Lesson 4: Applying Circumference

Goals

- Apply understanding of circumference to calculate the perimeter of a shape that includes circular parts, and explain (orally and in writing) the solution method.
- Compare and contrast (orally) values for the same measurements that were calculated using different approximations for π.
- Explain (orally) how to calculate the radius, diameter, or circumference of a circle, given one of these three measurements.

Learning Targets

- I can choose an approximation for π based on the situation or problem.
- If I know the radius, diameter, or circumference of a circle, I can find the other two.

Lesson Narrative

In this lesson, students use the equation $C = \pi d$ to solve problems in a variety of contexts. They compute the circumference of circles and parts of circles given diameter or radius, and vice versa. Students develop flexibility using the relationships between diameter, radius, and circumference rather than memorizing a variety of formulas. Understanding the equation $C = 2\pi r$ will help with the transition to the study of area in future lessons.

Students think strategically about how to decompose and recompose complex shapes (MP7) and need to choose an appropriate level of precision for π and for their final calculations (MP6).

Alignments

Addressing

- 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR5: Co-Craft Questions
- MLR7: Compare and Connect
- MLR8: Discussion Supports
- Notice and Wonder

- Think Pair Share

Required Materials

Four-function calculators

> ### Student Learning Goals
> Let's use π to solve problems.

4.1 What Do We Know? What Can We Estimate?

Warm Up: 5 minutes

The purpose of this warm-up is for students to reason about the various measurements of a circle. In each of the pictures, students are given a piece of information about the circle and asked to reason about the other measurements of the circle. Some of these measurements can be found based on the given information while others, without a calculator, would require an estimate.

Addressing

- 7.G.B.4

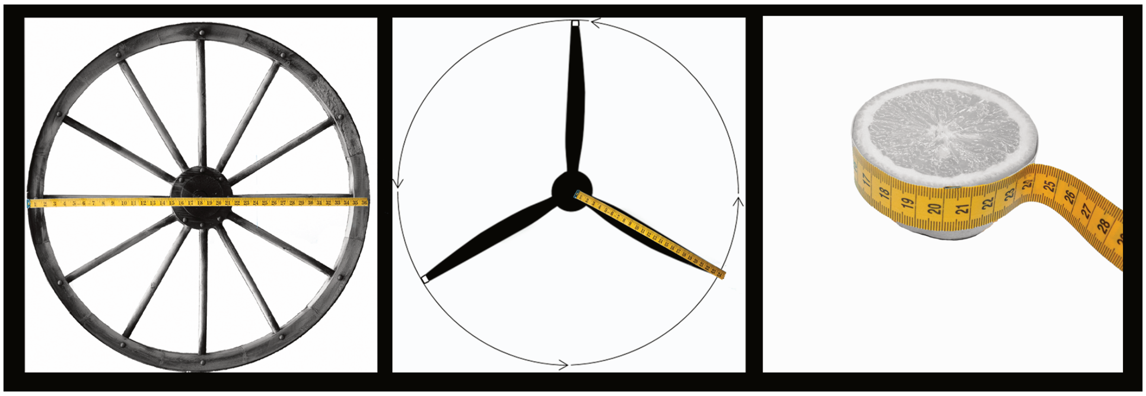
Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

Arrange students in groups of 2. Give students 1 minute of quiet think time followed by 2 minutes to discuss their estimates with a partner.

> ### Student Task Statement
> Here are some pictures of circular objects, with measurement tools shown. The measurement tool on each picture reads as follows:
>
> - Wagon wheel: 3 feet
> - Plane propeller: 24 inches
> - Sliced Orange: 20 centimeters



1. For each picture, which measurement is shown?
2. Based on this information, what measurement(s) could you estimate for each picture?

Student Response

1.
    - For the wagon wheel, the measurement of the diameter is shown.
    - For the plane propeller, the measurement of the radius is shown.
    - For the sliced orange, the circumference is shown.
2.
    - For the wagon wheel, I could estimate the circumference.
    - For the plane propeller, I could estimate the circumference.
    - For the sliced orange, I could estimate the diameter and the radius.

Activity Synthesis

For each picture, ask selected students to share the measurements shown in the pictures. Ask students to explain the strategies or calculations they would use for computing or estimating other measurements (for example, the radius and circumference of the wheel). Record and display student responses on the actual pictures. Refer to MLR 2 (Collect and Display) to highlight student responses and language about what they know and what they estimated.

