Lesson 13: More Solutions to Linear Equations

Goals

- Calculate the solution to a linear equation given one variable, and explain (orally) the solution method.
- Determine whether a point is a solution to an equation of a line using a graph of the line.

Learning Targets

- I can find solutions (x, y) to linear equations given either the x- or the y-value to start from.

Lesson Narrative

The previous lesson focused on the relationship between a linear equation in two variables, its solution set, and its graph. These themes continue to develop in this lesson. In the first activity after the warm-up, students analyze statements about a collection of three graphs, deciding whether or not certain ordered pairs are solutions to the equations defining the lines. In particular, students realize that values $x = a$ and $y = b$ satisfy two different linear equations simultaneously when the point (a, b) lies on both lines represented by the equations. This is important preparation for thinking about what it means to be a solution to a system of equations in the next unit.

In the second activity, students consider equations given in many different forms, ask their partner for either the x- or y-coordinate of a solution to the equation, and then give the other coordinate. This activity prepares students for finding solutions to systems of equations, because it gets them to look at the structure of an equation and decide whether it would be easier to solve for y given x, or to solve for x given y (MP7).

Alignments

Addressing

- 8.EE.C: Analyze and solve linear equations and pairs of simultaneous linear equations.
- 8.EE.C.8.a: Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

Required Materials

Pre-printed slips, cut from copies of the blackline master

Required Preparation

One copy of the I'll Take an X Please blackline master for every pair of students.

> ### Student Learning Goals
>
> Let's find solutions to more linear equations.

13.1 Coordinate Pairs

Warm Up: 5 minutes

The purpose of this warm-up is for students to practice solving an equation for an unknown value while thinking about a coordinate pair, (x, y), that makes the equation true. While the steps to solve the equation are the same regardless of which value of x students choose, there are strategic choices that make solving the resulting equation simpler. This should be highlighted in the discussion.

Addressing

- 8.EE.C

Launch

Encourage students to not pick 0 for x each time.

> ### Student Task Statement
>
> For each equation choose a value for x and then solve to find the corresponding y value that makes that equation true.
>
> 1. $6x = 7y$
> 2. $5x + 3y = 9$
> 3. $y + 5 - \frac{1}{3}x = 7$

Student Response

Answers vary. Sample responses:

1. $x = 7, y = 6$
2. $x = 3, y = \text{-}2$
3. $x = 3, y = 3$

Activity Synthesis

Collect the pairs of x's and y's students calculated and graph them on a set of axes. For each equation, they form a different line. Have students share how they picked their x values. For example:

- For the first problem, choosing x to be a multiple of 7 makes y an integer.

- For the last problem, picking x to be a multiple of 3 makes y an integer.

13.2 True or False: Solutions in the Coordinate Plane

15 minutes

In the previous lesson, students studied the set of solutions to a linear equation, the set of all values of x and y that make the linear equation true. They identified that this was a line in the coordinate plane. In this activity, they are given graphs of lines and then are asked whether or not different x-y coordinate pairs are solutions to equations that define the lines. This helps students solidify their understanding of the relationship between a linear equation and its graph in the coordinate plane.

Consider asking students to work on the first 5 problems only if time is an issue. Since this activity largely reinforces the material of the previous lesson, it is not essential to do all 8 problems.

Addressing

- 8.EE.C.8.a

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Arrange students in groups of 2. Students work through the eight statements individually and then compare and discuss their answers with their partner. Tell students that if they disagree, they should work to come to an agreement.

Access for Students with Disabilities

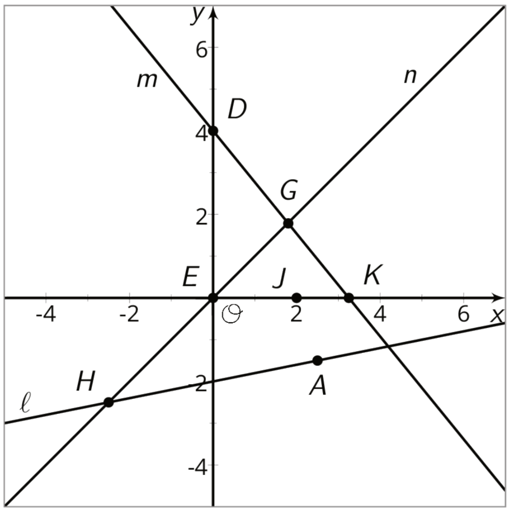
Action and Expression: Internalize Executive Functions. Check for understanding by inviting students to rephrase directions in their own words. Provide the following sentence frame to support student explanations: “Statement ___ is true/false because . . .”
Supports accessibility for: Organization; Attention

Anticipated Misconceptions

Some students may try to find equations for the lines. There is not enough information to accurately find these equations, and it is not necessary since the questions only require understanding that a coordinate pair lies on a line when it gives a solution to the corresponding linear equation. Ask these students if they can answer the questions without finding equations for the lines.

