# Lesson 11: Equations of All Kinds of Lines

## Goals

- Comprehend that for the graph of a vertical or horizontal line, one variable does not vary, while the other can take any value.
- Create multiple representations of linear relationship, including a graph, equation, and table.
- Generalize (in writing) that a set of points of the form $(x, b)$ satisfy the equation $y = b$ and that a set of points of the form $(a, y)$ satisfy the equation $x = a$.

## Learning Targets

- I can write equations of lines that have a positive or a negative slope.
- I can write equations of vertical and horizontal lines.

## Lesson Narrative

In previous lessons, students have studied lines with positive and negative slope and have learned to write equations for them, usually in the form $y = mx + b$. In this lesson, students extend their previous work to include equations for horizontal and vertical lines. Horizontal lines can still be written in the form $y = mx + b$ but because $m = 0$ in this case, the equation simplifies to $y = b$. Students interpret this to mean that, for a horizontal line, the $y$ value does not change, but $x$ can take *any* value. This structure is identical for vertical lines except that now the equation has the form $x = a$ and it is $x$ that is determined while $y$ can take any value.

Note that the equation of a vertical line cannot be written in the form $y = mx + b$. It can, however, be written in the form $Ax + By = C$ (with $B$ = 0). This type of linear equation will be studied in greater detail in upcoming lessons. In this lesson, students encounter a context where this form arises naturally: if a rectangle has length $\ell$ and width $w$ and its perimeter is 50, this means that $2\ell + 2w = 50$.

### Alignments

### Building On

- 7.G.A: Draw, construct, and describe geometrical figures and describe the relationships between them.

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.6: Use similar triangles to explain why the slope $m$ is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at $b$.

### Instructional Routines

- MLR2: Collect and Display
- MLR7: Compare and Connect
- Which One Doesn't Belong?

### Required Materials

**String**

### Required Preparation

Take a piece of string 50 centimeters long and tie the ends together to be used as demonstration in the third activity.

> **Student Learning Goals**
>
> Let's write equations for vertical and horizontal lines.

## 11.1 Which One Doesn't Belong: Pairs of Lines

**Warm Up: 5 minutes**

This warm-up prompts students to compare four pairs of lines. It invites students to explain their reasoning and hold mathematical conversations, and allows you to hear how they use terminology and talk about lines. To allow all students to access the activity, each figure has one obvious reason it does not belong. Encourage students to find reasons based on geometric properties (e.g., only one set of lines are not parallel, only one set of lines have negative slope).

### Building On

- 7.G.A

### Instructional Routines

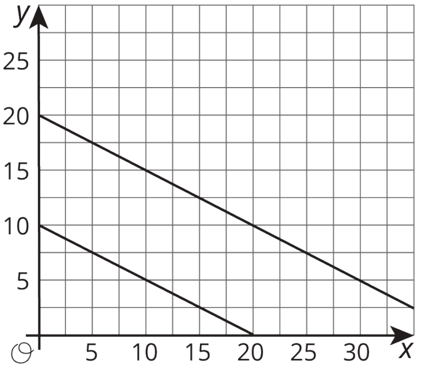
- Which One Doesn't Belong?

### Launch

Arrange students in groups of 2–4. Display the image of the four graphs for all to see. Ask students to indicate when they have noticed one graph that does not belong and can explain why. Give students 2 minutes of quiet think time and then time to share their thinking with their group. After everyone has conferred in groups, ask the group to offer at least one reason *each* graph doesn't belong.

> **Student Task Statement**
>
> Which one doesn't belong?

A

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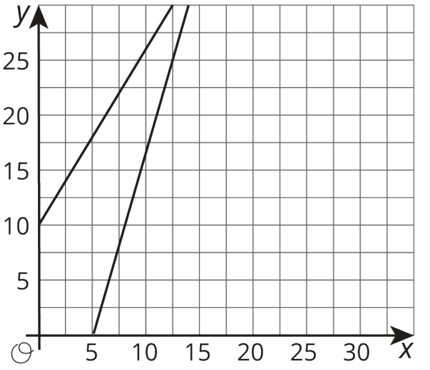


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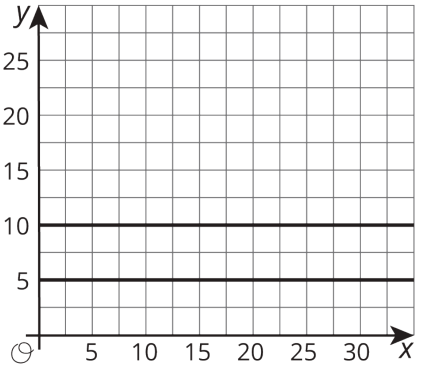


