Lesson 14: Alternate Interior Angles

Goals

- Calculate angle measures using alternate interior, adjacent, vertical, and supplementary angles to solve problems.
- Justify (orally and in writing) that “alternate interior angles” made by a “transversal” connecting two parallel lines are congruent using properties of rigid motions.

Learning Targets

- If I have two parallel lines cut by a transversal, I can identify alternate interior angles and use that to find missing angle measurements.

Lesson Narrative

In this lesson, students justify that **alternate interior angles** are congruent, and use this and the vertical angle theorem, previously justified, to solve problems.

Thus far in this unit, students have studied different types of rigid motions, using them to examine and build different figures. This work continues here, with an emphasis on examining angles. In a previous lesson, 180 degree rotations were employed to show that vertical angles, made by intersecting lines, are congruent. The warm-up recalls previous facts about angles made by intersecting lines, including both vertical and adjacent angles. Next a third line is added, parallel to one of the two intersecting lines. There are now 8 angles, 4 each at the two intersection points of the lines. At each vertex, vertical and adjacent angles can be used to calculate all angle measures once one angle is known. But how do the angle measures compare at the two vertices? It turns out that each angle at one vertex is congruent to the corresponding angle (via translation) at the other vertex and this can be seen using rigid motions: translations and 180 degree rotations are particularly effective at revealing the relationships between the angle measures.

One mathematical practice that is particularly relevant for this lesson is MP8. Students will notice as they calculate angles that they are repeatedly using vertical and adjacent angles and that often they have a choice which method to apply. They will also notice that the angles made by parallel lines cut by a **transversal** are the same and this observation is the key structure in this lesson.

Alignments

Building On

- 7.G.B.5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Addressing

- 8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:

- 8.G.A.5: Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. For example, arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.

Instructional Routines

- MLR2: Collect and Display
- MLR8: Discussion Supports
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Students need rulers and tracing paper from the geometry toolkits.

> **Student Learning Goals**
>
> Let’s explore why some angles are always equal.

14.1 Angle Pairs

Warm Up: 5 minutes
This task is designed to prompt students to recall their prior work with supplementary angles. While they have seen this material in grade 7, this is the first time it has come up explicitly in grade 8. As students work on the task, listen to their conversations specifically for the use of vocabulary such as supplementary and vertical angles. If no students use this language, make those terms explicit in the discussion.

Some students may wish to use protractors, either to double check work or to investigate the different angle measures. This is an appropriate use of technology (MP5), but ask these students what *other* methods they could use instead.

Building On

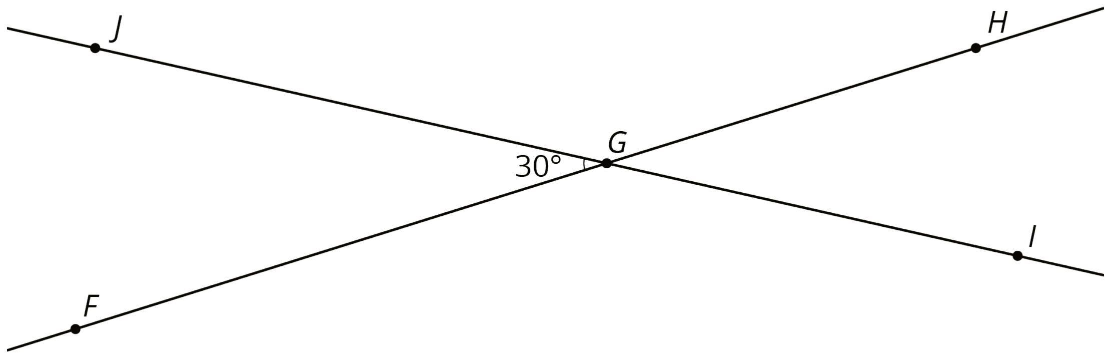
- 7.G.B.5

Launch

Provide access to geometry toolkits. Before students start working, make sure they are familiar with the convention for naming an angle using three points, where the middle letter denotes the angle's vertex.

Student Task Statement

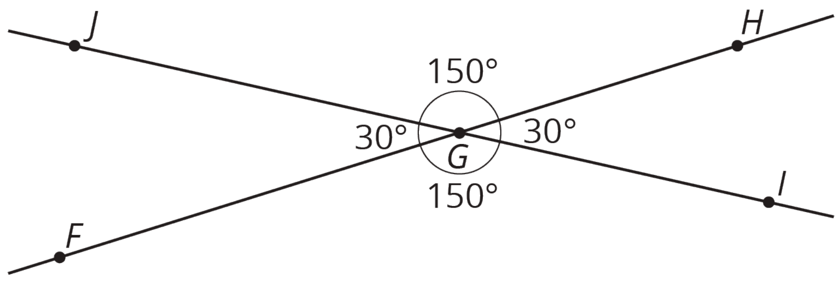
1. Find the measure of angle JGH. Explain or show your reasoning.



2. Find and label a second 30° degree angle in the diagram. Find and label an angle congruent to angle JGH.

Student Response

1. 150°. Sample response: In the diagram, the given 30° angle and angle JGH are supplementary, so they add up to 180°.



2. Angles are labeled as shown using reasoning about vertical or supplementary angles.

Activity Synthesis

Display the image for all to see. Invite students to share their responses, adding onto the image as needed to help make clear student thinking. If not mentioned by students, make sure to highlight the term *supplementary angles* to describe, for example, angles FGJ and JGH, and *vertical angles* to describe, for example, angles JGF and HGI.

14.2 Cutting Parallel Lines with a Transversal

15 minutes

In this task, students explore the relationship between angles formed when two parallel lines are cut by a transversal line. Students investigate whether knowing the measure of one angle is sufficient to figure out all the angle measures in this situation. They also consider whether the relationships they found hold true for any two lines cut by a transversal.

The last two questions in this activity are optional, to be completed if time allows. Make sure to leave enough time for the next activity, "Alternate Interior Angles are Congruent."

As students work with their partners, they begin to fill in supplementary angles and vertical angles. To find the measures of corresponding and alternate interior, students may use tracing paper and some of the strategies found earlier in the unit. For example, they may use tracing paper to translate vertex B to vertex E. They might try to translate B to E in the third picture and observe that the angles at those two vertices are not congruent. Similarly, to find measures of vertical angles students may use a 180° rotation like they did earlier in this unit when showing that vertical angles are congruent. Monitor for students who use these different strategies and select them to share during the discussion.

