a situation like the one in the task, there may be multiple months where the average temperature is 0 degree Celsius.)

- "When the average monthly temperature goes up, what seems to be happening to sales?" (They are decreasing.)

Access for English Language Learners

Writing, Speaking: MLR1 Stronger and Clearer Each Time. Use this routine to give students a structured opportunity to revise and refine their response to the last question. Ask each student to meet with 2–3 other partners in a row for feedback. Provide students with prompts for feedback that will help students strengthen their ideas and clarify their language (e.g., "Can you give an example?", "Why do you think...?", "How do you know...?", etc.). Students can borrow ideas and language from each partner to strengthen their final version.
Design Principle(s): Optimize output (for explanation)

Lesson Synthesis

Revisit the idea that a point in a scatter plot represents an individual in a population. Ask students:

- "What kind of information does a point in a scatter plot represent?" (Two measurements for an individual in a population.)
- "How do we know what information is captured by a point in a scatter plot?" (The axis labels tell us how to interpret the coordinates of the points.)
- "Do trends in a scatter plot necessarily apply to individuals in the population?" (No.)

3.4 Quarterbacks

Cool Down: 5 minutes
Students continue to practice comparing data from a table and scatter plot by identifying points in the scatter plot and their corresponding entries in a table. Additionally, students show their understanding of the connection by plotting an additional point to add to the data shown.

Addressing

- 8.SP.A.1

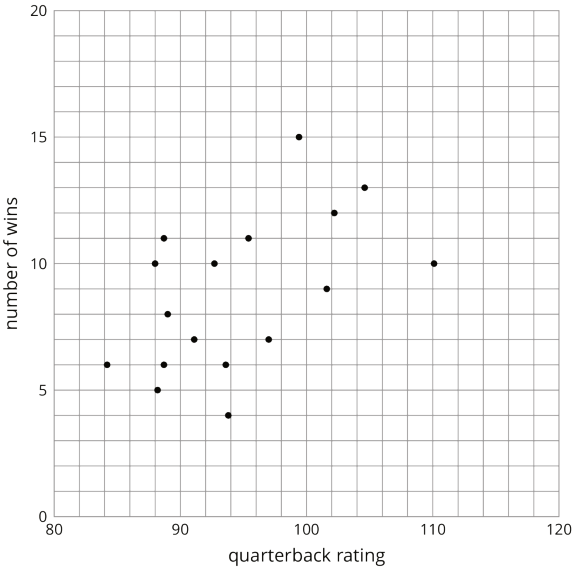
Launch

Explain that quarterback rating is a way of rating a quarterback in football on a scale of 0 to 158.3 based on many variables (such as pass attempts, interceptions, and touchdown passes). A higher rating generally represents a quaterback with better passing statistics. When the quarterback starts a game, he is playing from the beginning of the game and is usually the best option available for the team at that position. The data from this activity refers to quarterbacks who were able to start in at least 16 games for their team in a season.

Student Task Statement

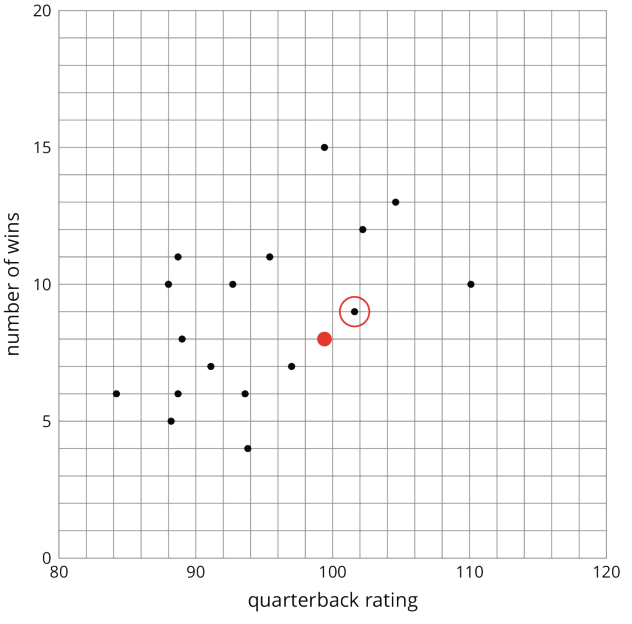
Here are a table and scatter plot that show ratings and wins for quarterbacks who started 16 games this season.