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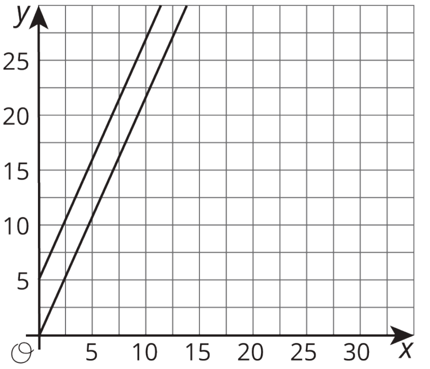


### Student Response

Answers vary. Some sample responses:

A: The lines have a negative slope.

B: The lines are not parallel.

C: The lines have 0 slope.

D: Neither line goes through the point $(0, 10)$.

### Activity Synthesis

After students have conferred in groups, invite each group to share one reason why a particular pair of lines might not belong. Record and display the responses for all to see. After each response, ask the rest of the class if they agree or disagree. Since there is no single correct answer to the question asking which shape does not belong, attend to students' explanations and ensure the reasons given are correct.

During the discussion, prompt students to explain the meaning of any terminology they use, such as parallel, intersect, origin, coordinate, ordered pair, quadrant or slope. Also, press students on claims of lines being parallel to one another. Ask students how they know they are parallel and highlight ideas about slope.

# 11.2 All the Same

**15 minutes (there is a digital version of this activity)**
In previous lessons, students have studied lines with positive slope, negative slope, and 0 slope and have written equations for lines with positive and negative slope. In this activity, they write equations for horizontal lines (lines of slope 0) and vertical lines and they graph horizontal and vertical lines from equations. Students explain their reasoning (MP3).

Horizontal lines can be thought of as being described by equations of the form $y = mx + b$ where $m = 0$. In other words, a horizontal line can be thought of as a line with slope 0. Vertical lines, on the other hand, cannot be described by an equation of the form $y = mx + b$.

### Addressing

- 8.EE.B.6

### Instructional Routines

- MLR2: Collect and Display

### Launch

Allow students quiet think time. Instruct them to pause their work after question 2 and discuss which equation makes sense and why. Tell students to resume working and pause again after question 4 for discussion. Discuss why the equations only contain one variable and what this means about the relationship between the quantities represented by $x$ and $y$. After students complete questions 5 and 6, ask if they can think of some real-world situations that can be represented by vertical and horizontal lines.

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**Access for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* Chunk this task into more manageable parts to support students who benefit from support with organizational skills. For example, present one question at a time.
*Supports accessibility for: Organization; Attention*

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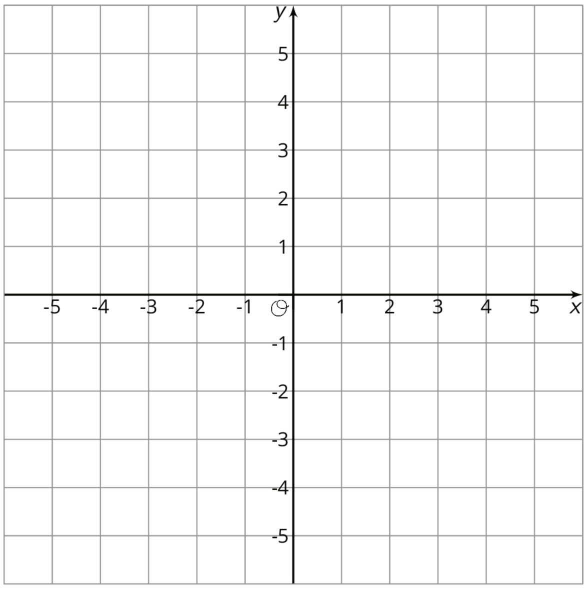
## Student Task Statement




1. Plot at least 10 points whose $y$-coordinate is -4. What do you notice about them?

2. Which equation makes the most sense to represent all of the points with $y$-coordinate -4? Explain how you know.

   $x = -4$ $\qquad$ $y = -4x$ $\qquad$ $y = -4$ $\qquad$ $x + y = -4$

3. Plot at least 10 points whose $x$-coordinate is 3. What do you notice about them?

4. Which equation makes the most sense to represent all of the points with $x$-coordinate 3? Explain how you know.

   $x = 3$ $\qquad$ $y = 3x$ $\qquad$ $y = 3$ $\qquad$ $x + y = 3$

5. Graph the equation $x = -2$.

6. Graph the equation $y = 5$.

## Student Response

1. Answers vary. Sample responses: Points all lie on a horizontal line that crosses the $y$-axis at -4. Points all lie on a line parallel to and 4 units down from $x$-axis.

