## Lesson 15: Features of Trigonometric Graphs (Part 1)

* Let’s compare graphs and equations of trigonometric functions.

### 15.1: Notice and Wonder: Musical Notes

Here are pictures of sound waves for two different musical notes:



What do you notice? What do you wonder?

### 15.2: Equations and Graphs

Match each equation with its graph. More than 1 equation can match the same graph.

Equations:

1. $y=-cos(θ)$
2. $y=2sin(θ)−3$
3. $y=cos\left(θ+\frac{π}{2}\right)$
4. $y=3sin(θ)−2$
5. $y=sin(θ−\frac{π}{2})$
6. $y=sin(θ+π)$

A



B



C



D



#### Are you ready for more?

1. Find an equation for this graph using the sine function.
2. Find another equation for the same graph using a cosine function.



### 15.3: Double the Sine

1. Complete the table of values for the expression $sin(2θ)$

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| * $θ$
 | * 0
 | * $\frac{π}{12}$
 | * $\frac{π}{6}$
 | * $\frac{π}{4}$
 | * $\frac{π}{2}$
 | * $\frac{3π}{4}$
 | * $π$
 | * $\frac{5π}{4}$
 | * $\frac{3π}{2}$
 | * $\frac{7π}{4}$
 | * $2π$
 |
| * $sin(2θ)$
 | *
 | *
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1. Plot the values and sketch a graph of the equation $y=sin(2θ)$. How does the graph of $y=sin(2θ)$ compare to the graph of $y=sin(θ)$?
* 
1. Predict what the graph of $y=cos(4θ)$ will look like and make a sketch. Explain your reasoning.
* 

### Lesson 15 Summary

We can find the amplitude and midline of a trigonometric function using the graph or from an equation. For example, let’s look at the function given by the equation $y=3cos\left(θ+\frac{π}{4}\right)+2$. We can see that the midline of this function is 2 because of the vertical translation up by 2. This means the horizontal line $y=2$ goes through the middle of the graph. The amplitude of the function is 3. This means the maximum value it takes is 5, 3 more than the midline value, and the minimum value it takes is -1, 3 less than the midline value. The horizontal translation is $\frac{π}{4}$ to the left, so instead of having, for example, a minimum at $π$, the minimum is at $\frac{3π}{4}$. Here is what the graph looks like:



Another type of transformation is one that affects the period and that is when a horizontal scale factor is used. For example, let's look at the equation $y=cos(2θ)$ where the variable $θ$ is multiplied by a number. Here, 2 is the scale factor affecting $θ$. When $θ=0$, we have $2θ=0$ so the graph of this cosine equation starts at $(0,1)$, just like the graph of $y=cos(θ)$. When $x=π$, we have $2θ=2π$ so the graph of $y=cos(2θ)$ goes through two full periods in the same horizontal span it takes $y=cos(θ)$ to complete one full period, as shown in their graphs.



Notice that the graph of $y=cos(2θ)$ has the same general shape as the graph of $y=cos(θ)$ (same midline and amplitude) but the waves are compressed together. And what if we wanted to give the graph of cosine a stretched appearance? Then we could use a horizontal scale factor between 0 and 1. For example, the graph of $y=cos(\frac{θ}{6})$ has a period of $12π$.



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