Lesson 1: How Well Can You Measure?

Goals

- Create and describe (in writing) graphs that show measurements of squares.

- Justify (orally and in writing) whether the relationship shown on a graph is close enough to a straight line through the origin that it might be a proportional relationship with some measurement error.

- Recognize that when we measure the quantities in a proportional relationship, measurement error can cause the graph to be not perfectly straight and the quotients to be not exactly constant.

Learning Targets

- I can examine quotients and use a graph to decide whether two associated quantities are in a proportional relationship.

- I understand that it can be difficult to measure the quantities in a proportional relationship accurately.

Lesson Narrative

The purpose of this lesson is for students to apply what they have learned about proportional relationships to describing geometric figures. The work in this lesson focuses on squares. In the first activity, students see that there is a proportional relationship between the length of the diagonal and the perimeter for squares of different sizes. They use a graph and a table to estimate the constant of proportionality and recognize that measurement error means they can only find an approximate value. This prepares students for future lessons when they will explore the relationship between diameter and circumference of circles.

In the second activity, students see that even taking measurement error into account, the relationship between the length of the diagonal and the area of a square is not a proportional relationship, in preparation for investigating area of circles in future lessons.

Alignments

Building On

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Addressing

- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Building Towards

- 7.G.B.4: Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

- 7.RP.A.3: Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Copies of blackline master   **Rulers marked with centimeters**
Four-function calculators

Required Preparation

Make enough copies of the Perimeter of a Square blackline master for each group of 3 students to get one copy. Prepare to distribute rulers.

> **Student Learning Goals**
>
> Let's see how accurately we can measure.

1.1 Estimating a Percentage

Warm Up: 5 minutes
This warm-up gets students ready to think about making estimates of quotients. Estimation will be central in how students learn about the circumference and area of a circle.

Building On

- 6.RP.A.3.c

Building Towards

- 7.RP.A.3

Launch

Instruct students to find a method of estimation other than performing long division.

> **Student Task Statement**
>
> A student got 16 out of 21 questions correct on a quiz. Use mental estimation to answer these questions.
>
> 1. Did the student answer less than or more than 80% of the questions correctly?
>
> 2. Did the student answer less than or more than 75% of the questions correctly?

Student Response

1. Less than 80%. Sample reasoning:
   - 16 is 80% of 20, so 16 out of 21 will be less than 80%.

   - 80% of 21 is 80% of 20 and 80% of 1. That makes $16\frac{4}{5}$ so 16 out of 21 is less than 80%.

2. More than 75%. Sample reasoning:
   - 75% is 3 out of 4: $\frac{3}{4}$ of 21 is $\frac{3}{4}$ of 20 and $\frac{3}{4}$ of 1, which is $15\frac{3}{4}$ so 16 out of 21 is more than 75%.

   - 15 is 75% of 20. If you have made 15 free throws out of 20 and then you make one more, your percentage of free throws made should go up. That means that 16 out of 21 is more than 75%.

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their explanations for all to see. Ask students if or how the factors in the problem impacted the strategy choice. To involve more students in the conversation, consider asking:

- "Who can restate ___'s reasoning in a different way?"

- "Did anyone solve the problem the same way but would explain it differently?"

- "Did anyone solve the problem in a different way?"

- "Does anyone want to add on to _____'s strategy?"

- "Do you agree or disagree? Why?"

1.2 Perimeter of a Square

15 minutes (there is a digital version of this activity)
In this activity, students examine the relationship between the length of the diagonal and the perimeter for squares of different sizes. This prepares students for examining the relationship between the diameter and circumference of circles in a future lesson.

Students graph their measurements and find that the points look like they are close to lying on a line through the origin, suggesting that there is a proportional relationship between these quantities. This makes sense because the squares can all be viewed as scale copies of one square and both the side length and the diagonal should change by the same scale factor.

