Lesson 2: Naming the Moves

Goals

- Describe (orally and in writing) the movement of shapes informally and formally using the terms “clockwise,” “counterclockwise,” “translations,” “rotations,” and “reflections” of figures.

Learning Targets

- I can identify corresponding points before and after a transformation.
- I know the difference between translations, rotations, and reflections.

Lesson Narrative

In this lesson, students begin to describe a given translation, rotation, or reflection with greater precision and are introduced to the terms **translation**, **rotation**, and **reflection**. The collective terms “transformation” and “rigid transformation” are not used until later lessons. Students are introduced to the terms **clockwise** and **counterclockwise**. Students then use this language to identify the individual moves on various figures.

Students engage in MP6 as they experiment with ways to describe moves precisely enough for another to understand their meaning.

Alignments

Building On

- 4.MD.C.5: Recognize angles as geometric shapes that are formed wherever two rays share a common endpoint, and understand concepts of angle measurement:

Addressing

- 8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:

Instructional Routines

- MLR8: Discussion Supports
- Take Turns
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed cards, cut from copies of the blackline master

Required Preparation

Print and cut up cards from the Translations, Rotations, and Reflections blackline master. Prepare 1 copy for every 3 students.

Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles.

Access to tracing paper is particularly important. Each student will need about 10 small sheets of tracing paper (commercially available “patty paper” is ideal). If using large sheets of tracing paper, such as 8.5 inches by 11 inches, cut each sheet into fourths.

Student Learning Goals

Let’s be more precise about describing moves of figures in the plane.

2.1 A Pair of Quadrilaterals

Warm Up: 10 minutes
Students estimate an angle of rotation. While they do not need to use a protractor, a protractor is an ideal tool and allows them to estimate the angle measure more accurately. Monitor for how students report the measure of the angle: do they round to the nearest degree, to the nearest 5 degrees?

Building On

- 4.MD.C.5

Addressing

- 8.G.A.1

Instructional Routines

- Think Pair Share

Launch

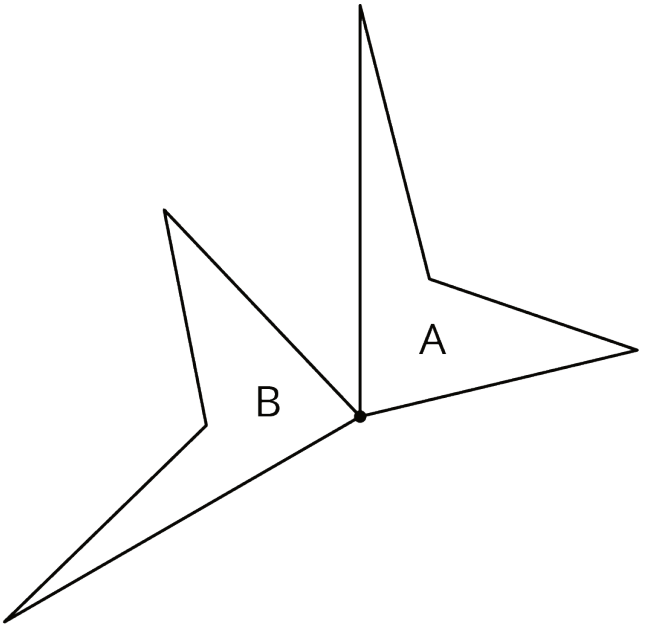
Arrange students in groups of 2–4. Provide access to geometry toolkits. Display the two quadrilateral figures for all to see. (They should also look at the task statement in their workbooks.) Ask students to give a discreet hand signal when they have an estimate for the angle of rotation. Give students 2 minutes of quiet think time and then time to share their thinking with their group before a whole-class discussion.

Anticipated Misconceptions

Students may not be sure which angle to measure. They may measure the acute angle between Shape A and Shape B. Ask these students to trace Shape A on tracing paper and rotate it by that angle to see that this does not give Shape B.

Student Task Statement

Quadrilateral A can be rotated into the position of Quadrilateral B.

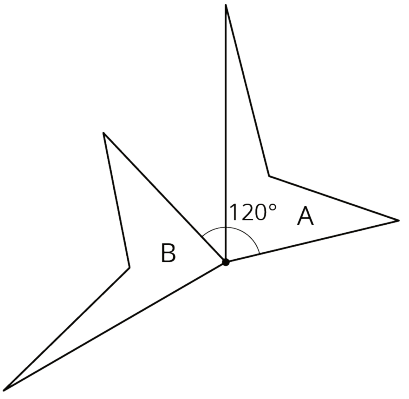


Estimate the angle of rotation.