player	quarterback rating	number of wins
A	93.8	4
B	102.2	12
C	93.6	6
D	89	8
E	88.2	5
F	97	7
G	88.7	6
H	91.1	7
I	92.7	10
J	88	10
K	101.6	9
L	104.6	13
M	84.2	6
N	99.4	15
O	110.1	10
P	95.4	11
Q	88.7	11



1. Circle the point in the scatter plot that represents Player K's data.
2. Which quarterback's data are represented by the point farthest to the left?
3. Player R is not included in the table because he did not start 16 games this year. He did have a quarterback rating of 99.4 and his team won 8 games. On the scatter plot, plot a point that represents Player R's data.

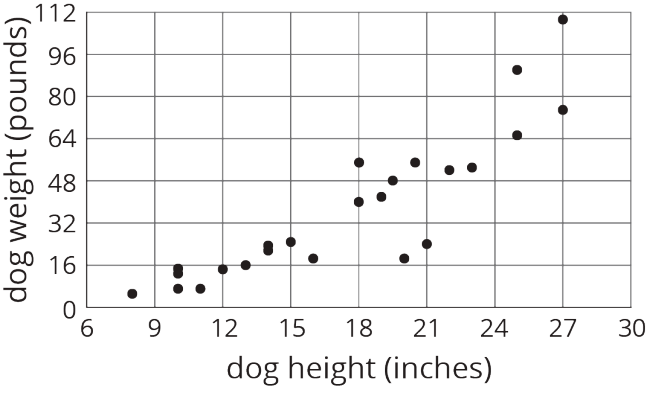
Student Response



1. The circled point on the scatter plot
2. Player M
3. The added point to the scatter plot, plotted larger for visibility

Student Lesson Summary

Scatter plots show two measurements for each individual from a group. For example, this scatter plot shows the weight and height for each dog from a group of 25 dogs.



We can see that the tallest dogs are 27 inches, and that one of those tallest dogs weighs about 75 pounds while the other weighs about 110 pounds. This shows us that dog weight is not a function of dog height because there would be two different outputs for the same input. But we can see a general trend: Taller dogs tend to weigh more than shorter dogs. There are exceptions. For example, there is a dog that is 18 inches tall and weighs over 50 pounds, and there is another dog that is 21 inches tall but weighs less than 30 pounds.

When we collect data by measuring attributes like height, weight, area, or volume, we call the data *numerical data* (or measurement data), and we say that height, weight, area, or volume is a *numerical variable*. Upcoming lessons will discuss how to identify and describe trends in data that has been collected.

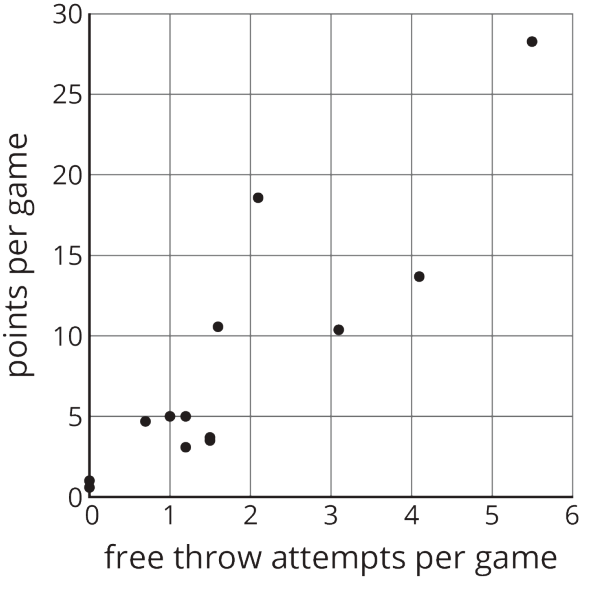
Lesson 3 Practice Problems

Problem 1

Statement

Here is a table and a scatter plot that compares points per game to free throw attempts for a basketball team during a tournament.