For students who finish early, ask them to think of *different* methods they could use to determine the angles: For example, all of the congruent angles can be shown to be congruent with transformations.

Addressing

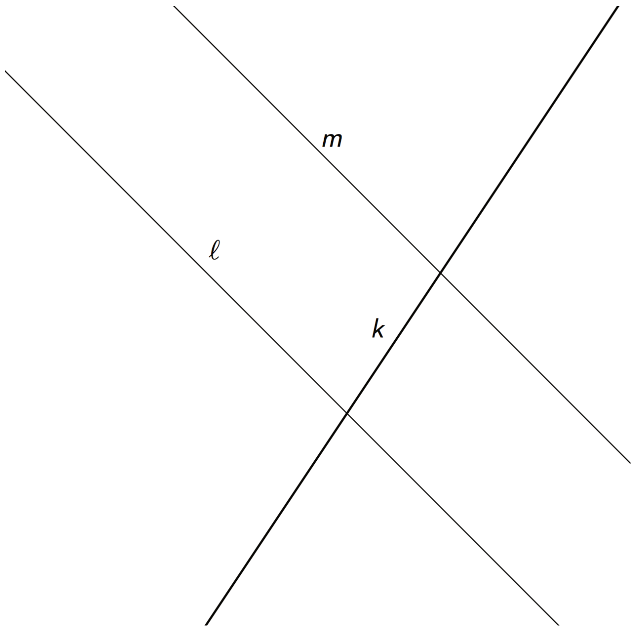
- 8.G.A.1
- 8.G.A.5

Instructional Routines

- MLR2: Collect and Display
- Think Pair Share

Launch

A **transversal** (or *transversal line*) for a pair of parallel lines is a line that meets each of the parallel lines at exactly one point. Introduce this idea and provide a picture such as this picture where line k is a transversal for parallel lines ℓ and m:



Arrange students in groups of 2. Provide access to geometry toolkits. Give students 1 minute of quiet think time to plan on how to find the angle measures in the picture then time to share their plan with their partner. Give partners time for the rest of the task, followed by a whole-class discussion. Instruct students to stop after the third question if you've decided to skip the last two questions.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Invite students to suggest language or diagrams to include that will support their understanding of these terms. Include the following term and maintain the display for reference throughout the unit: transversal.
Supports accessibility for: Conceptual processing; Language

Access for English Language Learners

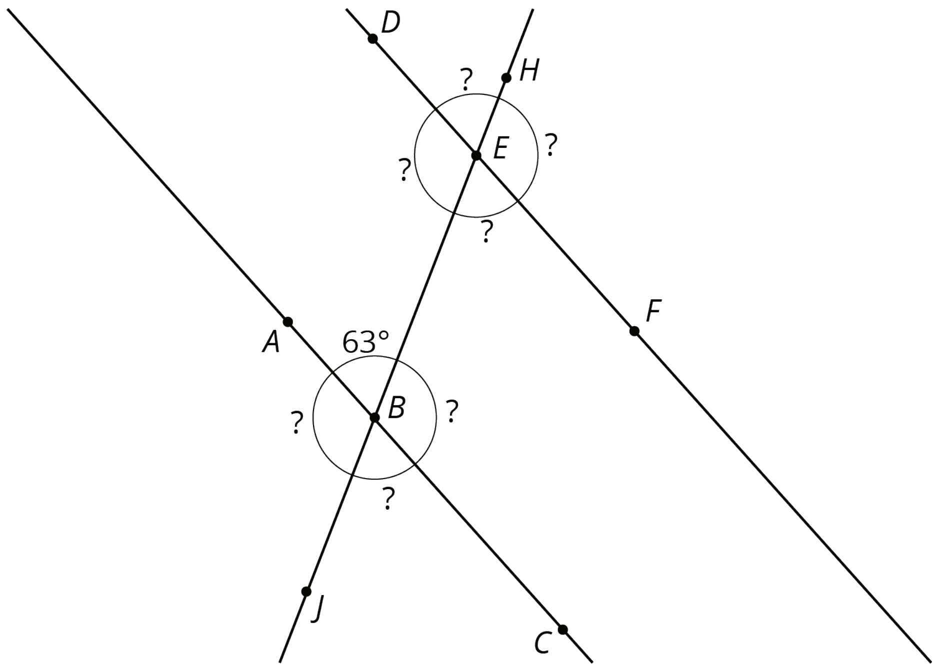
Conversing, Representing: MLR2 Collect and Display. As students discuss with a partner, listen for and collect vocabulary, gestures, and diagrams students use to describe the relationships they notice between the angles formed when two parallel lines are cut by a transversal. Capture student language that reflects a variety of ways to determine determine angle congruence. Record students’ words, phrases, and diagrams onto a visual display and update it throughout the lesson. Remind students to borrow language from the display as needed. This will help students use mathematical language during small-group and whole-class discussions.
Design Principle(s): Support sense-making