2. $y = -4$ is the only equation that is true for every point we graphed and all points for which the $y$-coordinate is -4.

3. Answers vary. Sample responses: Points all lie on a vertical line that crosses the $x$-axis at 3. Points all lie on a line parallel to and 3 units to the right of the $y$-axis.

4. $x = 3$.

5. A vertical line through $(-2, 0)$.

6. A horizontal line through $(0, 5)$.

### Are You Ready for More?

1. Draw the rectangle with vertices $(2, 1)$, $(5, 1)$, $(5, 3)$, $(2, 3)$.

2. For each of the four sides of the rectangle, write an equation for a line containing the side.

3. A rectangle has sides on the graphs of $x = -1$, $x = 3$, $y = -1$, $y = 1$. Find the coordinates of each vertex.

### Student Response

1. Figure: Graph showing first quadrant, axes marked in integers from 1 to 6 horizontally and 1 to 4 vertically, points marked and labeled at $(2, 1)$, $(5, 1)$, $(5, 3)$, $(2, 3)$, points connected by horizontal and vertical lines to form a rectangle.

2. $x = 2$, $x = 5$, $y = 1$, $y = 3$.

3. $(-1, -1)$, $(3, -1)$, $(3, 1)$, $(-1, 1)$

### Activity Synthesis

In order to highlight the structure of equations of vertical and horizontal lines, ask students:

- "Why does the equation for the points with $y$-coordinate -4 not contain the variable $x$?" ($x$ can take any value while $y$ is always -4. The only constraint is on $y$ and there is no dependence of $x$ on $y$.)

- "Why does the equation for the points with $x$-coordinate 3 not contain the variable $y$?" ($y$ can take any value while $x$ is always 3. The only constraint is on $x$ and there is no dependence of $y$ on $x$.)

- "What does this say about the relationship between the quantities represented by $x$ and $y$ in these situations?" (Changes in one do not affect the other. One is not dependent on the other. They don't change together according to a formula.)

- "What would be some real-world examples of situations that could be represented by these types of equations?" (Examples: You pay the same fee regardless of your age; bus tickets cost the same no matter how far you travel; you remain the same distance from home as the hours pass during the school day.)

### Access for English Language Learners

*Representing, Speaking, Listening: MLR2 Collect and Display.* As students discuss which equation makes sense and why with a partner, create a 2-column table with the headings "horizontal lines" and "vertical lines". Circulate through the groups and record student language in the appropriate column. Look for phrases such as "$x$ (or $y$) is always the same," "$x$ (or $y$) is always changing," and "the slope is 0." This will help students make sense of the structure of equations for horizontal and vertical lines.
*Design Principle(s): Support sense-making; Maximize meta-awareness*

# 11.3 Same Perimeter

**15 minutes (there is a digital version of this activity)**
In this activity, students analyze a line and an equation defining the line in a geometric context. Students find pairs of numbers for the width and length of rectangles that all have the same perimeter. Next, they draw some of the rectangles with a vertex at the origin in the coordinate plane, and discover that the set of opposite vertices lie on a line. Finally, they write an equation for the line and consider how the slope of the line relates to the changing lengths and widths of the rectangles.

Identify students who come up with different equations for the line. An equation of the form $2\ell + 2w = 50$, where $\ell$ is the length of the rectangle and $w$ is its width, is natural if they are thinking about the context, namely, that the perimeter of the rectangle is 50 units. In fact, students have likely seen this equation in this context in grade 6. On the other hand, students who follow the steps in the task and draw the line connecting the rectangle vertices are likely to write an equation in the form $w = 25 - \ell$ (or perhaps $2w = 50 - 2\ell$) because the $w$-intercept of the graph is 25 and its slope is -1. Invite students who wrote these different forms for an equation to present during the discussion.