Because of measurement error, the points do not lie exactly on a line. Whenever analyzing proportional relationships through experimentation, small errors can be expected.

Addressing
- 7.RP.A.2.a

Building Towards

- 7.G.B.4

Launch

Arrange students in groups of 3. Distribute copies of the blackline master and rulers.

Explain that students will only work with 3 of the squares for now and can fill in the other rows of the table at the end of the activity. Assign each group to work with either squares A, B, C, squares D, E, F, or squares G, H, I.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to talk about what they notice in the graph with their group before writing their ideas down. Display sentence frames to support students when they explain their ideas and ask peers for their ideas. For example, "It looks like....", "I notice that....", and "What do you notice?"
Supports accessibility for: Language; Organization

Student Task Statement

Your teacher will give you a picture of 9 different squares and will assign your group 3 of these squares to examine more closely.

1. For each of your assigned squares, measure the length of the diagonal and the perimeter of the square in centimeters.

   Check your measurements with your group. After you come to an agreement, record your measurements in the table.

	diagonal (cm)	perimeter (cm)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		

2. Plot the diagonal and perimeter values from the table on the coordinate plane.
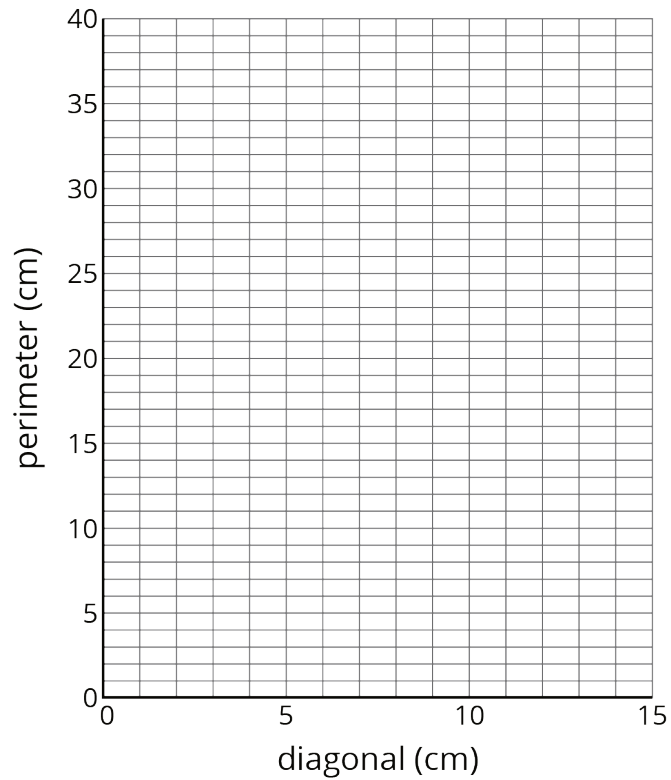
3. What do you notice about the points on the graph?

Pause here so your teacher can review your work.

4. Record measurements of the other squares to complete your table.

Student Response

1. Answers vary. Each group will only have 3 of these rows filled in, and the numbers may be slightly different. The important thing is that the perimeter of each square should be a little less than 3 times the length of the diagonal. Sample responses:

	diagonal (cm)	perimeter (cm)
square A	5.7	16.1
square B	8.2	23.2
square C	3.4	9.6
square D	4.2	11.9
square E	6.8	19.2
square F	10.5	30
square G	2.2	6.2
square H	12.6	35.2
square I	5.5	15.6
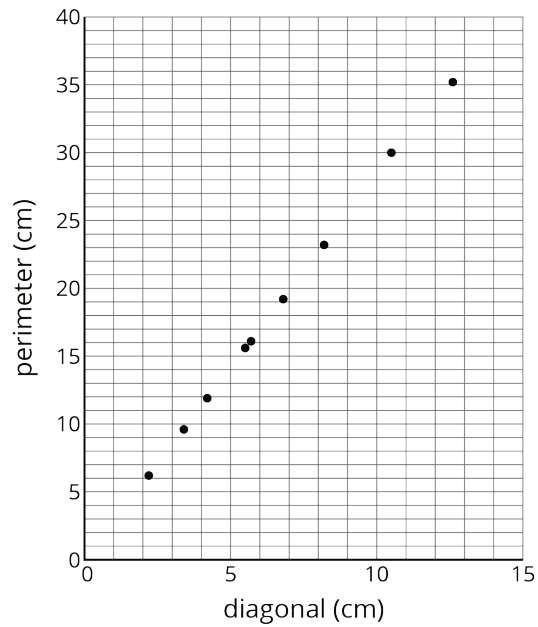
2.