Anticipated Misconceptions

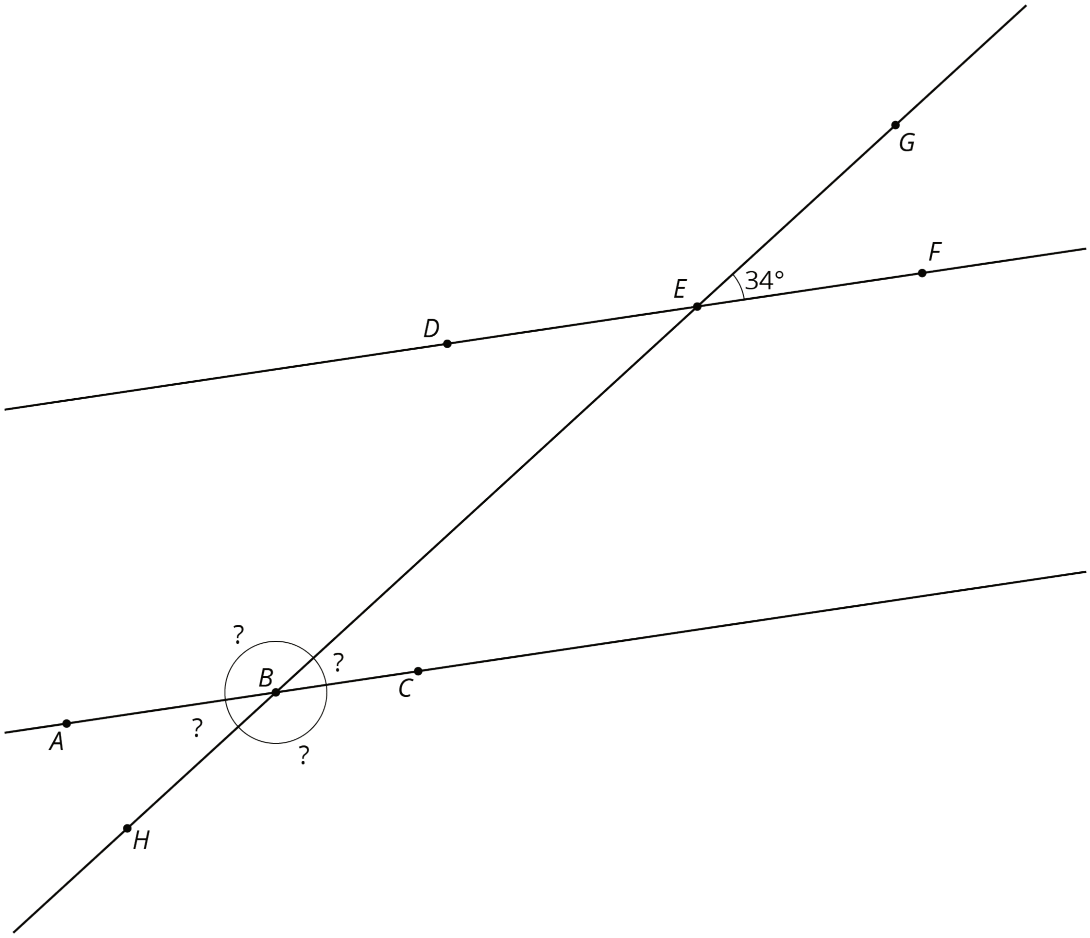
In the first image, students may fill in congruent angle measurements based on the argument that they look the same size. Ask students how they can be certain that the angles don't differ in measure by 1 degree. Encourage them a way to explain how we can know for sure that the angles are exactly the same measure.

Student Task Statement

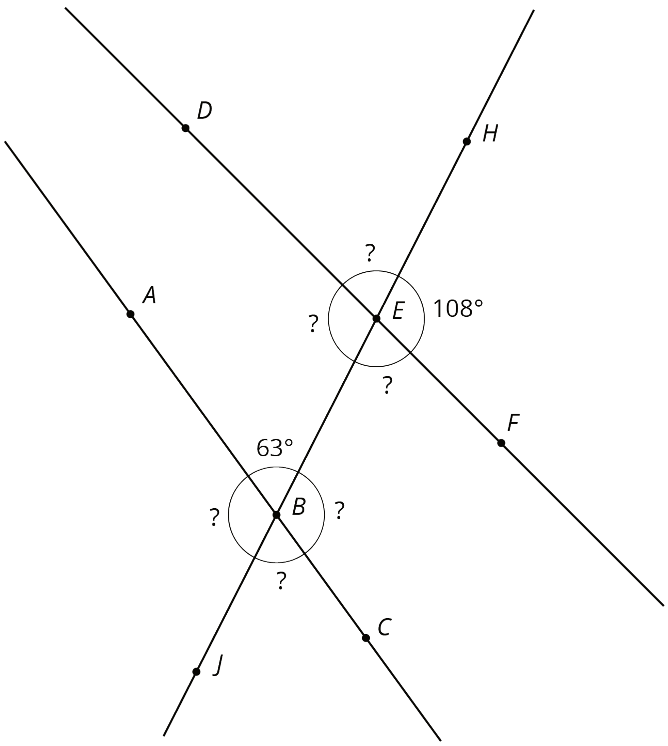
Lines AC and DF are parallel. They are cut by **transversal** HJ.



1. With your partner, find the seven unknown angle measures in the diagram. Explain your reasoning.
2. What do you notice about the angles with vertex B and the angles with vertex E?
3. Using what you noticed, find the measures of the four angles at point B in the second diagram. Lines AC and DF are parallel.