## Addressing

- 8.EE.B

## Instructional Routines

- MLR7: Compare and Connect

## Launch

Ask students to sketch a rectangle whose perimeter is 50 units and label the lengths of its sides. After giving them a minute to come up with their rectangle, ask them to share some of the lengths and widths they found. Examples might be 10 and 15, 5 and 20, or 1 and 24. Then demonstrate with a 50-cm long string with its ends tied together that a given perimeter can yield several different rectangles by varying the width and length. Ensure that everyone understands that rectangles have

4 sides, that rectangles have two pairs of congruent sides, and that there is more than one rectangle whose perimeter is 50 units.

### Access for Students with Disabilities

*Action and Expression: Develop Expression and Communication.* Maintain a display of important terms and vocabulary. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: width, length, perimeter, rectangle, intercept, and slope.
*Supports accessibility for: Memory; Language*

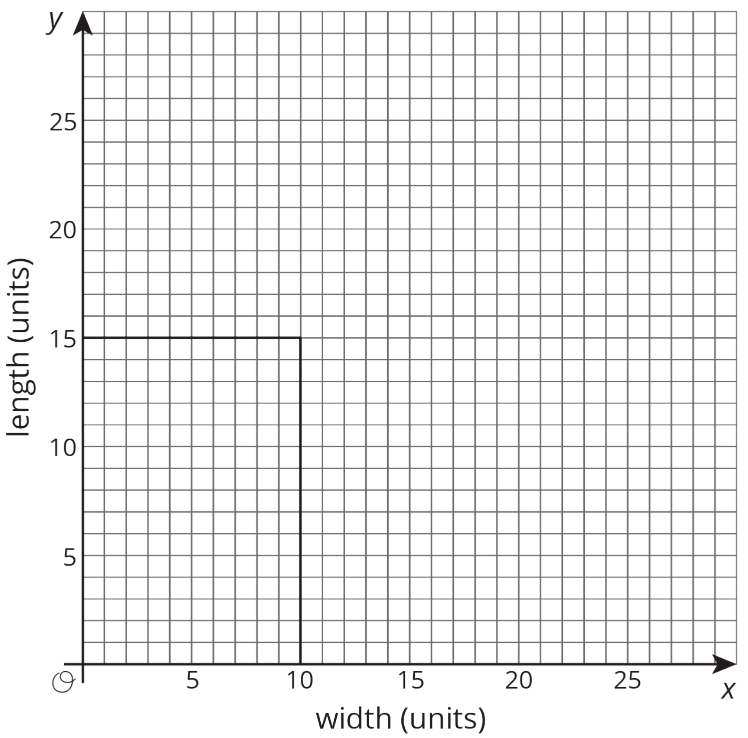
## Student Task Statement

1. There are many possible rectangles whose perimeter is 50 units. Complete the table with lengths, $\ell$, and widths, $w$, of at least 10 such rectangles.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\ell$ | | | | | | | | | | |
| $w$ | | | | | | | | | | |

2. The graph shows one rectangle whose perimeter is 50 units, and has its lower left vertex at the origin and two sides on the axes.

On the same graph, draw more rectangles with perimeter 50 units using the values from your table. Make sure that each rectangle has a lower left vertex at the origin and two sides on the axes.



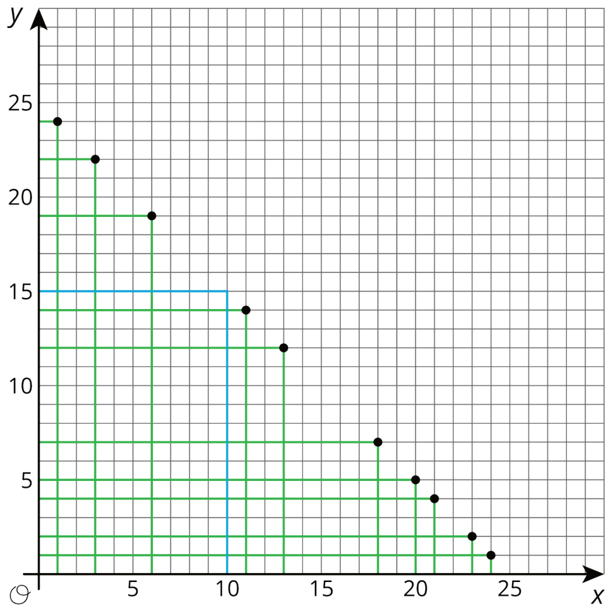
3. Each rectangle has a vertex that lies in the first quadrant. These vertices lie on a line. Draw in this line and write an equation for it.
4. What is the the slope of this line? How does the slope describe how the width changes as the length changes (or vice versa)?