3. Answers vary. Sample responses:

- The points almost lie on a straight line through the origin.

- From the graph it appears that the perimeter is about 3 times as large as the diameter.

4. See the completed table above.

Activity Synthesis

The goal of this discussion is for students to see that there is a proportional relationship between the length of the diagonal and the perimeter of a square, even though it is difficult to measure accurately enough to get an exact constant of proportionality.

First, ask students to share what they noticed about their graphs. Next, display a table like the one in students' books or devices and poll the class on the measurements they got for each square. If desired, give students 1–2 minutes to plot more points on their graph for the values they added to their tables.

Use questions like the following to lead students to the idea that the length of the diagonal and the perimeter are proportional to each other. Even though the data does not lie perfectly on a line, the inconsistencies are caused by measurement error.

- "Would it makes sense to have $(0, 0)$ as a possible point?" (Yes, since a square with diagonal 0 cm would have a perimeter of 0 cm.)

- "The data is not perfectly lined up. Do you think it should be? What could be causing the inconsistencies?" (Measurement error.)

- "If you had a square with diagonal 1 cm, what would the perimeter be?" (About 2.8 cm.)

Consider adding a column to the table with the quotient of the perimeter of the square divided by the length of the diagonal to help students identify the constant of proportionality for the relationship. Sample data is included in the table below. While the level of precision of their measurements does not justify including this many decimals places for the quotients, they are shown in this sample table because it is important that students notice the variability in the numbers and come to their own realization that these quotients are all approximately 2.8.

	diagonal (cm)	perimeter (cm)	perimeter ÷ diagonal
square A	5.7	16.1	2.825
square B	8.2	23.2	2.841
square C	3.4	9.6	2.824
square D	4.2	11.9	2.833
square E	6.8	19.2	2.823
square F	10.5	30	2.857
square G	2.2	6.2	2.818
square H	12.6	35.2	2.793
square I	5.5	15.6	2.836

When we collect data through measurement, we usually will introduce small errors into the data. Even though the data will look a little bit "bumpy," it will often show the underlying relationships between two quantities. In this case, we were able to use the plotted data to estimate the constant of proportionality. Once we have an estimate for the constant of proportionality, we can find other diagonal lengths given the perimeter or other perimeters given the diagonal length. In a situation like this, analyzing the graph is a powerful method to check visually to see if a relationship looks like it may be proportional. The table not only helps to confirm this but also gives the approximate constant of proportionality.