4.2 Using π

10 minutes (there is a digital version of this activity)

In the previous lesson, students identified the constant of proportionality relating the circumference and diameter of a circle. The purpose of this activity is for students to calculate measurements of a circle using different approximations of π. Students are given either the radius, diameter, or circumference of a circle and use calculators to compute the other two measurements. Different students use different approximations of π: $\frac{22}{7}$, 3.14, and 3.1415927. The last approximation for π is the precision that many calculators show.

The different approximations for π lead to different estimates for the missing measurements, and they should be encouraged to think about which approximation is the most useful in these cases.

Addressing

- 7.G.B.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Divide students into 3 groups. Assign each group a different approximation for π to use in their calculations: 3.1415927, 3.14, and $\frac{22}{7}$. Give students 4–5 minutes of quiet work time followed by whole-class discussion.

If using the digital activity, students can work in small groups to complete the task. The applet demonstrates the relationship between rotation and circumference.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organizational skills in problem solving. For example, after students have completed the first 1–2 rows of the table, check for understanding before moving on.
Supports accessibility for: Organization; Attention

Student Task Statement

In the previous activity, we looked at pictures of circular objects. One measurement for each object is listed in the table.

Your teacher will assign you an approximation of π to use for this activity.

1. Complete the table.

object	radius	diameter	circumference
wagon wheel		3 ft	
airplane propeller	24 in		
orange slice			20 cm

2. A bug was sitting on the tip of the propeller blade when the propeller started to rotate. The bug held on for 5 rotations before flying away. How far did the bug travel before it flew off?

Student Response

1.

object	radius	diameter	circumference
wagon wheel	1.5 ft	3 ft	9.4247781 ft 9.42 ft $9\frac{3}{7}$ ft
airplane propeller	24 in	48 in	150.7964496 in 150.72 in $150\frac{6}{7}$ in
orange slice	3.183098815 cm 3.184713376 cm $3\frac{2}{11}$ cm	6.36619763 cm 6.36942675 cm $6\frac{4}{11}$ cm	20 cm

2. The bug travels about 754 inches, because $(150.7964496) \cdot 5 = 753.982248$.

Activity Synthesis

Compare the measurements that students calculated using the different approximations of π. Point out that while the approximation for π influences the values of the radius and diameter, it does not affect the relationship $d = 2 \cdot r$. Use the circumference of the wagon wheel to point out that all three results (9.4285714 ft, 9.42 ft, and 9.4247781 ft) agree in the first three digits, i.e., to within a hundredth of a foot. Explain that even when using 3.1415927, the measurements are still approximations.

Discuss whether 9.4247781 ft for the circumference of a wagon wheel or 3.183098815 cm for the radius of an orange slice is a reasonable number of decimal places to report the measurement. Could these objects actually be measured that precisely? Explain that people use different approximations for π depending on the situation and the precision of the measurement. For situations like these where the measurements themselves do not have too much accuracy either 3.14 or $\frac{22}{7}$ is probably the most appropriate value of π to use. Using a more accurate value for π is always acceptable, but the final answer should not be reported with more accuracy than the measurements.

When working with circles, sometimes it is more natural to work with the diameter and sometimes with the radius. But we can always go quickly from one to the other, if needed. Emphasize the progression in solving the propeller problem: use the radius to get the diameter, then the diameter to get the circumference. It is not necessary for students to learn the rule $C = 2\pi r$, but this would be the place to do so if desired.

Access for English Language Learners

Speaking, Writing: MLR7 Compare and Connect. As students complete their calculations in small groups, ask them to leave their work out on display. Ask students to do a gallery walk around the room to examine the work of their peers, and consider why, specifically, each approximation for π did or did not produce a different result. Ask students, “How does (this) response compare with your team’s?” “What do the answers have in common?” (For example, up to what decimal place.) This routine will help students to focus the discussion on why one approximation did or did not produce a different result.
Design Principle(s): Support sense-making; Cultivate conversation

4.3 Around the Running Track

Optional: 15 minutes
In this activity, students compute the length of a figure that is composed of half-circles and straight line segments. For the first question, they are given the length of the line segments and the diameter of the circle. For the second question, students have to compute the diameter of the circle.

As students work, monitor for students who decomposed the figure in different ways and used different approximations of π. The discussion focuses on whether the differences lead to meaningful differences in the estimate for the distance around the track.