### Student Response

1. Answers vary. Sample response:

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\ell$ | 1 | 4 | 7 | 12 | 24 | 19 | 2 | 5 | 14 | 22 |
| $w$ | 24 | 21 | 18 | 13 | 1 | 6 | 23 | 20 | 11 | 3 |

2. Answers vary. Sample response:




3. Answers vary. Some possibilities: $2x + 2y = 50$, $x + y = 25$, $y = 25 - x$, $y = -x + 25$
4. The slope is -1. Answers vary. Sample response: If you take away one from the width you have to add one to the length.

### Activity Synthesis

Invite students who write $2\ell + 2w = 50$ for an equation of the line to share their response (if no one has written this equation for the third question, ask students if it is correct). This equation has the advantage of directly modeling the context: the perimeter of a rectangle is $2\ell + 2w$, so if the perimeter is 50, then $2\ell + 2w = 50$. An additional advantage to this form of the equation is that *every* line, including horizontal and vertical lines, can be written in this form: $\ell = 5$ is an equation for a horizontal line while $w = 3$ is an equation for a vertical line.

Invite students who write $\ell = 25 - w$ (or some variant) to share. This equation is not apparent from the scenario, but it reveals a few interesting aspects of the problem:

- The rectangle has to have length less than 25 (since the width has to be positive)
- For each unit the length increases, the width decreases by one unit (in order to balance out when the sides of the rectangle are added to get the perimeter)

Ask students if the lengths and widths need to be whole numbers. In the next two lessons, students will encounter equations where the contexts determine what values the variables can take on. If students agree that the lengths and widths can take on any measurable value, ask how many different rectangles can be drawn. Practically speaking, the number is limited by what we can measure and draw with reasonable precision, while in theory, there are an infinite number of rectangles with perimeter 50.

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**Access for English Language Learners**

*Representing, Conversing: MLR7 Compare and Connect.* As the selected students show and describe their equations for the line connecting the vertices of the rectangles, invite pairs to discuss "What is the same and what is different?" about their own equation and reasoning. Highlight connections between the equations by amplifying use of the targeted language (i.e., horizontal and vertical lines, perimeter, slope, length is less than, or width is less than). This will help students to better understand that different forms of the equation represent the same geometrical context.
*Design Principle(s): Maximize meta-awareness; Support sense-making*

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## Lesson Synthesis

Students have spent considerable time in the 7th and 8th grades solving problems with proportional relationships and non-proportional relationships that can be represented by equations and graphs with positive slopes. Ask students to now consider real-world situations where slopes are not positive.

Ask students, "how can you tell from a real-world situation that the graph of the equation that represents it will be a horizontal line? Be a vertical line? Have a negative slope?"

For horizontal and vertical lines, the key feature is that one of the two variables does *not* vary while the other one can take *any* value. In the $x$-$y$ plane, when the variable $x$ can take any value, it is a vertical line, and when the variable $y$ can take any value, it is a horizontal line.

# 11.4 Line Design

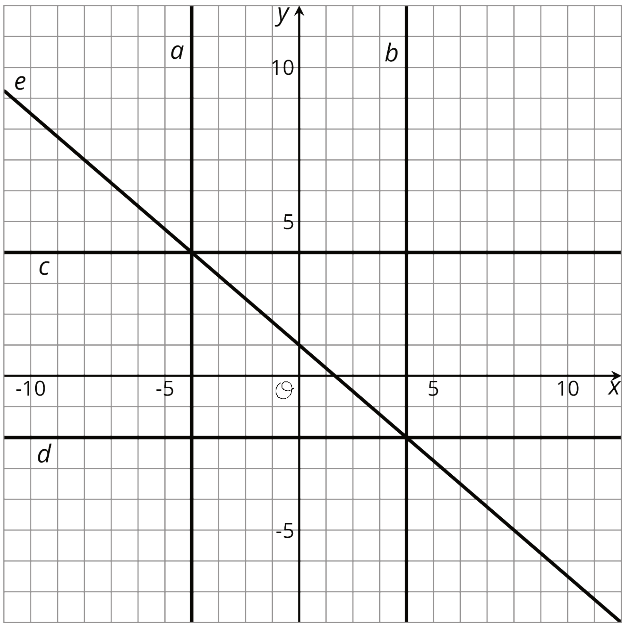
**Cool Down: 5 minutes**
Students write equations for lines that are horizontal, vertical, or have negative slope.