player	free throw attempts	points
player A	5.5	28.3
player B	2.1	18.6
player C	4.1	13.7
player D	1.6	10.6
player E	3.1	10.4
player F	1	5
player G	1.2	5
player H	0.7	4.7
player I	1.5	3.7
player J	1.5	3.5
player K	1.2	3.1
player L	0	1
player M	0	0.8
player N	0	0.6



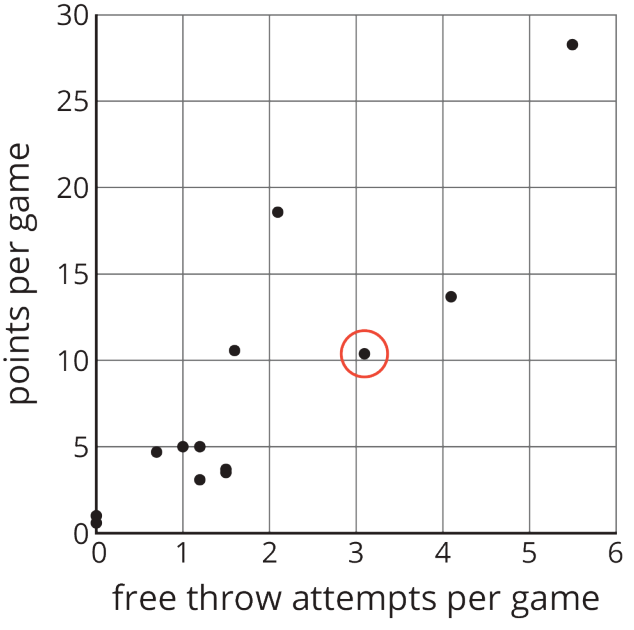
a. Circle the point that represents the data for Player E.

b. What does the point $(2.1, 18.6)$ represent?

c. In that same tournament, Player O on another team scored 14.3 points per game with 4.8 free throw attempts per game. Plot a point on the graph that shows this information.

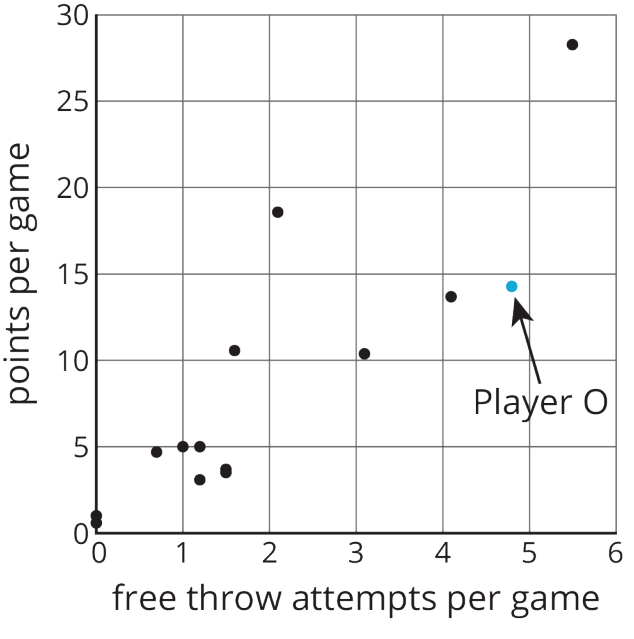
Solution

a.



b. The point $(2.1, 18.6)$ represents the free throw attempts and points per game for Player B.

c.



Problem 2

Statement

Select **all** the representations that are appropriate for comparing exam score to number of hours of sleep the night before the exam.

A. Histogram

B. Scatter plot

C. Dot plot

D. Table

E. Box plot

Solution

["B", "D"]
(From Unit 6, Lesson 2.)

Problem 3

Statement

A cylinder has a volume of 36π cm^3 and height h. Complete this table for the volume of other cylinders with the same radius but different heights.

height (cm)	volume (cm^3)
h	36π
$2h$	
$5h$	
$\frac{h}{2}$	
$\frac{h}{5}$	

Solution

height (cm)	volume (cm^3)
h	36π
$2h$	72π
$5h$	180π
$\frac{h}{2}$	18π
$\frac{h}{5}$	$\frac{36}{5}\pi$

(From Unit 5, Lesson 17.)