1.3 Area of a Square

15 minutes
This activity builds on the previous one as students plot a different relationship, comparing the length of each square diagonal to its area. Students use the length measurements from the previous activity and use them to calculate the area of the squares. Unlike in the previous activity, this time both the table and the graph show that the relationship between diagonal length and area is *not* close to being proportional. As a result, the measurement error does not play a role in this activity. Rather, the focus is on identifying that a relationship is not proportional: this is an example of MP7 because students will identify that there is no constant of proportionality in the table (not even allowing for measurement error) and will correspondingly notice that the plotted points are not close to lying on a line.
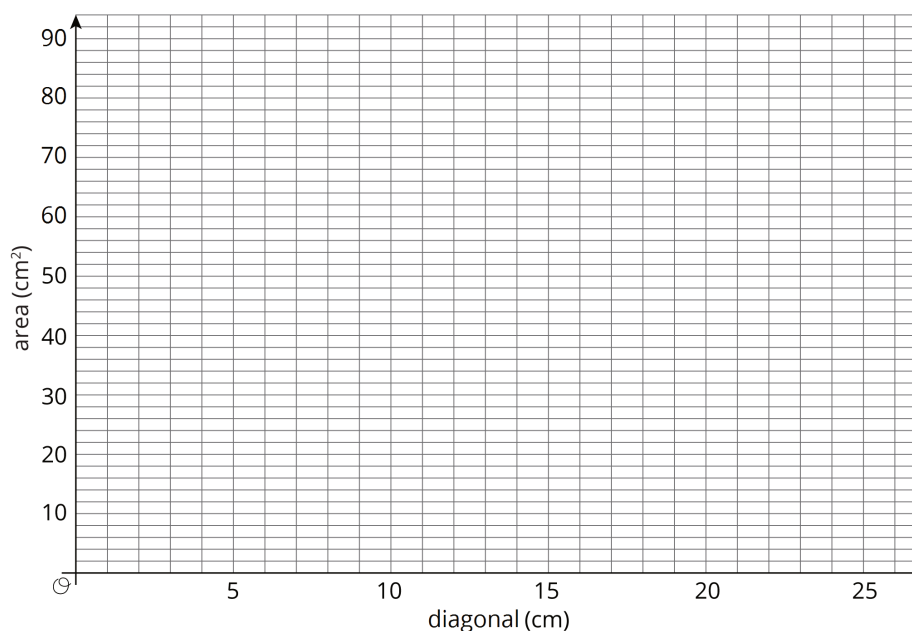
Addressing

- 7.RP.A.2.a

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups. Tell them to calculate the area of the same 3 squares they measured in the previous activity. Give students 4 minutes of group work time. Display this blank grid and have students plots points for their measurements.

Give students 2–3 more minutes of group work time followed by whole-class discussion.

Anticipated Misconceptions

Some students may struggle to calculate the area of the squares, or may use the length of the diagonal as if it were the side length. Prompt them with questions like "How can you calculate the area of a rectangle?" "What is the length and width of your square?"

Some students may measure the side length of each square again, instead of dividing the perimeter from the previous activity by 4. This strategy is allowable, although you can also prompt them to consider if they no longer had access to a ruler, is there a way they could use the information they already recorded to find this measurement.

Some students may struggle to organize the information. Prompt them to add a column to the table in the previous activity to record the side length of the squares.

Student Task Statement

1. In the table, record the length of the diagonal for each of your assigned squares from the previous activity. Next, calculate the area of each of your squares.

	diagonal (cm)	area (cm^2)
square A		
square B		
square C		
square D		
square E		
square F		
square G		
square H		
square I		

Pause here so your teacher can review your work. Be prepared to share your values with the class.

2. Examine the class graph of these values. What do you notice?

3. How is the relationship between the diagonal and area of a square the same as the relationship between the diagonal and perimeter of a square from the previous activity? How is it different?