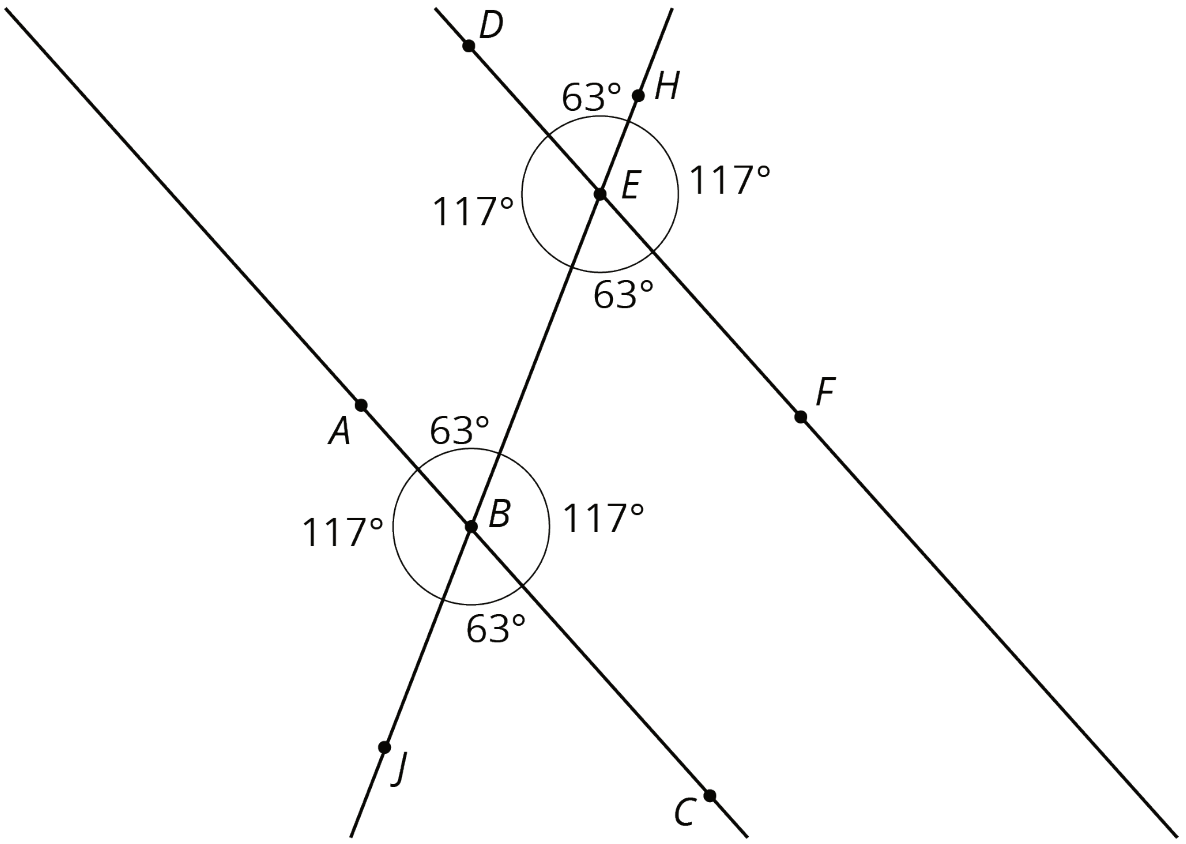


4. The next diagram resembles the first one, but the lines form slightly different angles. Work with your partner to find the six unknown angles with vertices at points B and E.



5. What do you notice about the angles in this diagram as compared to the earlier diagram? How are the two diagrams different? How are they the same?

Student Response

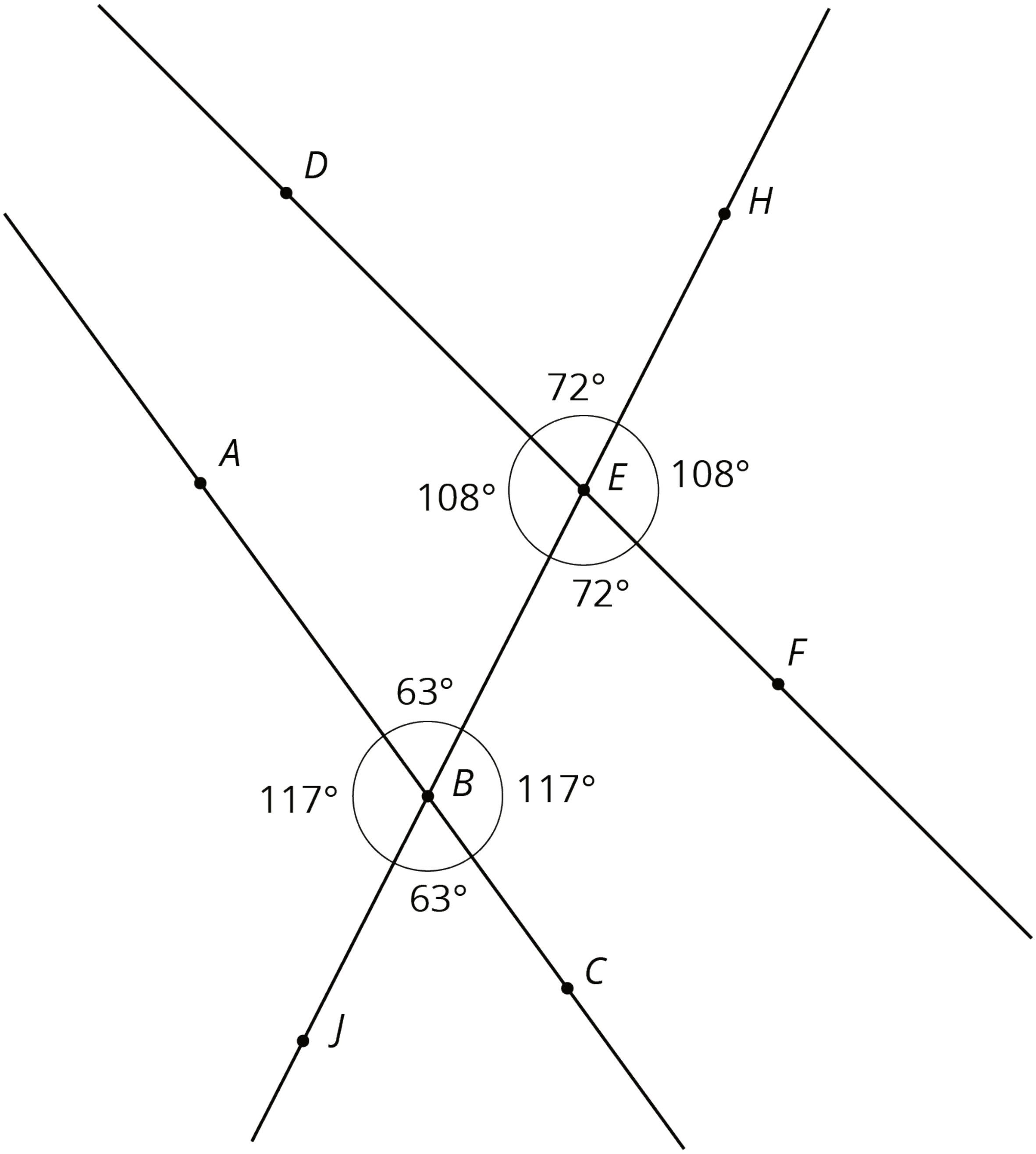
1. 

   Explanations vary. Sample strategy 1: Tracing paper helped find the three 117 degree angles. Each of the other four angles is supplementary to a 117 degree angle, so they are all 63 degree angles. Sample strategy 2: Using pairs of vertical angles shows that angle CBJ is a 63 degree angle. The other angles at vertex B can be found using supplementary angles. The angles at vertex E can be found the same way after using tracing paper to find one of them.

2. Answers vary. Sample response: The angles in the same place relative to the transversal have the same measure.

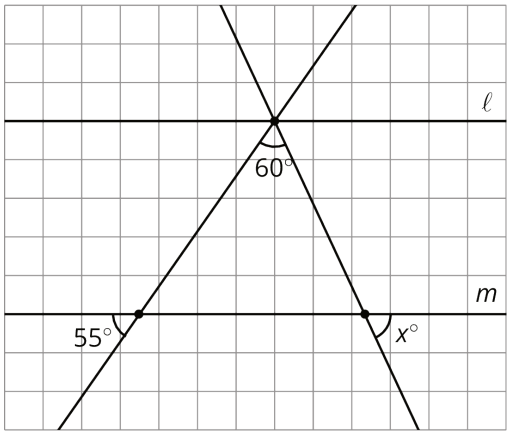
3. Answers vary. Sample response: Angle ABH is a 34 degree angle because it forms a vertical pair with the marked 34 degree angle after translating E to B. Angle HBC is a 146 degree angle because it is supplementary to the 34 degree angle found by translating E to B.