Student Task Statement

Here are graphs representing three linear relationships. These relationships could also be represented with equations.



For each statement below, decide if it is true or false. Explain your reasoning.

1. $(4, 0)$ is a solution of the equation for line m.
2. The coordinates of the point G make both the equation for line m and the equation for line n true.
3. $x = 0$ is a solution of the equation for line n.
4. $(2, 0)$ makes both the equation for line m and the equation for line n true.
5. There is no solution for the equation for line ℓ that has $y = 0$.
6. The coordinates of point H are solutions to the equation for line ℓ.
7. There are exactly two solutions of the equation for line ℓ.
8. There is a point whose coordinates make the equations of all three lines true.

After you finish discussing the eight statements, find another group and check your answers against theirs. Discuss any disagreements.

Student Response

1. False. The point $(4, 0)$ does not lie on line m, so it is not a solution to the equation for line m. The point $(0, 4)$ does lie on the line and is a solution to the equation for line m.
2. True. Since point G lies on both lines, its coordinates are a solution to both equations.
3. False. Since the equation has two variables, a solution must be a pair of numbers, or both coordinates of a point on the line. $x = 0$, $y = 0$ is a solution to the equation for line n.
4. False. $(2, 0)$ does not lie on either line and is therefore not a solution to either equation.

5. False. We don't see the solution here but can see that line ℓ and the x-axis (where $y = 0$) will meet in a point when the lines are extended.
6. True, because point H lies on line ℓ.
7. False. There are infinitely many solutions for line ℓ: the coordinates of every point on the line.
8. False. There is no point that lies on all three lines: the picture shows all intersection points of these lines.

Activity Synthesis

Display the correct answer to each question, and give students a few minutes to discuss any discrepancies with their partner. For the third question, some students might say yes because there is a solution to the equation for line n, which has $x = 0$, namely if $y = 0$ as well. For the fifth question, make sure students understand that the line ℓ meets the x-axis even if that point is not shown on the graph.

Some key points to highlight, reinforcing conclusions from the previous lesson as well as this activity:

- A solution of an equation in two variables is an ordered pair of numbers.
- Solutions of an equation lie on the graph of the equation.

Access for English Language Learners

Reading, Conversing: MLR3 Clarify, Critique, Correct. After students have had time to make decisions about whether the statements are true or false, offer an incorrect response such as: "The statement '$x = 0$ is a solution of the equation for line n' is true because I see the line go through the x value at 0." Invite students to work with a partner to clarify the meaning of this incorrect response and then critique it. Invite pairs to offer a correct response by asking, "What language might you add or change to make this statement more accurate?" Listen for key phrases such as "$x = 0$ is a line, not a point." This will help students to solidify their understanding that the solution to an equation with two variables is both coordinates of the point on the line.
Design Principle(s): Cultivate conversation; Maximize meta-awareness

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13.3 I'll Take an X, Please

15 minutes
Students are given linear equations—some of which represent proportional relationships—in various forms, and are also given solutions to their partner's equations in the form of coordinates of a point. The student with the equation decides which quantity they would like to know, x or y, and requests this information from their partner. They then solve for the other quantity. The activity reinforces the concept that solutions to equations with two variables are a pair of numbers, and

that knowing one can give you the other by using the value you know and solving the equation. Students also have a chance to think about the most efficient way to find solutions for equations in different forms.

You will need the I'll Take An x Please blackline master for this activity.

Addressing

- 8.EE.C.8.a

Instructional Routines

- MLR8: Discussion Supports

Launch

Consider demonstrating the first step with a student. Write the equation $y = 5x - 11$ on one slip of paper and the point $(1, \text{-}6)$ on another slip of paper. Give the slip with the coordinate pair to the student. You can ask for the x or y coordinate of a point on the graph and then need to find the other coordinate. Ask the student helper for either x or y. (In this case, asking for the x coordinate is wise because then you can just plug it into the equation to find the corresponding y-coordinate for the point on the graph.) Display your equation for all to see and demonstrate substituting the value in and solving for the other variable. Alternatively, ask students how they might use the information given (one of the values for x or y to find the other given your equation.