Student Response

Answers vary. Sample response: About 120 degrees (counterclockwise)

This figure doesn't need to be part of students' responses but is provided as an example of an angle between two segments that could be measured to find the angle of rotation from A to B.



Activity Synthesis

Invite students to share their estimates for the angle of rotation. Ask students how they knew, for example, that the angle is *more* than 90 degrees (because the angle is obtuse) but *less* than 180 degrees (because the angle is less than a straight line).

Introduce or reiterate the language of **clockwise** (for rotating in the direction the hands on a clock move) and **counterclockwise** (for rotating in the opposite direction). In this case, the direction of rotation is not specified but it is natural to view Figure A being rotated counterclockwise onto Figure B. Make sure to introduce the language of the *center* of rotation (the vertex shared by A and B is the center of rotation).

It may be helpful to display the picture from the task statement to support this discussion, and if possible, show the 120° counterclockwise turn dynamically.

2.2 How Did You Make That Move?

10 minutes

This activity informally introduces reflections, which appear in addition to some translations and rotations (that were introduced informally in the previous lesson). Students are given a 6-frame cartoon showing the change in position of a polygon. As in the previous lesson, they describe the moves, but this time there are reflections, which may seem impossible as physical moves unless you allow the shape to leave the plane. Students identify the new moves and try to describe them.

After the end of this activity, the three basic moves have been introduced and the next activity will introduce their names (translations, rotations, and reflections).

Addressing

- 8.G.A.1

Instructional Routines

- Think Pair Share

Launch

Keep students in the same groups, and maintain access to geometry toolkits. Give students 5 minutes of quiet work time, and then invite them to share their responses with their group. Follow with a whole-class discussion. Tell students that they will be describing moves as they did in the previous lesson, but this time there is a new move to look out for. Recall the words the class used to describe slides and turns.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. During the launch, take time to review terms that students will need to access for this activity. Invite students to suggest language or diagrams to include that will support their understanding of moving, sliding, and turning.
Supports accessibility for: Conceptual processing; Language

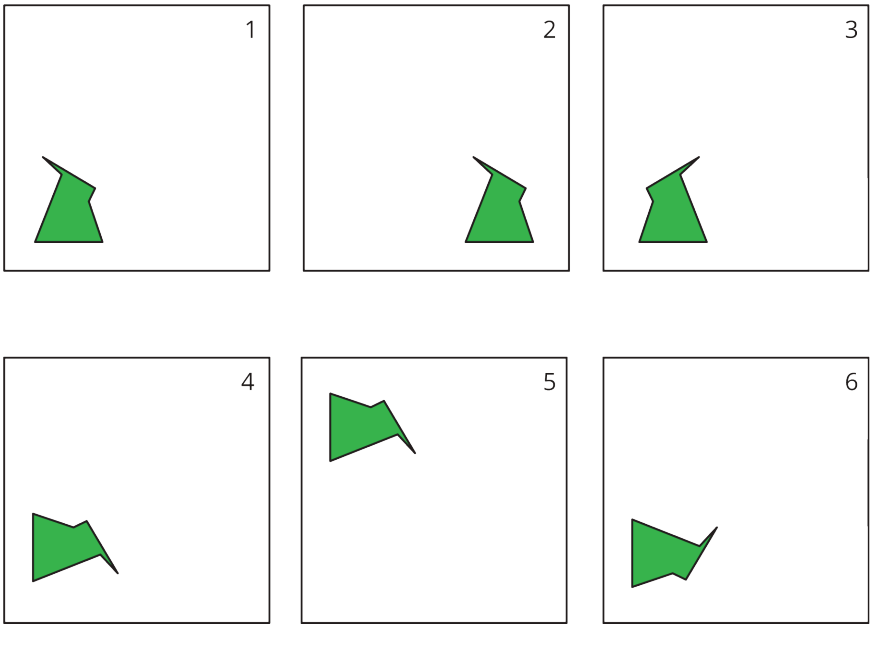
Anticipated Misconceptions

Students may see a reflection as a translation especially since the figures are not on the same frame. Ask these students to trace Frame 2 on tracing paper. Is there any way to turn it into Frame 3 by sliding it? What do they have to do to turn it into Frame 3? (They have to flip the tracing paper over, so, this is a new kind of move.)