4.



5. Answers vary. Sample response: In both pictures, the two pair of vertical angles at each vertex are congruent. Also adjacent angles at each vertex are supplementary. In the first picture, the angle measures at the two vertices are the same while in the second picture they are different.

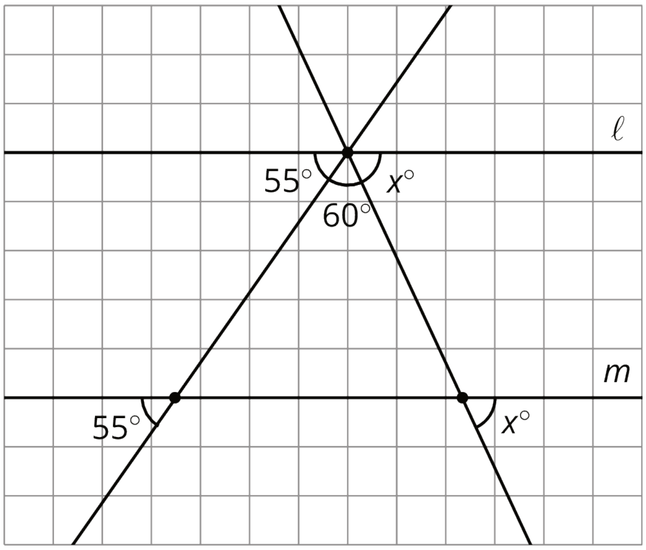
Are You Ready for More?



Parallel lines ℓ and m are cut by two transversals which intersect ℓ in the same point. Two angles are marked in the figure. Find the measure x of the third angle.

Student Response

$x = 65^\circ$.



Adding in two alternate interior angles, we see that the angles marked 55°, 60°, and x° make a straight angle, so $55 + 60 + x = 180$.

Activity Synthesis

The purpose of this discussion is to make sure students noticed relationships between the angles formed when two parallel lines are cut by a transversal and to introduce the term alternate interior angles to students. Display the images from the Task Statement for all to see one at a time and invite groups to share their responses. Encourage students to use precise vocabulary, such as supplementary and vertical angles, when describing how they figured out the different angle measurements. After students point out the matching angles at the two vertices, define the term **alternate interior angles** and ask a few students to identify some pairs of angles from the activity.

Consider asking some of the following questions:

- "What were some tools you used to find or confirm angle measures?" (Tracing paper, protractor, transformations)
- "What were some angle relationships you used to find missing measures?" (Vertical angles, supplementary angles)
- "What do you notice about the angles at vertex B compared to the angles at vertex E?" (They have the same angle measures for angles in the same position relative to the transversal.)
- "Which angle relationships were true for all three pictures? Which were true for only one or two of the pictures?" (Congruent vertical and supplementary angles around a vertex were always true. Congruent angles in corresponding positions at the two vertices were only true in the first two pictures, which had parallel lines.)

14.3 Alternate Interior Angles Are Congruent

15 minutes
The goal of this task is to experiment with rigid motions to help visualize why alternate interior angles (made by a transversal connecting two parallel lines) are congruent. This result will be used in a future lesson to establish that the sum of the angles in a triangle is 180 degrees. The second question is optional if time allows. This provides a deeper understanding of the relationship between the angles made by a pair of (not necessarily parallel) lines cut by a transversal.

Expect informal arguments as students are only beginning to develop a formal understanding of parallel lines and rigid motions. They have, however, studied the idea of 180 degree rotations in a previous lesson where they used this technique to show that a pair of vertical angles made by intersecting lines are congruent. Consider recalling this technique especially to students who get stuck and suggesting the use of tracing paper.

Given the diagram, the tracing paper, and what they have learned in this unit, students should be looking for ways to demonstrate that alternate interior angles are congruent using transformations. Make note of the different strategies (including different transformations) students use to show that the angles are congruent and invite them to share their strategies during the discussion. Approaches might include:

- A 180 degree rotation about M
- First translating P to Q and then applying a 180-degree rotation with center Q

Addressing

- 8.G.A.1
- 8.G.A.5

Instructional Routines

- MLR8: Discussion Supports

Launch

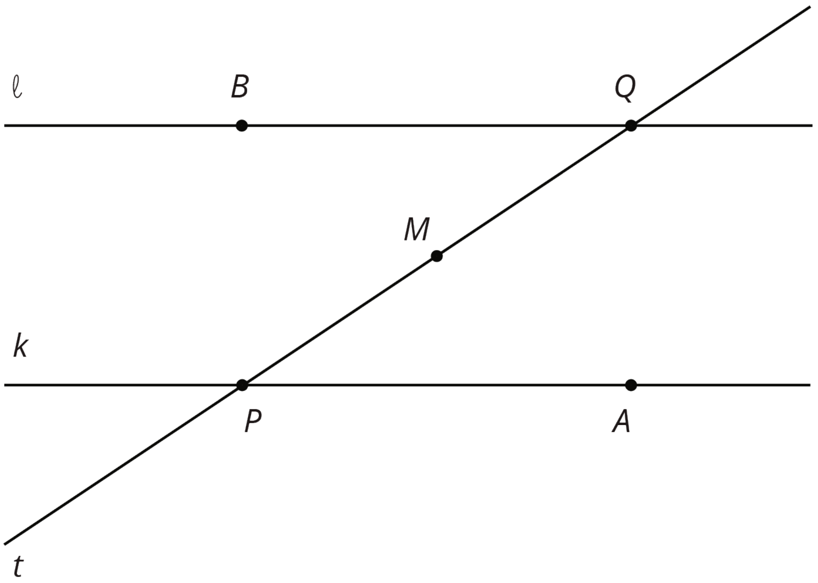
Provide access to geometry toolkits. Tell students that in this activity, we will try to figure out why we saw all the matching angles we did in the last activity.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, “First, I _____ because...”, “I noticed _____ so I...”, “Why did you...?”, “I agree/disagree because....”
Supports accessibility for: Language; Organization