Arrange students in groups of 2. One partner receives Cards A through F from the left side of the blackline master and the other receives Cards a through f from the right side. Students take turns asking for either x or y then solving their equation for the other, and giving their partner the information requested.

Students play three rounds, where each round consists of both partners having a turn to ask for a value and to solve their equation. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Provide Access for Perception. Display or provide students with a physical copy of the written directions and read them aloud. Check for understanding by inviting students to rephrase directions in their own words. Display directions throughout the activity.
Supports accessibility for: Language; Memory .

Student Task Statement

One partner has 6 cards labeled A through F and one partner has 6 cards labeled a through f. In each pair of cards (for example, Cards A and a), there is an equation on one card and a coordinate pair, (x, y), that makes the equation true on the other card.

1. The partner with the equation asks the partner with a solution for either the x-value or the y-value and explains why they chose the one they did.

2. The partner with the equation uses this value to find the other value, explaining each step as they go.
3. The partner with the coordinate pair then tells the partner with the equation if they are right or wrong. If they are wrong, both partners should look through the steps to find and correct any errors. If they are right, both partners move onto the next set of cards.
4. Keep playing until you have finished Cards A through F.

Student Response

The A through F cards have both coordinates for each point.

Are You Ready for More?

Consider the equation $ax + by = c$, where a, b, and c are positive numbers.

1. Find the coordinates of the x- and y-intercepts of the graph of the equation.
2. Find the slope of the graph.

Student Response

1. $(\frac{c}{a}, 0)$ and $(0, \frac{c}{b})$. Putting $y = 0$ in the equation we get $ax = c$, so $x = \frac{c}{a}$. So the x-intercept is $(\frac{c}{a}, 0)$. Putting $x = 0$ in the equation we get $by = c$, so $y = \frac{c}{b}$. So the y-intercept is $(0, \frac{c}{b})$.
2. Using the two intercepts to calculate the slope, we get $\text{slope} = \frac{\frac{c}{b} - 0}{0 - \frac{c}{a}} = -\frac{\frac{c}{b}}{\frac{c}{a}} = -\frac{a}{b}$

Activity Synthesis

The discussion should focus on using the given information to efficiently find a solution for the equation. Consider asking students:

- "How did you decide whether you wanted the value of x or the value of y?" (One might be more efficient to solve for: for example, with card B asking for x makes sense while with card d the arithmetic to perform is similar whether asking for x or for y.)
- "Which equations represent proportional relationships? How do you know? Which do not?" (C and F are proportional because they can be written as $y = kx$, although this is hidden at first glance in C.)
- "Once you have identified one solution to your equation, what are some ways you could find others?" (Use the constant rate of change to add/subtract to the solution you know, solve the equation for x or y, choose values for one variable and solve for the other, for proportional relationships you could find equivalent ratios.)

Point out that all of the equations in this activity are linear. They are given in many different forms, not just $y = mx + b$ or $Ax + By = C$.

Access for English Language Learners

Conversing: MLR8 Discussion Supports. Use this routine to support student discussions. Provide the following sentence frames, and invite the partner who had the equation to begin with: "I decided to ask for the value of x (or y) because.... The steps I took to find the other value were...." Encourage the listener to ask clarifying questions such as: "What would have happened if you chose the other variable?" or "Could you think of a more efficient way to solve?" This will help students justify their reasoning for choosing a certain value to start and explain the steps they took to obtain the other coordinate.
Design Principle(s): Support sense-making; Cultivate conversation (for explanation)

Lesson Synthesis

In order to highlight student thinking about different strategies for finding a solution to a linear equation, ask students:

- "What are different ways to find a solution to the linear equation $3y + x = 12$?" (Substitute in a value for one variable and solve for the other; graph the equation and find points that lie on the line; rearrange the equation so that one variable is written in terms of the other variable.)
- "How do you know when you have found a solution to the equation $3y + x = 12$?" (The coordinates of the point will make the statement true.)
- "What are some easy values to substitute into the equation?" (In this case, a good strategic choice is $x = 0$, which gives $y = 4$, and $y = 0$, which gives $x = 12$. This says that the y-intercept of the graph of the equation is $(0, 4)$. Similarly, the x-intercept of the equation's graph is $(12, 0)$.
- "How can you find the slope of the line?" (Graphing the line shows that the slope is negative, and we can verify this by rewriting the equation as $y = -\frac{1}{3}x + 4$.)

13.4 Intercepted

Cool Down: 5 minutes
Students show their understanding of solutions to linear equations in two variables and connections to the graph of the equation.