**Student Response**

1. Three rows of this table filled in.

	diagonal (cm)	area (cm^2)
square A	5.7	16.2
square B	8.2	33.6
square C	3.4	5.8
square D	4.2	9
square E	6.8	23
square F	10.5	56.2
square G	2.2	2.4
square H	12.6	77.4
square I	5.5	15.2
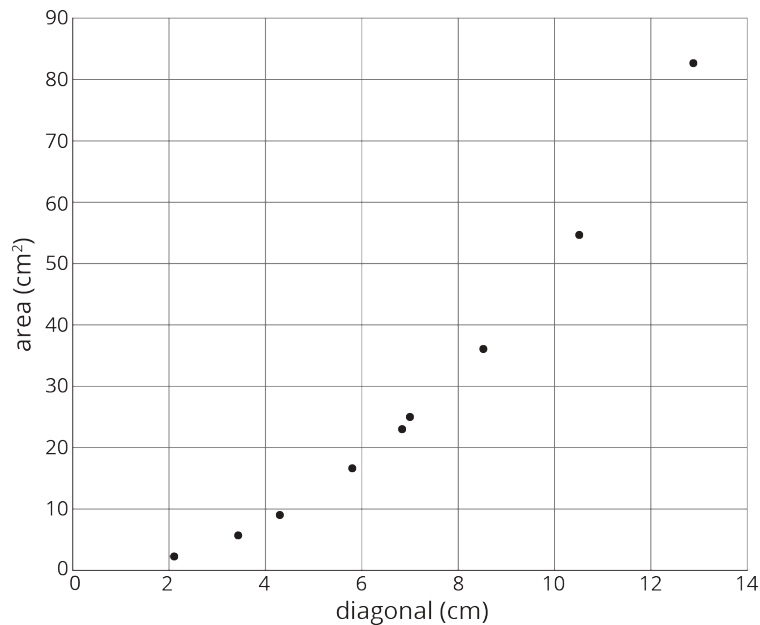
2. Answers vary. Sample responses: The graph appears to curve upward. The relationship is not proportional.

3. As the diagonal of a square increases, both perimeter and area increase. However, perimeter is proportional to the length of the diagonal, but area is not.

Are You Ready for More?

Here is a rough map of a neighborhood.
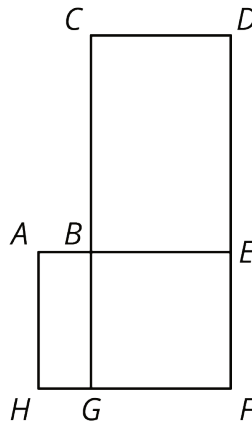
There are 4 mail routes during the week.

- On Monday, the mail truck follows the route A-B-E-F-G-H-A, which is 14 miles long.

- On Tuesday, the mail truck follows the route B-C-D-E-F-G-B, which is 22 miles long.

- On Wednesday, the truck follows the route A-B-C-D-E-F-G-H-A, which is 24 miles long.

- On Thursday, the mail truck follows the route B-E-F-G-B.

How long is the route on Thursdays?

Student Response

Thursday's route is 12 miles long.

Activity Synthesis

The goal of this discussion is for students to recognize that the relationship between the length of the diagonal and the area of squares of different sizes is not a proportional relationship.

Invite students to share their observations about the graph. Important points include:

- As the length of the diagonal increases, the area of the square increases.

- The points do not look like they lie on a line and definitely do not lie on a line that goes through $(0, 0)$.

- The area of the square looks to grow at a faster rate for larger diagonals than for the short diagonals.

To reinforce that the relationship is not proportional, instruct students to calculate the quotient of the area divided by the length of the diagonal for their square. Consider displaying a table of their measurements and adding a third column to record their quotients. Students should realize that the differences between these quotients are too large to be accounted for by just measurement error.

diagonal (cm)	area (cm^2)	area ÷ diagonal
8.2	33.6	4.1
4.2	9	2.1
6.8	23	3.4
2.2	2.4	1.1
12.6	77.4	6.1

If time permits, consider using this picture to reinforce that length scales linearly while area does not:
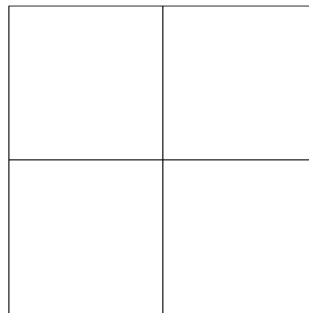
- "If the side length of the small square is s, what is the side length of the large square?" ($2s$)

- "If the diagonal of the small square is d, what is the diagonal of the large square?" ($2d$)

- "If the area of the small square is A, what is the area of the large square?" ($4A$)

- "Is the relationship between the diagonal and the area proportional?" (No, when the diagonal doubles, the area quadruples; if the relationship were proportional, both the diagonal and the area would double.)