### Addressing

- 8.EE.B.6

### Student Task Statement

Here are 5 lines on a coordinate grid:




Write equations for lines $a$, $b$, $c$, $d$, and $e$.

### Student Response

a: $x = \text{-}4$ b: $x = 4$ c: $y = 4$ d: $y = \text{-}2$ e: $y = \frac{-3}{4}x + 1$ or $\frac{y-1}{x} = \frac{-3}{4}$ (or equivalent equation)

## Student Lesson Summary

Horizontal lines in the coordinate plane represent situations where the $y$ value doesn't change at all while the $x$ value changes. For example, the horizontal line that goes through the point $(0, 13)$ can be described in words as "for all points on the line, the $y$ value is always 13." An equation that says the same thing is $y = 13$.

Vertical lines represent situations where the $x$ value doesn't change at all while the $y$ value changes. The equation $x = \text{-}4$ describes a vertical line through the point $(\text{-}4, 0)$.

# Lesson 11 Practice Problems

## Problem 1

## Statement

Suppose you wanted to graph the equation $y = \text{-}4x - 1$.

a. Describe the steps you would take to draw the graph.

b. How would you check that the graph you drew is correct?

## Solution

Answers vary. Sample response:

a. Start with the intercept $(0, -1)$, and use the slope of $-4$ to move down 4 and 1 to the right (or up 4 and 1 to the left) to find other points. Or, find two or more solutions to the equation and graph the points whose coordinates are the ordered pairs of the solutions, then draw a line connecting the points.

b. Check the intercept and slope. Identify the coordinates of some points on the line, and substitute them into the equation to make sure they make the equation true.

## Problem 2

### Statement

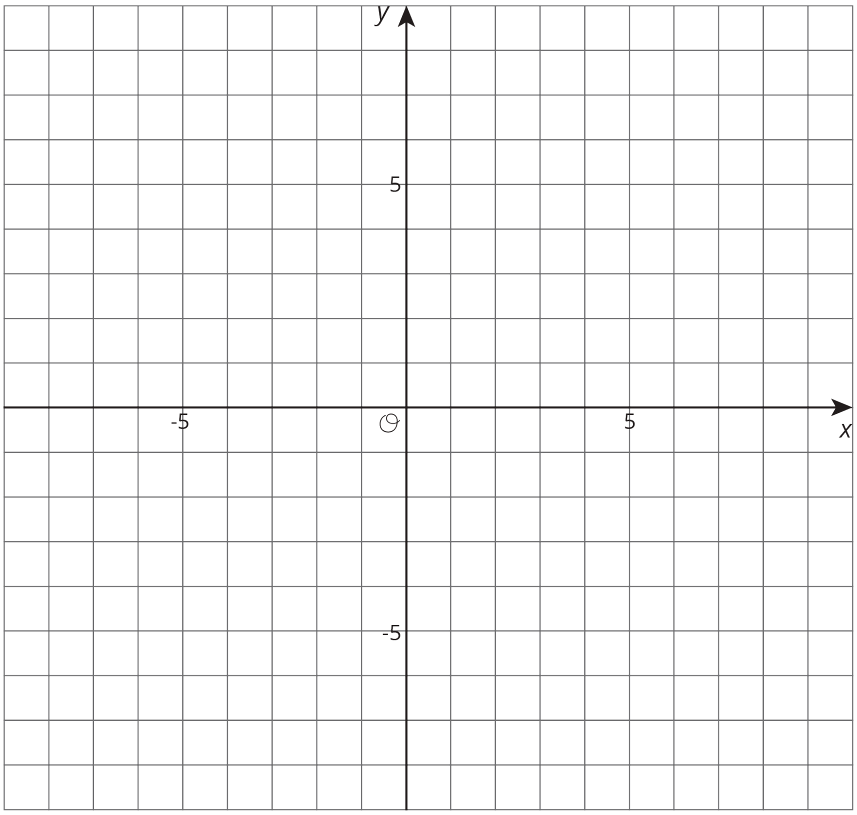
Draw the following lines and then write an equation for each.