Addressing

- 8.EE.C.8.a

Student Task Statement

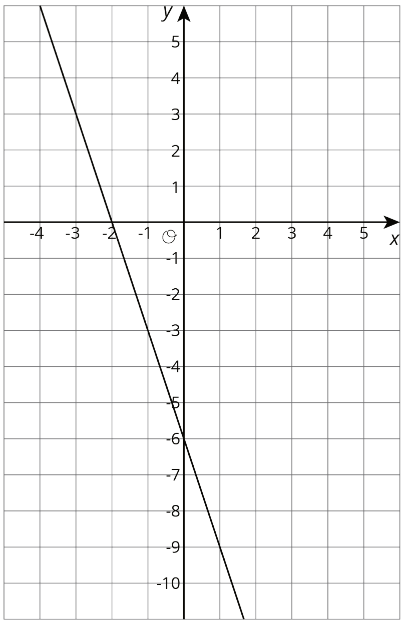
A graph of a linear equation passes through $(-2, 0)$ and $(0, -6)$.

1. Use the two points to sketch the graph of the equation.

2. Is $3x - y = -6$ an equation for this graph? Explain how you know.

Student Response

1.



2. No. Answers vary. Sample response: Test the two given pairs: $3(-2) - 0 = -6 - 0 = -6$ so the coordinates of this point represent a solution to the equation. $3(0) - (-6) = 0 + 6 = 6$, not -6, so the coordinates of this point do not represent a solution to the equation. The graph of a linear equation contains only ordered pairs whose coordinates are solutions to the equation, so the equation is not represented by the line with the two given points.

Student Lesson Summary

Let's think about the linear equation $2x - 4y = 12$. If we know $(0, -3)$ is a solution to the equation, then we also know $(0, -3)$ is a point on the graph of the equation. Since this point is on the y-axis, we also know that it is the vertical intercept of the graph. But what about the coordinate of the horizontal intercept, when $y = 0$? Well, we can use the equation to figure it out.

$$\begin{aligned} 2x - 4y &= 12 \\ 2x - 4(0) &= 12 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

Since $x = 6$ when $y = 0$, we know the point $(6, 0)$ is on the graph of the line. No matter the form a linear equation comes in, we can always find solutions to the equation by starting with one value and then solving for the other value.

Lesson 13 Practice Problems

Problem 1

Statement

For each equation, find y when $x = \text{-}3$. Then find x when $y = 2$

a. $y = 6x + 8$

b. $y = \frac{2}{3}x$

c. $y = \text{-}x + 5$

d. $y = \frac{3}{4}x - 2\frac{1}{2}$

e. $y = 1.5x + 11$

Solution

a. $y = \text{-}10$, $x = \text{-}1$

b. $y = \text{-}2$, $x = 3$

c. $y = 8$, $x = 3$

d. $y = \frac{\text{-}19}{4}$, $x = 6$

e. $y = 6.5$, $x = \text{-}6$

Problem 2

Statement

True or false: The points $(6, 13)$, $(21, 33)$, and $(99, 137)$ all lie on the same line. The equation of the line is $y = \frac{4}{3}x + 5$. Explain or show your reasoning.

Solution

True, all three points make the equation true.

Problem 3

Statement

Here is a linear equation: $y = \frac{1}{4}x + \frac{5}{4}$

a. Are $(1, 1.5)$ and $(12, 4)$ solutions to the equation? Explain or show your reasoning.

b. Find the x-intercept of the graph of the equation. Explain or show your reasoning.

Solution

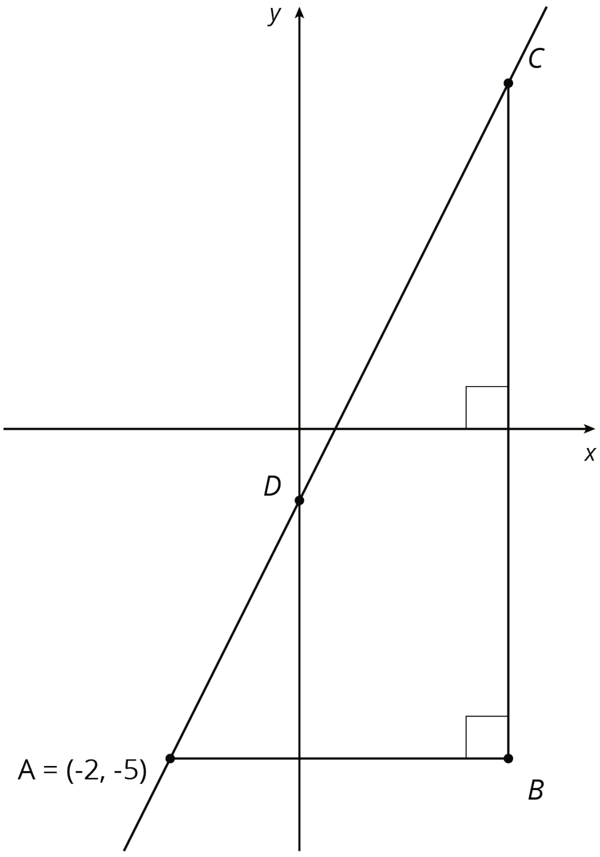
a. $(1, 1.5)$: Yes, check it with the equation; $(12, 4)$: No, when $x = 12$, y would be 4.25, not 4.

