# Lesson 16: Solving Systems by Elimination (Part 3)

- Let's find out how multiplying equations by a factor can help us solve systems of linear equations.

## 16.1: Multiplying Equations By a Number

Consider two equations in a system:

$$\begin{cases} 4x + \phantom{2}y = 1 & \text{Equation A} \\ x + 2y = 9 & \text{Equation B} \end{cases}$$

1. Use graphing technology to graph the equations. Then, identify the coordinates of the solution.

2. Write a few equations that are equivalent to equation A by multiplying both sides of it by the same number, for example, 2, -5, or $\frac{1}{2}$. Let's call the resulting equations A1, A2, and A3. Record your equations here:

   a. Equation A1:

   b. Equation A2:

   c. Equation A3:

3. Graph the equations you generated. Make a couple of observations about the graphs.

## 16.2: Writing a New System to Solve a Given System

Here is a system you solved by graphing earlier.

$$\begin{cases} 4x + \phantom{2}y = 1 & \text{Equation A} \\ x + 2y = 9 & \text{Equation B} \end{cases}$$

To start solving the system, Elena wrote:

$$\begin{aligned} 4x + \phantom{8}y &= 1 \\ 4x + 8y &= 36 \end{aligned}$$

And then she wrote:

$$\begin{array}{r} 4x + \phantom{8}y = \phantom{3}1 \\ 4x + 8y = 36 \; - \\ \hline -7y = -35 \end{array}$$

1. What were Elena's first two moves? What might be possible reasons for those moves?

2. Complete the solving process algebraically. Show that the solution is indeed $x = -1, y = 5$.

## 16.3: What Comes Next?

Your teacher will give you some slips of paper with systems of equations written on them. Each system represents a step in solving this system:

$$\begin{cases} \frac{4}{5}x + 6y = 15 \\ -x + 18y = 11 \end{cases}$$

Arrange the slips in the order that would lead to a solution. Be prepared to:

- Describe what move takes one system to the next system.
- Explain why each system is equivalent to the one before it.

### Are you ready for more?

This system of equations has solution $(5, -2)$: $\begin{cases} Ax - By = 24 \\ Bx + Ay = 31 \end{cases}$

Find the missing values $A$ and $B$.

## 16.4: Build Some Equivalent Systems

Here is a system of equations:

$$\begin{cases} 12a + 5b = -15 \\ 8a + b = 11 \end{cases}$$

1. To solve this system, Diego wrote these equivalent systems for his first two steps.

   Step 1:

   $$\begin{cases} 12a + 5b = -15 \\ -40a + -5b = -55 \end{cases}$$

   Step 2:

   $$\begin{cases} 12a + 5b = -15 \\ -28a = -70 \end{cases}$$

   Describe the move that Diego made to get each equivalent system. Be prepared to explain how you know the systems in Step 1 and Step 2 have the same solution as the original system.

2. Write another set of equivalent systems (different than Diego's first two steps) that will allow one variable to be eliminated and enable you to solve the original system. Be prepared to describe the moves you make to create each new system and to explain why each one has the same solution as the original system.

3. Use your equivalent systems to solve the original system. Then, check your solution by substituting the pair of values into the original system.

## Lesson 16 Summary

We now have two algebraic strategies for solving systems of equations: by substitution and by elimination. In some systems, the equations may give us a clue as to which strategy to use. For example:

$$\begin{cases} y = 2x - 11 \\ 3x + 2y = 18 \end{cases}$$

In this system, $y$ is already isolated in one equation. We can solve the system by substituting $2x - 11$ for $y$ in the second equation and finding $x$.

$$\begin{cases} 3x - y = \text{-}17 \\ \text{-}3x + 4y = 23 \end{cases}$$

This system is set up nicely for elimination because of the opposite coefficients of the $x$-variable. Adding the two equations eliminates $x$ so we can solve for $y$.

In other systems, which strategy to use is less straightforward, either because no variables are isolated, or because no variables have equal or opposite coefficients. For example:

$$\begin{cases} 2x + 3y = 15 & \text{Equation A} \\ 3x - 9y = 18 & \text{Equation B} \end{cases}$$

To solve this system by elimination, we first need to rewrite one or both equations so that one variable can be eliminated. To do that, we can multiply both sides of an equation by the same factor. Remember that doing this doesn't change the equality of the two sides of the equation, so the $x$- and $y$-values that make the first equation true also make the new equation true.

There are different ways to eliminate a variable with this approach. For instance, we could:

- Multiply Equation A by 3 to get $6x + 9y = 45$. Adding this equation to Equation B eliminates $y$.

$$\begin{cases} 6x + 9y = 45 & \text{Equation A1} \\ 3x - 9y = 18 & \text{Equation B} \end{cases}$$

- Multiply Equation B by $\frac{2}{3}$ to get $2x - 6y = 12$. Subtracting this equation from Equation A eliminates $x$.

$$\begin{cases} 2x + 3y = 15 & \text{Equation A} \\ 2x - 6y = 12 & \text{Equation B1} \end{cases}$$

- Multiply Equation A by $\frac{1}{2}$ to get $x + \frac{3}{2}y = 7\frac{1}{2}$ and multiply Equation B by $\frac{1}{3}$ to get $x - 3y = 6$. Subtracting one equation from the other eliminates $x$.

$$\begin{cases} x + \frac{3}{2}y = 7\frac{1}{2} & \text{Equation A2} \\ x - 3y = 6 & \text{Equation B2} \end{cases}$$

Each multiple of an original equation is equivalent to the original equation. So each new pair of equations is equivalent to the original system and has the same solution.

Let's solve the original system using the first equivalent system we found earlier.

$$\begin{cases} 6x + 9y = 45 & \text{Equation A1} \\ 3x - 9y = 18 & \text{Equation B} \end{cases}$$

- Adding the two equations eliminates $y$, leaving a new equation $9x = 63$, or $x = 7$.

$$\begin{array}{rl} 6x + 9y = 45 & \\ 3x - 9y = 18 & + \\ \hline 9x + 0 \ = 63 & \\ x = 7 & \end{array}$$

- Putting together $x = 7$ and the original $3x - 9y = 18$ gives us another equivalent system.

$$\begin{cases} x = 7 \\ 3x - 9y = 18 \end{cases}$$

- Substituting 7 for $x$ in the second equation allows us to solve for $y$.

$$\begin{aligned} 3(7) - 9y &= 18 \\ 21 - 9y &= 18 \\ \text{-}9y &= \text{-}3 \\ y &= \frac{1}{3} \end{aligned}$$

When we solve a system by elimination, we are essentially writing a series of **equivalent systems**, or systems with the same solution. Each equivalent system gets us closer and closer to the solution of the original system.

$$\begin{cases} 2x + 3y = 15 \\ 3x - 9y = 18 \end{cases} \qquad \begin{cases} 6x + 9y = 45 \\ 3x - 9y = 18 \end{cases} \qquad \begin{cases} x = 7 \\ 3x - 9y = 18 \end{cases} \qquad \begin{cases} x = 7 \\ y = \frac{1}{3} \end{cases}$$