a. Slope is 0, $y$-intercept is 5

b. Slope is 2, $y$-intercept is -1

c. Slope is -2, $y$-intercept is 1

d. Slope is $-\frac{1}{2}$, $y$-intercept is -1



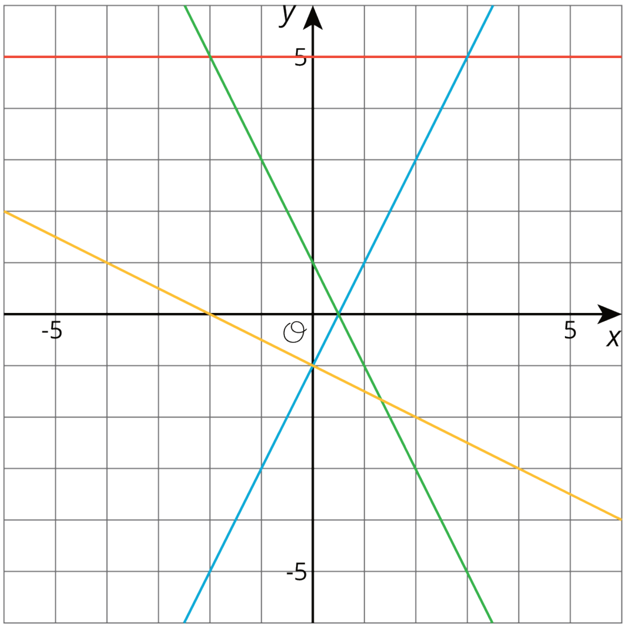

## Solution

a. Red line $y = 5$

b. Blue line $y = 2x - 1$ or equivalent

c. Green line $y = -2x + 1$ or equivalent

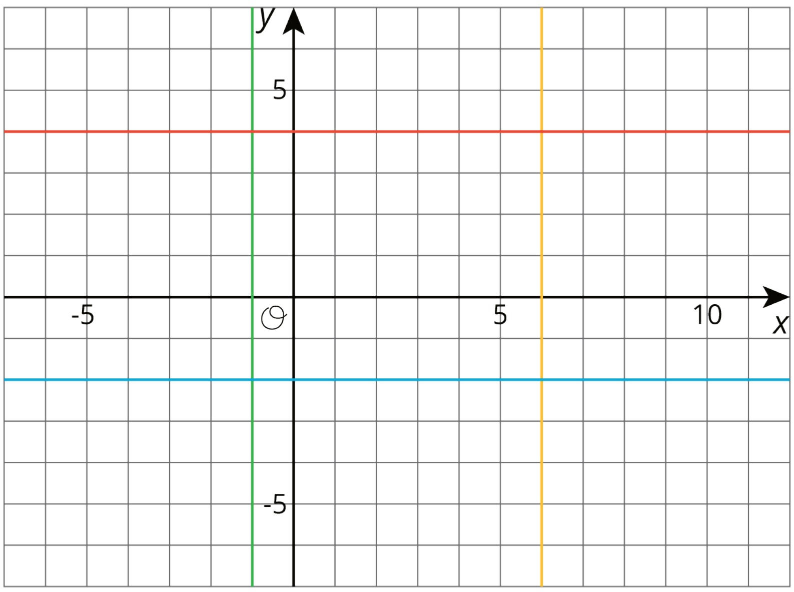
d. Yellow line $y = -\frac{1}{2}x - 1$ or equivalent




## Problem 3

### Statement

Write an equation for each line.




### Solution

Green line: $x = -1$, yellow line: $x = 6$, red line: $y = 4$, blue line: $y = -2$

## Problem 4

### Statement

A publisher wants to figure out how thick their new book will be. The book has a front cover and a back cover, each of which have a thickness of $\frac{1}{4}$ of an inch. They have a choice of which type of paper to print the book on.

a. Bond paper has a thickness of $\frac{1}{4}$ inch per one hundred pages. Write an equation for the width of the book, $y$, if it has $x$ hundred pages, printed on bond paper.

b. Ledger paper has a thickness of $\frac{2}{5}$ inch per one hundred pages. Write an equation for the width of the book, $y$, if it has $x$ hundred pages, printed on ledger paper.

c. If they instead chose front and back covers of thickness $\frac{1}{3}$ of an inch, how would this change the equations in the previous two parts?

## Solution

a. $y = \frac{1}{2} + \frac{1}{4}x$

b. $y = \frac{1}{2} + \frac{2}{5}x$

c. $y = \frac{2}{3} + \frac{1}{4}x$ and $y = \frac{2}{3} + \frac{2}{5}x$, respectively

(From Unit 3, Lesson 7.)