Addressing

- 7.G.B.4

Instructional Routines

- MLR5: Co-Craft Questions
- Think Pair Share

Launch

If desired, introduce the context of a running track. Allow students to choose what approximation to use for π. Quiet work time followed by partner discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Provide students with a graphic organizer to organize the information provided in the problem and to structure their problem-solving strategy. The graphic organizer should include the prompts: “What do I need to find out?”, “What do I need to do?”, and “How I solved the problem.”
Supports accessibility for: Language; Organization

Access for English Language Learners

Speaking, Writing: MLR5 Co-Craft Questions. To help students understand the situation, display the image of the running track, and ask students to write down possible mathematical questions they could ask. Listen for, and amplify language students use to describe different parts of the track (inside and outside), half-circles, straight line segments, etc. Then, invite pairs to share their questions with the class. This will help students produce the language of mathematical questions, and support common understanding of the situation.
Design Principle(s): Maximize meta-awareness; Support sense-making

Anticipated Misconceptions

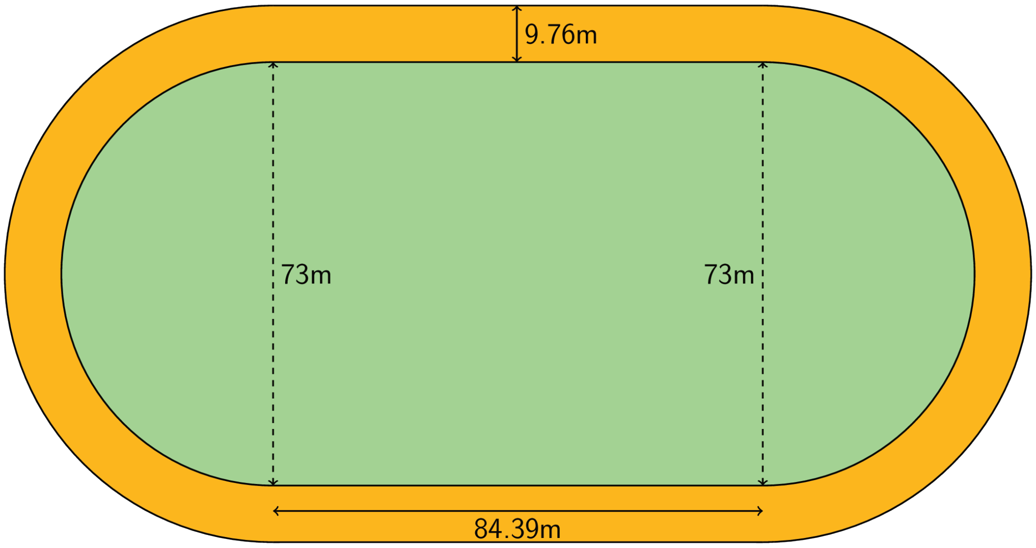
If students have trouble seeing the circle and rectangle that compose the figure, suggest that they draw additional lines to decompose the figure.

Students might apply the formula for the circumference of a circle to find the circumference of the oval-shaped field and track. If this happens, remind students that the track is not in the shape of a circle. Ask if they see a way to form a circle and a rectangle within the space.

Students who are familiar with the fact that this size track is referred to as a 400-meter track may be confused when their answer does not equal 400 meters. Explain that a runner does not run right on the edge of the track and possibly direct the student to look at the extension.

Student Task Statement

The field inside a running track is made up of a rectangle that is 84.39 m long and 73 m wide, together with a half-circle at each end.



1. What is the distance around the inside of the track? Explain or show your reasoning.
2. The track is 9.76 m wide all the way around. What is the distance around the outside of the track? Explain or show your reasoning.

Student Response

1. The inside of the track is 398 m long. The distance around each half-circle is 114.61 m, because $73 \cdot 3.14 \div 2 = 114.61$. Add two line segments and two half-circles: $84.39 + 84.39 + 114.61 + 114.61 = 398$.
2. The outside of the track is 459.3 m long. The diameter of the larger half-circles is 92.52 m, because $73 + 9.76 + 9.76 = 92.52$. The distance around the large half-circle is 145.26 m, because $92.52 \cdot 3.14 \div 2 = 145.26$. Add two line segments and two half-circles: $84.39 + 84.39 + 145.26 + 145.26 = 459.3$.