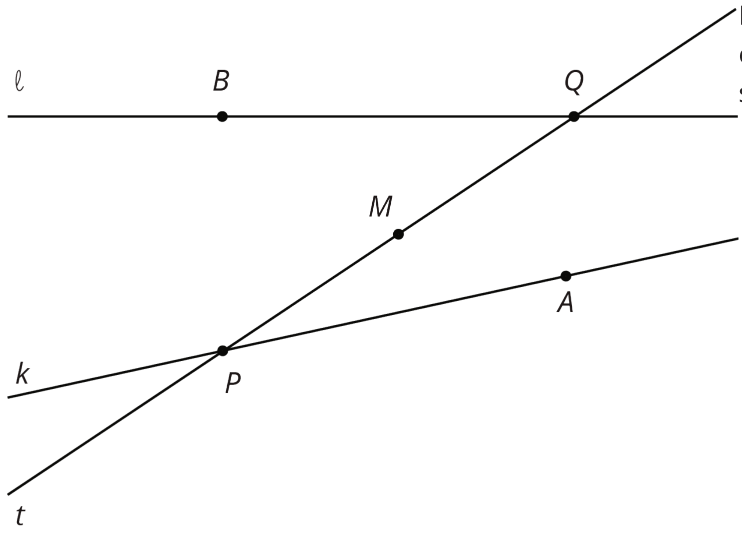
Student Task Statement

1. Lines ℓ and k are parallel and t is a transversal. Point M is the midpoint of segment PQ.



   Find a rigid transformation showing that angles MPA and MQB are congruent.

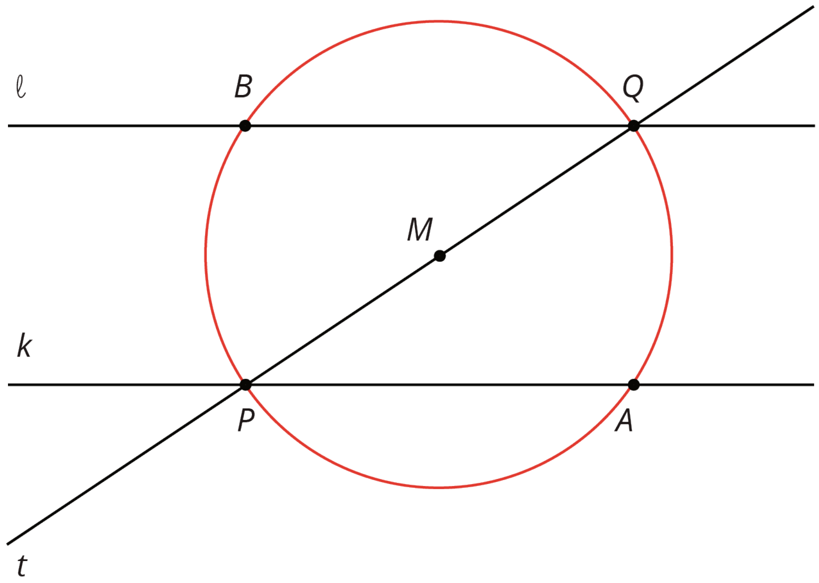
2. In this picture, lines ℓ and k are no longer parallel. M is still the midpoint of segment PQ.



Does your argument in the earlier problem apply in this situation? Explain.

Student Response

1. Rotate the picture 180° with center M.



   Since 180 degrees is half of a circle this takes each point on the circle to its "opposite." Point A maps to point B and B maps to A. So the 180 degree rotation will interchange P and Q. The rotation interchanges lines ell and m and also angles MQB and MPA so the angles are congruent.

2. If ℓ and m are not parallel, a 180 degree rotation around M still takes P to Q and Q to P. The problem is that it does not take ℓ to m, and it does not take m to ℓ because m is not parallel to ℓ. So this rotation does *not* take angle MQB to angle MPA and vice versa. The argument from 1 does not apply unless ℓ and m are parallel.

Activity Synthesis

Select students to share their explanations. Pay close attention to which transformations students use in the first question and make sure to highlight different possibilities if they arise. Ask students to describe and demonstrate the transformations they used to show that alternate interior angles are congruent.

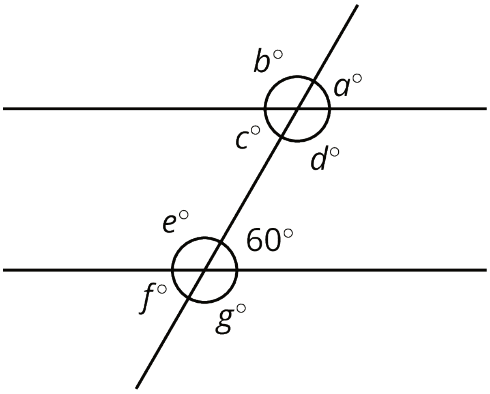
Highlight the fact that students are using many of the transformations from earlier sections of this unit. The argument here is especially close to the one used to show that vertical angles made by intersecting lines are congruent. In both cases a 180 degree rotation exchanges pairs of angles. For vertical angles, the center of rotation is the common point of intersecting lines. For alternate interior angles, the center of rotation is the midpoint of the transverse between the two parallel lines. But the structure of these arguments is identical.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to amplify students' mathematical uses of language when describing and demonstrating transformations used for showing alternate interior angles are congruent. After a student shares their response, invite another student to repeat the reasoning using the following mathematical language: vertical angles, 180 degree rotation, center of rotation, intersecting lines, midpoint, parallel lines. Invite all students to chorally repeat the phrases that include these words in context.
Design Principle(s): Support sense-making, Optimize output (for explanation)

Lesson Synthesis

Display the image of two parallel lines cut by a **transversal**. Tell students that in cases like this, translations and rotations can be particularly useful in figuring out angle measurements since they move angles to new positions, but the angle measure does not change.



Select students to point out examples of **alternate interior**, vertical, and supplementary angles in the picture. They should also be able to articulate which angles are congruent to one another and give an example of a rigid transformation that explains why.

In particular, make sure students can articulate:

- $c = 60$ because it is the measure of an angle forming an alternate interior angle with the given 60 degree angle.
- $e = d = 120$ because they are also alternate interior angles, each supplementary to a 60 degree angle.
- The rest of the angle measures can be found using vertical or supplementary angles.