Lesson Synthesis

Key takeaways:

- When we measure the quantities in a proportional relationship, measurement error may make it look like there is not an exact constant of proportionality.

- A graph can be helpful to decide whether the points are close to lying on a straight line.

Questions:

- "What does the graph of a proportional relationship look like?" (A straight line through the origin.)

- "If we measure the quantities in a proportional relationship and graph our measurements, what will the graph look like?" (The points will probably lie close to a straight line through the origin, but not exactly.)

Consider displaying various graphs and asking students whether they think the quantities are in a proportional relationship.
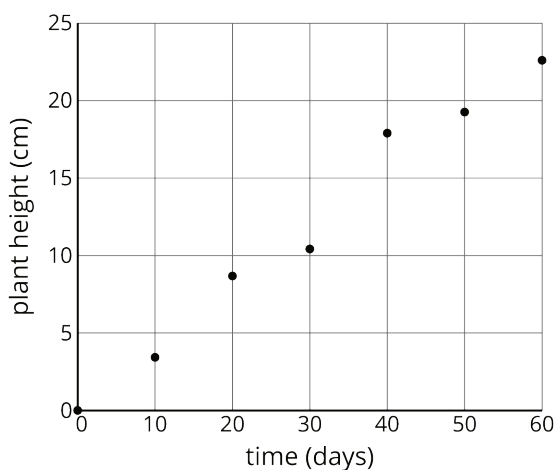
1.4 Examining Relationships

Cool Down: 5 minutes
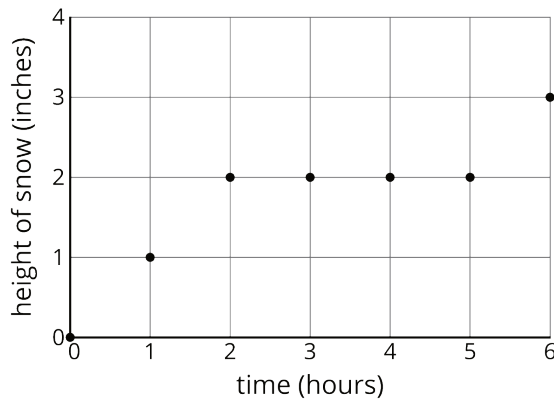**Addressing**

- 7.RP.A.2.a

> **Student Task Statement**
>
> 1. The graph shows the height of a plant after a certain amount of time measured in days.
>
> Do you think that there may be a proportional relationship between the number of days and the height of the plant? Explain your reasoning.
>
> 2. The graph shows how much snow fell after a certain amount of time measured in hours.

Do you think that there may be a proportional relationship between the number of hours and the amount of snow that fell? Explain your reasoning.

Student Response

1. Yes, there may be a proportional relationship. The point $(0,0)$ is on the graph, the points are close to being on a line, and there could be measurement error. It is also possible that the relationship is not proportional. It is not possible to decide for sure from the graph.

2. No, there is not a proportional relationship. For several hours there was no snow falling while some time at the beginning and toward the end there was some snowfall.

Student Lesson Summary

When we measure the values for two related quantities, plotting the measurements in the coordinate plane can help us decide if it makes sense to model them with a proportional relationship. If the points are close to a line through $(0,0)$, then a proportional relationship is a good model. For example, here is a graph of the values for the height, measured in millimeters, of different numbers of pennies placed in a stack.