Are You Ready for More?

This size running track is usually called a 400-meter track. However, if a person ran as close to the "inside" as possible on the track, they would run less than 400 meters in one lap. How far away from the inside border would someone have to run to make one lap equal exactly 400 meters?

Student Response

The length of the straight part of the track is not affected by the distance from the border that a person runs. Excluding the straight parts, the rest of the distance is 231.22 meters, because $400 - 2 \cdot 84.39 = 231.22$. The half-circles must have a diameter of 73.6 meters, because $231.22 \div \pi \approx 73.6$. The runner must run 0.3 meters in from the inside border of the track, because $(73.6 - 73) \div 2 = 0.3$.

Activity Synthesis

Most of the discussion will occur in small groups. However, the whole class can debrief on the following questions:

- "In what ways did you decompose the figure into different shapes?"
- "Why did you choose a particular approximation for π, and what was the resulting answer?"
- "How are different students' answers related, and are they all reasonable lengths for this situation?"

Students who use more digits for their approximation of π may come up with a slightly different answer, such as 398.1. In general, when making calculations, if only an estimate is desired, then using 3.14 for π is usually good enough. In a situation like this, where the given measurements are quite precise, it can be worth trying more digits in the expansion of π, but it turns out not to make much difference in this case.

4.4 Measuring a Picture Frame

10 minutes

The purpose of this activity is to calculate the length of a complex shape made out of parts of circles. Students are given a drawing of a picture frame that is made up of half-circles and three-quarter circles and are asked to find the total length required to make the frame out of wire. Next, students are asked what the radius would be of a circle with a circumference equal to the picture frame's perimeter. This activity marks the first time that the term *perimeter* is used in the context of circumference.

As students work, monitor and select students who approach the problem differently to share their solution methods in the discussion. There are many different methods to calculate the perimeter of the wire that show different ways of thinking about the circles and combining them (MP7): 10 full circles, 7 full circles from the half-circles and 3 full circles from the $\frac{3}{4}$ circles, 18 separate parts of circles.

Addressing

- 7.G.B.4

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR8: Discussion Supports
- Notice and Wonder

Launch

Display the image from the task statement and ask students to share what they notice and what they wonder. Confirm that the shapes are half-circles and $\frac{3}{4}$ of a circle in each corner. Also, tell them that all of these circles are the same size.

Give students quiet work time followed by whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Chunk this task into more manageable parts to support students who benefit from support with organization and problem solving. For example, break the first question into multiple parts. First, ask students to calculate the perimeter of the half circles on each side of the picture frame. Next, ask students to calculate the perimeter of the three-quarter circles at each corner of the picture frame. Lastly, ask students to calculate the total perimeter of the wire around the picture frame.
Supports accessibility for: Organization; Attention

Access for English Language Learners

Reading, Conversing: MLR8 Discussion Supports. Ask students to describe how the wire picture frame and rectangle are arranged. Students should work with a partner and take turns speaking/listening. Tell students, "Show your partner the shapes you notice in the picture. Draw the shapes in the air, explaining to your partner what shapes you see." Point out that the rectangle's measurements are listed. Say, "Explain to your partner how you can use the measurements listed with the shapes you see. How are the corner circles different?" This should help students focus in on the formulas they want to use to solve and how to justify their reasons for their choices.
Design Principle(s): Support sense-making; Maximize meta-awareness

Anticipated Misconceptions

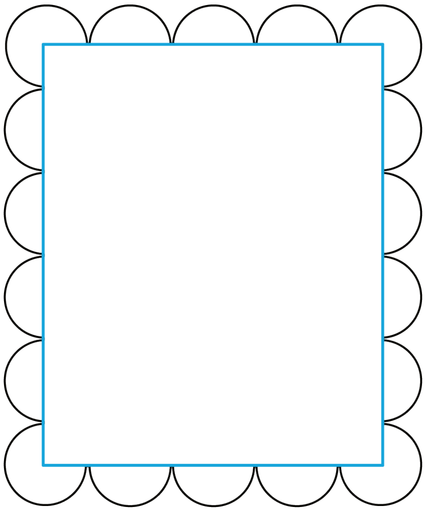
Some students may not have realized the connection between circumference and perimeter and will need to be prompted to use the circumference of the circular pieces to find the perimeter of the wire frame.