In describing reflections, students may confuse the terms horizontal and vertical. Consider posting the terms horizontal and vertical with examples in your room.

Student Task Statement

Here is another set of dance moves.



1. Describe each move or say if it is a new move.
    a. Frame 1 to Frame 2.
    b. Frame 2 to Frame 3.
    c. Frame 3 to Frame 4.
    d. Frame 4 to Frame 5.
    e. Frame 5 to Frame 6.
2. How would you describe the new move?

Student Response

Answers vary. Sample response:

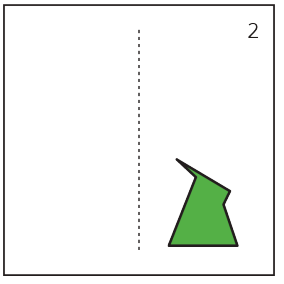
1. a. Frame 1 to Frame 2: Shift to the right
    b. Frame 2 to Frame 3: New move
    c. Frame 3 to Frame 4: Turn 90° clockwise
    d. Frame 4 to Frame 5: Shift up
    e. Frame 5 to Frame 6: New move
2. The new move is like becoming your mirror image through a mirror placed at the center of the frame. For the second move, the mirror is vertical and for the last move it is horizontal.

Activity Synthesis

The purpose of this discussion is an initial understanding that there is a third type of move that is fundamentally different from the moves encountered in the previous lesson, because it reverses directions. Some possible discussion questions to help them identify these are:

- "How is the motion from panel 2 to panel 3 different than the ones we discussed yesterday?"
- "Is there anywhere else that happens in this cartoon?"
- "What features of the image help us to see that this move is happening?"

To help answer these questions, tell students to pay attention to the direction that the "beak" of the polygon is pointing, left or right. Draw a dotted vertical line in the middle of Frame 2, and say, "Here is a mirror. The polygon in Frame 3 is what the polygon in Frame 2 sees when it looks in the mirror."



Demonstrate using tracing paper or transparencies to show they are mirror images. Then ask students if there are any other mirror lines in other frames. For the second reflection, from Frame 5 to Frame 6, point out that the mirror line is now a horizontal line: in Frame 5 the beak is pointing down, and in Frame 6 the beak is pointing up, with the head on the right of the body in both cases. Contrast this with a rotation through 180°, which would put the head on the left of the body. Demonstrate with tracing paper or transparencies.

2.3 Card Sort: Move

15 minutes (there is a digital version of this activity)
The purpose of this card sort activity is to give students further practice identifying translations, rotations, and reflections, and in the discussion after they have completed the task, introduce those terms. In groups of 3 they sort 9 cards into categories. There are 3 translations, 3 rotations, and 3 reflections. Students explain their categories and come to agreement on them.

On the blackline master, there are actually 12 cards. The last three show slightly more complicated moves than the first 9. These can be withheld, at first, and used if time permits.

Students might identify only 2 categories, putting the reflections with the translations (in the case of Card 3) or the rotations (in the case of Card 5). As students work, monitor for groups who have sorted the cards into translations, rotations, and reflections (though not necessarily using those words). Also monitor for descriptions of corresponding points such as “these points go together” or “here are before and after points.”

Addressing

- 8.G.A.1

Instructional Routines

- MLR8: Discussion Supports
- Take Turns

Launch

Arrange students into groups of 3, and provide access to geometry toolkits. Give each group the first 9 cards. Reserve the last 3 cards for use if time permits.

Tell students that their job is to sort the cards into categories by the type of move that they show. After they come to consensus about which categories to use, they take turns placing a card into a category and explaining why they think their card goes in that category. When it is not their turn, their job is to listen to their partner's reasoning and make sure they understand. Consider conducting a short demonstration with a student of productive ways to communicate during this activity. For example, show what it looks like to take turns, explain your thinking, and listen to your partner's thinking.

Give students about 10 minutes to sort the cards. Do *not* explicitly instruct students at the beginning to use the words translations, rotations, and reflections. Monitor for a group who uses these categories, even if they use different names for them. If time permits, distribute the remaining 3 cards. Follow with whole-class discussion.