b. $(\text{-}5, 0)$ Explanations vary. Sample response: Set $y = 0$ in the equation.

Problem 4

Statement

Find the coordinates of B, C, and D given that AB = 5 and BC = 10.



Solution

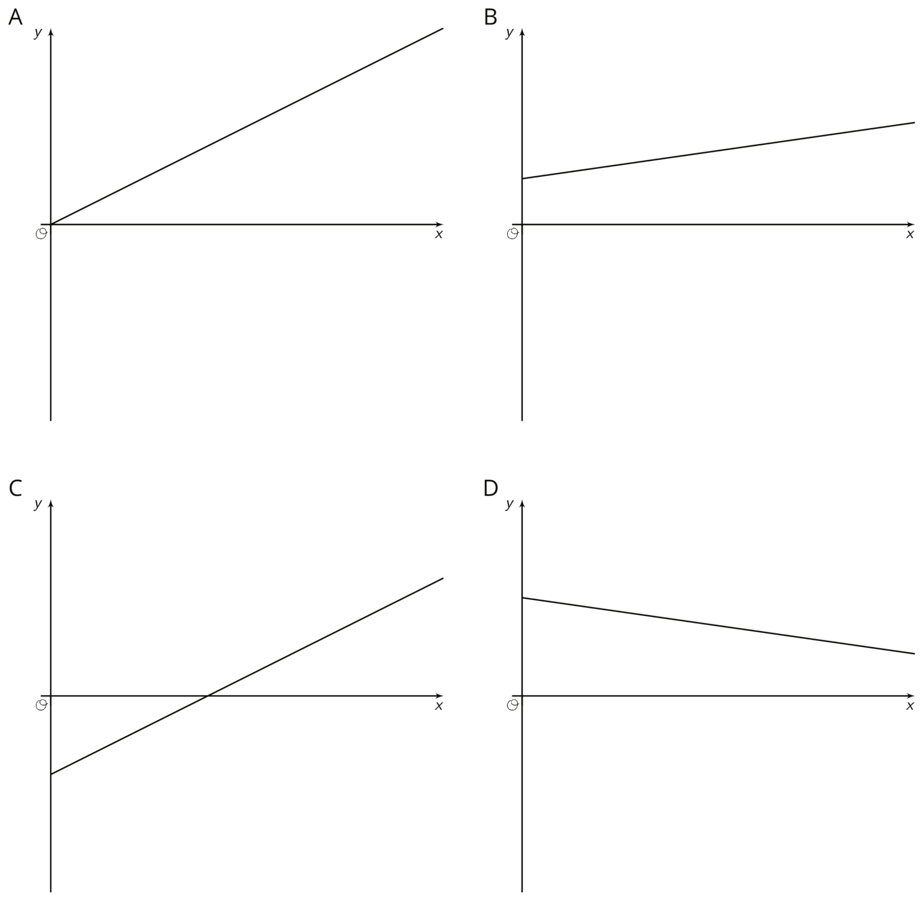
$B = (3, -5)$, $C = (3, 5)$, $D = (0, -1)$

(From Unit 2, Lesson 11.)

Problem 5

Statement

Match each graph of a linear relationship to a situation that most reasonably reflects its context.



A. Graph A

B. Graph B

C. Graph C

D. Graph D

1. y is the weight of a kitten x days after birth.

2. y is the distance left to go in a car ride after x hours of driving at a constant rate toward its destination.

3. y is the temperature, in degrees C, of a gas being warmed in a laboratory experiment.

4. y is the amount of calories consumed eating x crackers.

Solution

- A: 4
- B: 1
- C: 3
- D: 2

(From Unit 3, Lesson 9.)