Students might try to include the perimeter of the rectangle when calculating the perimeter of the wire frame. Explain that we are only looking for the length of all the circular pieces.

Once they have figured out the size of each half-circle's diameter, it might be challenging for some students to imagine the frame being stretched to form one complete circle. Suggest that the entire length of the frame becomes the circumference of the complete circle. It might be helpful to have some string and an index card available for students to explore the idea of bending and stretching the frame.

Student Task Statement

Kiran bent some wire around a rectangle to make a picture frame. The rectangle is 8 inches by 10 inches.



1. Find the perimeter of the wire picture frame. Explain or show your reasoning.
2. If the wire picture frame were stretched out to make one complete circle, what would its radius be?

Student Response

1. Each half-circle has a diameter of 2 inches, because $8 \div 4 = 2$. Each half-circle has a length of about 3.14 inches, because $2 \cdot \pi \div 2 \approx 3.14$. Each three-quarters circle has a length of about 4.71 inches, because $2 \cdot \pi \cdot 0.75 \approx 4.71$. The frame's perimeter is about 62.8 inches, because it has 14 half-circles and 4 three-quarters circles, and $14\pi + 4.71 \cdot 4 \approx 62.8$.
2. The radius of the circle made out of the frame would be about 10 inches, because $62.8 \div \pi \div 2 \approx 10$.

Activity Synthesis

Ask the students how they used the dimensions of the picture frame to find the radii of the circles. The long side, for example, is made up of 4 diameters and 2 radii of circles. That is the same as 10 radii so if that side measures 10 inches, then the radius of the circle is 1 inch.

Ask select students to share different ways they decomposed the wire frame into parts. Possible methods include:

- 14 half-circles + 4 three-quarter circles
- 7 full circles + 4 three-quarter circles

- 7 full circles + 3 full circles
- 10 full circles

Ask students to discuss how they used the circumference of the circle in their work. If it does not arise in discussing the second question, make explicit the idea that knowing the circumference of the circle allows you to work backwards to find the circle's radius.

Lesson Synthesis

The main ideas are:

- The proportional relationship between diameter and circumference of a circle can be applied in more complex situations that require multi-step solutions.
- Because the diameter is twice the radius, we can write the relationship between the circumference of a circle and its radius like this: $C = 2\pi r$.

Discussion questions:

- What are some approximations of π?
- If I know the radius of a circle, how do I find its diameter and circumference?
- What if I know the circumference? How do I find diameter or radius?

4.5 Circumferences of Two Circles

Cool Down: 5 minutes

Addressing

- 7.G.B.4

Anticipated Misconceptions

Some students may not notice that the radius was given for Circle B rather than the diameter. They will likely answer the first question incorrectly, but they may still get the correct answer of about 9.42 cm for the second question, because $12 - 9 = 3$ and also $9 - 6 = 3$.

Student Task Statement

Circle A has a diameter of 9 cm. Circle B has a radius of 5 cm.

1. Which circle has the larger circumference?
2. About how many centimeters larger is it?

Student Response

1. Circle B has the larger circumference. Circle A has a diameter of 9 cm, and Circle B has a diameter of $5 \cdot 2$, or 10 cm. Since Circle B's diameter is larger than Circle A's diameter, and circumference is proportional to diameter, that means Circle B's circumference is also larger.

2. The difference is about 3.14 cm because the circumference of Circle A is 9π, or about 28.26 cm, and the circumference of Circle B is 10π, or about 31.4 cm. The difference is $31.4 - 28.26$, or about 3.14 cm.

Student Lesson Summary

The circumference of a circle, C, is π times the diameter, d. The diameter is twice the radius, r. So if we know any one of these measurements for a particular circle, we can find the others. We can write the relationships between these different measures using equations:

$$d = 2r$$
$$C = \pi d$$
$$C = 2\pi r$$

If the diameter of a car tire is 60 cm, that means the radius is 30 cm and the circumference is $60 \cdot \pi$ or about 188 cm.

If the radius of a clock is 5 in, that means the diameter is 10 in, and the circumference is $10 \cdot \pi$ or about 31 in.

If a ring has a circumference of 44 mm, that means the diameter is $44 \div \pi$, which is about 14 mm, and the radius is about 7 mm.