Because the points are close to a line through $(0,0)$, the height of the stack of pennies appears to be proportional to the number of pennies in a stack. This makes sense because we can see that the heights of the pennies only vary a little bit.
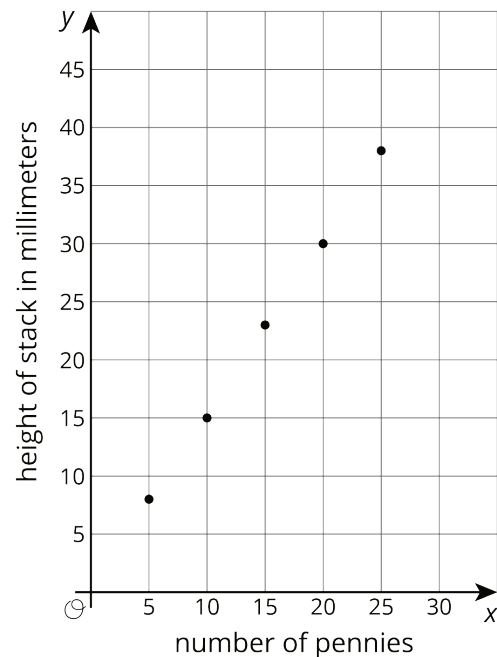
An additional way to investigate whether or not a relationship is proportional is by making a table. Here is some data for the weight of different numbers of pennies in grams, along with the corresponding number of grams per penny.

number of pennies	weight in grams	grams per penny
1	3.1	3.1
2	5.6	2.8
5	13.1	2.6
10	25.6	2.6

Though we might expect this relationship to be proportional, the quotients are not very close to one another. In fact, the metal in pennies changed in 1982, and older pennies are heavier. This explains why the weight per penny for different numbers of pennies are so different!

Lesson 1 Practice Problems
Problem 1
Statement
Estimate the side length of a square that has a 9 cm long diagonal.

Solution

6.3 cm, because the perimeter of the square is approximately $9 \cdot 2.8$ or 25.2 cm and $\frac{25.2}{4} = 6.3$ cm.

Problem 2
Statement
Select **all** quantities that are proportional to the diagonal length of a square.

   A. Area of the square

   B. Perimeter of the square

   C. Side length of the square

Solution
["B", "C"]

Problem 3

Statement

Diego made a graph of two quantities that he measured and said, "The points all lie on a line except one, which is a little bit above the line. This means that the quantities can't be proportional." Do you agree with Diego? Explain.
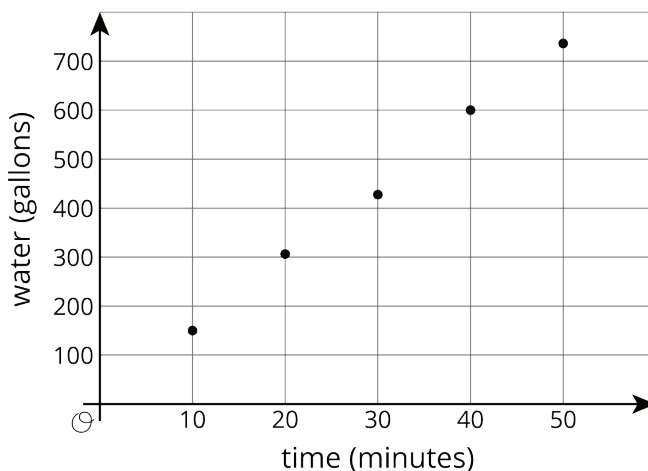
Solution

Answers vary. Sample response: I don't agree with Diego, since the quantities could be proportional if the line goes through the origin. Measurements are not perfect and the relationship could be proportional.

Problem 4

Statement

The graph shows that while it was being filled, the amount of water in gallons in a swimming pool was approximately proportional to the time that has passed in minutes.

a. About how much water was in the pool after 25 minutes?

b. Approximately when were there 500 gallons of water in the pool?

c. Estimate the constant of proportionality for the gallons of water per minute going into the pool.

Solution

a. About 380 gallons

b. After about 35 minutes

c. About 15