14.4 All The Rest

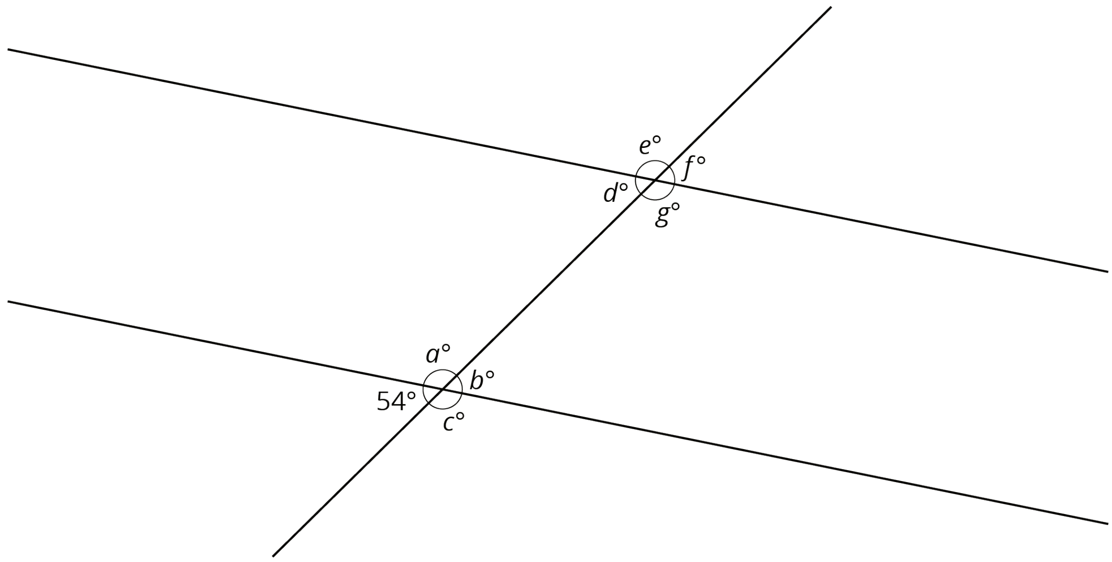
Cool Down: 5 minutes
Students use what they have learned about vertical and alternate interior angles in this lesson and earlier lessons, applying it to a diagram in order to fill in angle measurements without needing to measure.

Addressing

- 8.G.A.5

Student Task Statement

The diagram shows two parallel lines cut by a transversal. One angle measure is shown.



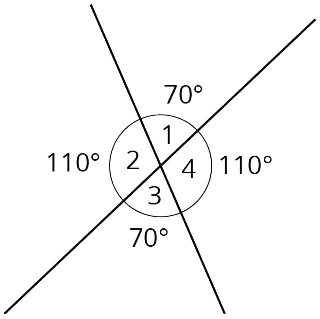
Find the values of a, b, c, d, e, f, and g.

Student Response

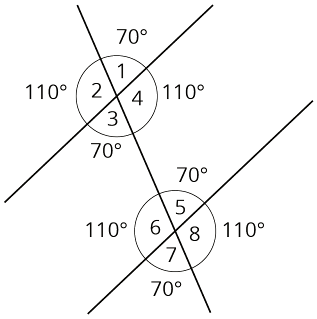
a: 126, b: 54, c: 126, d: 54, e: 126, f: 54, g: 126

Student Lesson Summary

When two lines intersect, vertical angles are equal and adjacent angles are supplementary, that is, their measures sum to 180°. For example, in this figure angles 1 and 3 are equal, angles 2 and 4 are equal, angles 1 and 4 are supplementary, and angles 2 and 3 are supplementary.



When two parallel lines are cut by another line, called a **transversal**, two pairs of **alternate interior angles** are created. ("Interior" means on the inside, or between, the two parallel lines.) For example, in this figure angles 3 and 5 are alternate interior angles and angles 4 and 6 are also alternate interior angles.



Alternate interior angles are equal because a 180° rotation around the midpoint of the segment that joins their vertices takes each angle to the other. Imagine a point M halfway between the two intersections—can you see how rotating 180° about M takes angle 3 to angle 5?

Using what we know about vertical angles, adjacent angles, and alternate interior angles, we can find the measures of any of the eight angles created by a transversal if we know just one of them. For example, starting with the fact that angle 1 is 70° we use vertical angles to see that angle 3 is 70°, then we use alternate interior angles to see that angle 5 is 70°, then we use the fact that angle 5 is supplementary to angle 8 to see that angle 8 is 110° since $180-70=110$. It turns out that there are only two different measures. In this example, angles 1, 3, 5, and 7 measure 70°, and angles 2, 4, 6, and 8 measure 110°.

Glossary

- alternate interior angles
- transversal

Lesson 14 Practice Problems

Problem 1

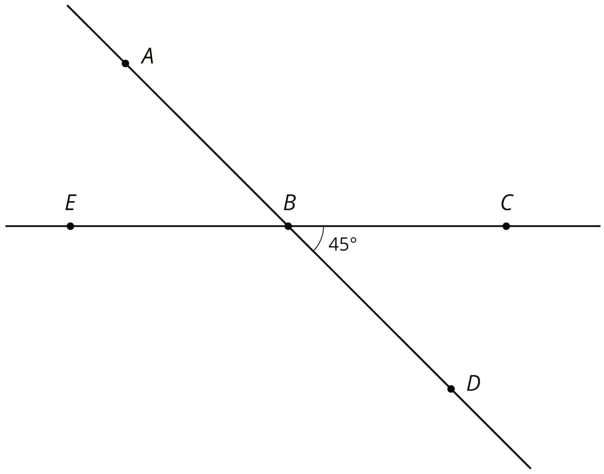
Statement

Use the diagram to find the measure of each angle.

a. $m\angle ABC$

b. $m\angle EBD$

c. $m\angle ABE$



Solution

a. 135 degrees

b. 135 degrees

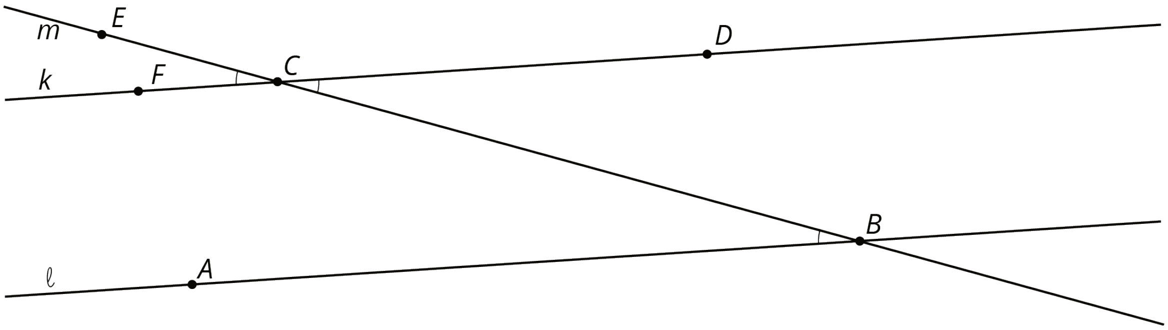
c. 45 degrees

(From Unit 1, Lesson 9.)