Lesson 4 Practice Problems

Problem 1

Statement

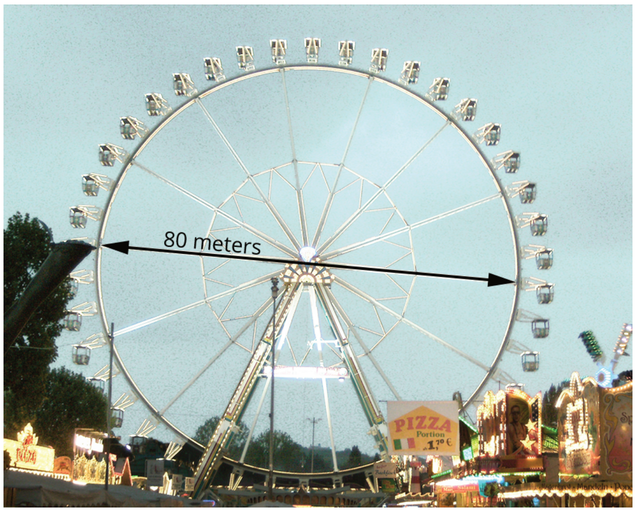
Here is a picture of a Ferris wheel. It has a diameter of 80 meters.

a. On the picture, draw and label a diameter.

b. How far does a rider travel in one complete rotation around the Ferris wheel?

Solution

a. Answers vary. Possible response:



b. In one complete rotation, a rider travels the circumference of the Ferris wheel. This distance is $80 \cdot \pi$, or about 251 meters. Since the gondola where the rider is seated is a little bit further from the center of the Ferris wheel than 40 meters, the distance the rider travels is actually a little more.

Problem 2

Statement

Identify each measurement as the diameter, radius, or circumference of the circular object. Then, estimate the other two measurements for the circle.

a. The length of the minute hand on a clock is 5 in.

b. The distance across a sink drain is 3.8 cm.

c. The tires on a mining truck are 14 ft tall.

d. The fence around a circular pool is 75 ft long.

e. The distance from the tip of a slice of pizza to the crust is 7 in.

f. Breaking a cookie in half creates a straight side 10 cm long.

g. The length of the metal rim around a glass lens is 190 mm.

h. From the center to the edge of a DVD measures 60 mm.

Solution

a. Radius; diameter: 10 in, circumference: about 31 in

b. Diameter; radius: 1.9 cm, circumference: about 12 cm

c. Diameter; radius: 7 ft, circumference: about 44 ft

d. Circumference; diameter: about 24 ft, radius: about 12 ft

e. Radius; diameter: 14 in, circumference: about 44 in

f. Diameter; radius: 5 cm, circumference: about 31 cm

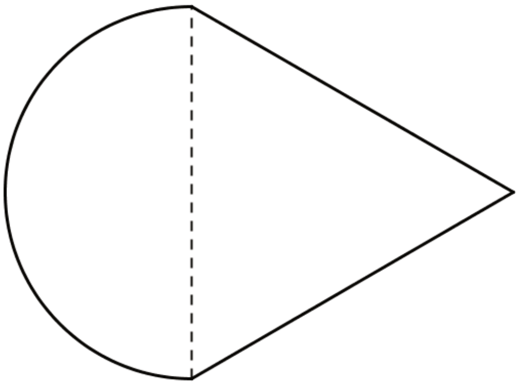
g. Circumference; diameter: about 60 mm, radius: about 30 mm

h. Radius; diameter: 120 mm, circumference: about 380 mm

Problem 3

Statement

A half circle is joined to an equilateral triangle with side lengths of 12 units. What is the perimeter of the resulting shape?



Solution

about 42.84 units. The two sides of the triangle each contribute 12 units and the semi-circle has a perimeter of $6 \cdot \pi$ or about 18.84 units.

Problem 4

Statement

Circle A has a diameter of 1 foot. Circle B has a circumference of 1 meter. Which circle is bigger? Explain your reasoning. (1 inch = 2.54 centimeters)

Solution

Circle B is bigger. Answers vary. Possible explanation: There are 12 inches in 1 foot. The circumference of Circle A is about 95.8 cm because $1 \cdot 12 \cdot 2.54 \cdot \pi \approx 95.8$. The circumference of Circle B is 100 cm because there are 100 cm in 1 m.

Problem 5

Statement

The circumference of Tyler's bike tire is 72 inches. What is the diameter of the tire?

Solution

$72 \div \pi$ or about 23 inches.

(From Unit 3, Lesson 3.)