## Lesson 2: Funding the Future

* Let’s look at some other things that polynomials can model.

### 2.1: Notice and Wonder: Writing Numbers

What do you notice? What do you wonder?



3 100s, 2 10s, 9 1s

### 2.2: Polynomials in the Integers

Consider the polynomial function given by .

1. Evaluate the function at and .
2. How does knowing that help you solve the equation ?

#### Are you ready for more?

Han notices:

* and
* while

The digits in the powers of 11 correspond to the coefficients of the polynomials.

1. Is this still true for and ? What about and ?
2. Give a mathematical justification of Han’s observation.

### 2.3: A Yearly Gift

At the end of 12th grade, Clare’s aunt started investing money for her to use after graduating from college four years later. The first deposit was $300. If is the annual interest rate of the account, then at the end of each school year the balance in the account is multiplied by a growth factor of .

1. After one year, the total value is . After two years, the total value is . Write an expression for the total value after graduation in terms of .
2. If Clare’s aunt had invested another $500 at the end of her freshman year, what would the expression be for the total value after graduation in terms of ?

* Pause here for a whole-class discussion.

1. Suppose that $250 was invested at the end of sophomore year, and $400 at the end of junior year in addition to the original $300 and the $500 invested at the end of freshman year. Write an expression for the total value after graduation in terms of .
2. The total amount , in dollars, after four years is a function of the growth factor . If the total Clare receives after graduation is , use a graph to find the interest rate that the account earned.

### Lesson 2 Summary

Let’s say we’re going to invest $200 at an annual interest rate of . This means at the end of a year, the balance in the account is multiplied by a growth factor of . After the first year, the amount in the account can be expressed as , which is a polynomial. Similarly, after the second year, the amount will be , after three years, the amount will be , etc.

If an additional $350 is invested at the end of the first year, we can revise the polynomial. The amount of money in the account after 1 year is the same, but now the amount of money after two years is .

What will the polynomial expression look like if $400 more is invested at the end of the second year and $150 more is invested at the end of the third year? .

Let be the amount of money in dollars in the account after four years and be the growth factor where . A graph of helps us visualize how the amount in the account after four years depends on different values of .



We can use this polynomial model to examine the effect of different annual interest rates, or to estimate what the annual interest rate needs to be to achieve a specific quantity at the end of the four years. For example, point A is at . From this, we know that the amount in the account after 4 years with an interest rate of 4% each year is approximately $1,216. Similarly, if we want the account to have $2,000 after four years, that corresponds to point B, and at that point the growth rate is approximately 1.25 each year, since . So an interest rate of 25% will get us there, though we are not likely to find a bank that would offer that rate. Note also that the values correspond to negative rates, which are also unlikely!

Polynomial models are adaptable to a variety of situations even as they grow in complexity.



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