Problem 2

Statement

Lines k and ℓ are parallel, and the measure of angle ABC is 19 degrees.



a. Explain why the measure of angle ECF is 19 degrees. If you get stuck, consider translating line ℓ by moving B to C.

b. What is the measure of angle BCD? Explain.

Solution

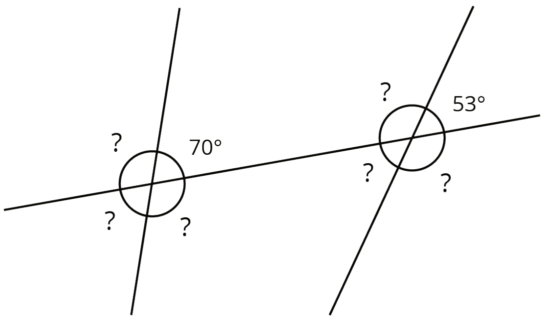
a. If ℓ is translated so that B goes to C, then l goes to k because k is parallel to ℓ. Angle ABC matches up with angle FCE after this translation, so FCE (and ECF) is also a 19 degree angle.

b. Angles ECF and BCD are congruent because they are vertical angles. Since angle ECF is a 19 degree angle, so is angle BCD.

Problem 3

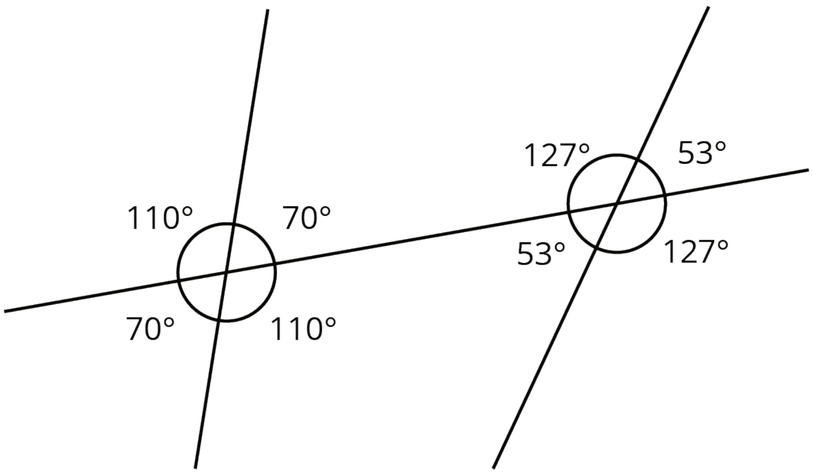
Statement

The diagram shows three lines with some marked angle measures.



Find the missing angle measures marked with question marks.

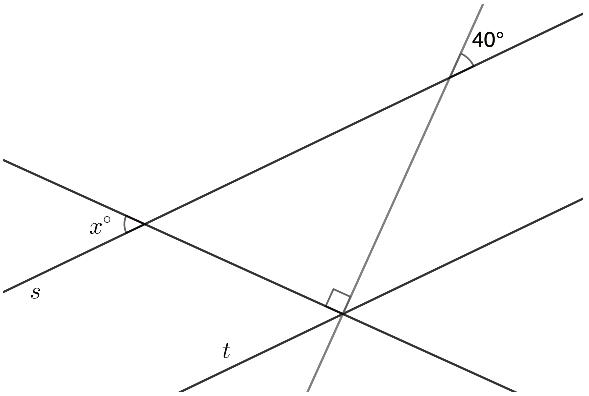
Solution



Problem 4

Statement

Lines s and t are parallel. Find the value of x.



Solution

$x = 50$. The measure of the given angle is 40 degrees. The corresponding angle on line t also measures 40 degrees. This angle is adjacent to the indicated 90-degree angle, on its right side. Similarly, the angle that measures $x°$ corresponds to the angle that is adjacent to the indicated 90-degree angle, on its left side. This gives the equation $40 + 90 + x = 180$. x is 50 degrees, because $180 - (90 + 40) = 50$.

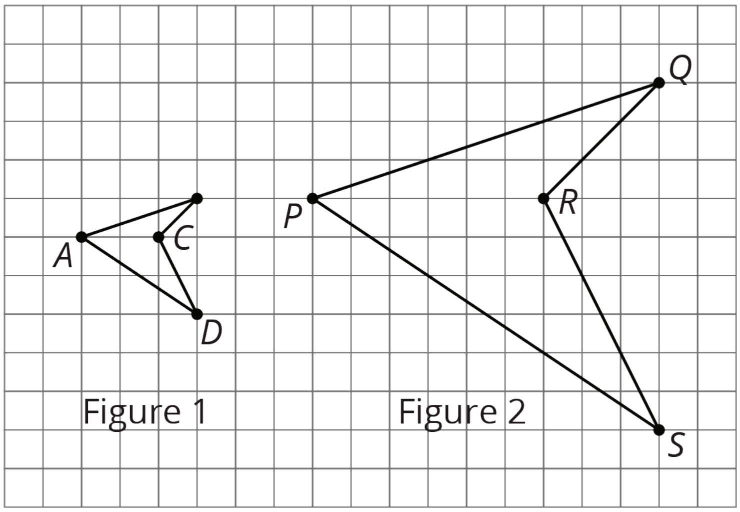
Problem 5

Statement

The two figures are scaled copies of each other.

a. What is the scale factor that takes Figure 1 to Figure 2?

b. What is the scale factor that takes Figure 2 to Figure 1?



Solution

a. 3

b. $\frac{1}{3}$