If using the digital activity, ask the students to close their devices, at first. After they have come to agreement about how their cards should be sorted, they can open their devices and use the applets to help them refine the way they describe the moves.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Display sentence frames to support students when they explain their strategy. For example, "This card belongs in _____ category because . . ." or "I noticed that this image _____ so I . . ."
Supports accessibility for: Language; Social-emotional skills

Anticipated Misconceptions

Students may struggle to differentiate between the three moves, confusing reflections with either translations or rotations. After they make their best decision, encourage these students to use tracing paper to justify their response. In Card 10, students may be confused when the translated figure overlaps the original. For Card 4, students may first think that this is a rotation (much like Cards 6 and 9). Encourage these students here to use tracing paper to check their answers.

Student Task Statement

Your teacher will give you a set of cards. Sort the cards into categories according to the type of move they show. Be prepared to describe each category and why it is different from the others.

Student Response

Translations: 1, 7, 8, 10

Rotations: 2, 6, 9, 12

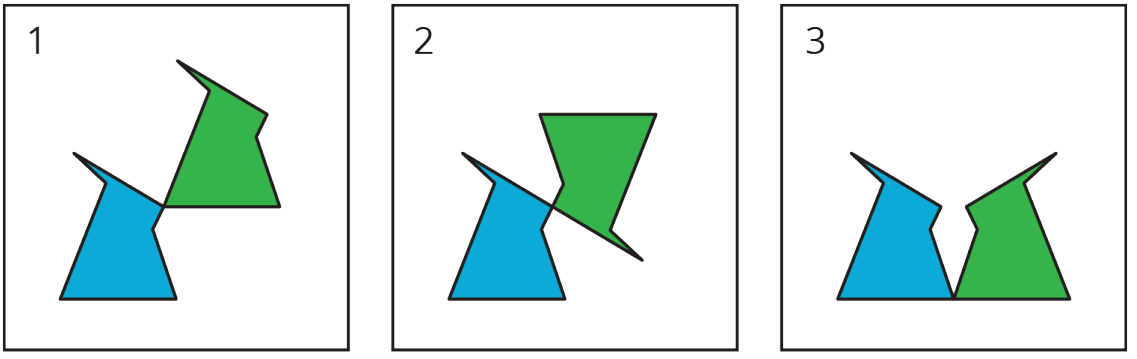
Reflections: 3, 4, 5, 11

To detect if one figure is a translation of another, look to see if it is still sitting in exactly the same way, e.g., the two figures have the same orientation and are sitting on the same base. To detect if one figure is a rotation of another, look to see if one figure is not standing up in exactly the same way as the other but appears to be turned. Reflections can be confused with both translations (if the two figures are still on the same base) and rotations (if they appear to be turned). The way to detect a reflection in these examples is to choose a feature of the figure that exists on one side of it but not the other (e.g., the sharp "rabbit ears" in this activity) and see if it is pointing to the left in one figure and to the right in the other. (Alternatively, up and down if the line of reflection is horizontal.)

Activity Synthesis

Select one or more groups to share the names of their categories. Select one or more groups to share how they sorted the cards into the categories. Ask the class if they disagree with any of the choices, and give students opportunities to justify their reasoning (MP3).

Introduce the terms **translation**, **rotation**, and **reflection**. It may be helpful to display an example of each to facilitate discussion:



Alternatively, you may wish to display the geogebra applets used in the digital version of the student materials to facilitate discussion:

- Translation: ggbm.at/wYYvZH7A
- Rotation: ggbm.at/RUtdpQmN
- Reflection: ggbm.at/nKQmSnDW

Point out ways to identify which type of move it is. Translations are a slide with no turning. Rotations are a turn. Reflections face the opposite direction. If desired, introduce the terms *image* and *corresponding points*. If we see the figures as rabbits, then the ear tips in the original figure and the ear tips in its image are *corresponding points*, for example. The *image* is the figure after a transformation is applied: for each of the cards, one figure is the image of the other figure after a translation, rotation, or reflection has been applied.

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this routine to support the introduction of new terms. As groups share how they categorized and sorted the shapes, revoice their ideas using the terms translation, rotation, and reflection. Some students may benefit from practicing words or phrases or words in context through choral repetition.
Design Principle(s): Optimize output (for explanation)

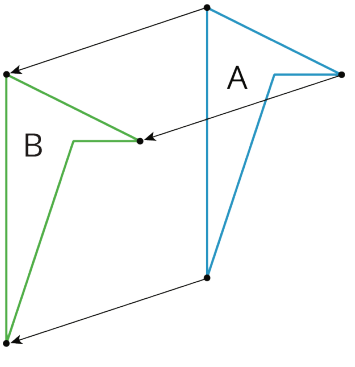
Lesson Synthesis

Questions for discussion:

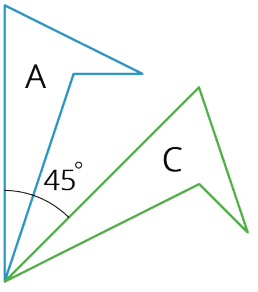
- "We encountered a new type of move that was different from yesterday. What can you tell me about it?" (It's like a mirror image, you can't make the move by sliding or turning, the figure faces the opposite direction.)
- "We gave mathematical names to the three types of moves we have seen. What are they called?" (The "slide" is called a translation, the "turn" is called a rotation, and the mirror image is called a reflection.)

Consider creating a semi-permanent display that shows these three terms and their definitions for reference throughout the unit.

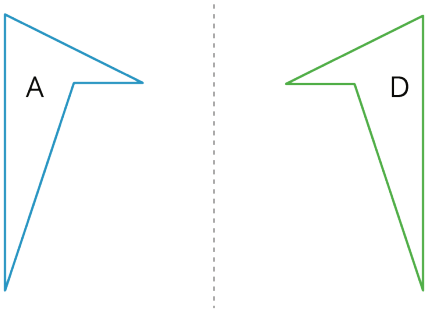
A **translation** slides a figure without turning it. Every point in the figure goes the same distance in the same direction. For example, Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.



A **rotation** turns a figure about a point, called the center of the rotation. Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be **clockwise**, going in the same direction as the hands of a clock, or **counterclockwise**, going in the other direction. For example, Figure A was rotated 45° clockwise around its bottom vertex. Figure C is a rotation of Figure A.



A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.



2.4 Is It a Reflection?

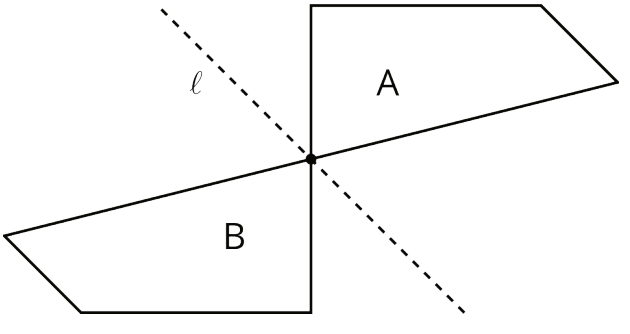
Cool Down: 5 minutes

Addressing

- 8.G.A.1

Student Task Statement

What type of move takes Figure A to Figure B?



Explain your reasoning.

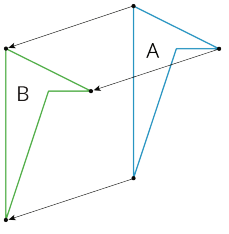
Student Response

Answers vary. Sample response: It is a rotation. If Figure A is turned around the point shared by Figures A and B, it can land on Figure B.

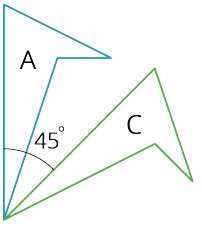
Student Lesson Summary

Here are the moves we have learned about so far:

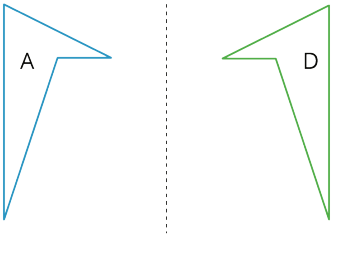
- A **translation** slides a figure without turning it. Every point in the figure goes the same distance in the same direction. For example, Figure A was translated down and to the left, as shown by the arrows. Figure B is a translation of Figure A.



- A **rotation** turns a figure about a point, called the center of the rotation. Every point on the figure goes in a circle around the center and makes the same angle. The rotation can be **clockwise**, going in the same direction as the hands of a clock, or **counterclockwise**, going in the other direction. For example, Figure A was rotated 45° clockwise around its bottom vertex. Figure C is a rotation of Figure A.



- A **reflection** places points on the opposite side of a reflection line. The mirror image is a backwards copy of the original figure. The reflection line shows where the mirror should stand. For example, Figure A was reflected across the dotted line. Figure D is a reflection of Figure A.



We use the word *image* to describe the new figure created by moving the original figure. If one point on the original figure moves to another point on the new figure, we call them *corresponding* points.

Glossary

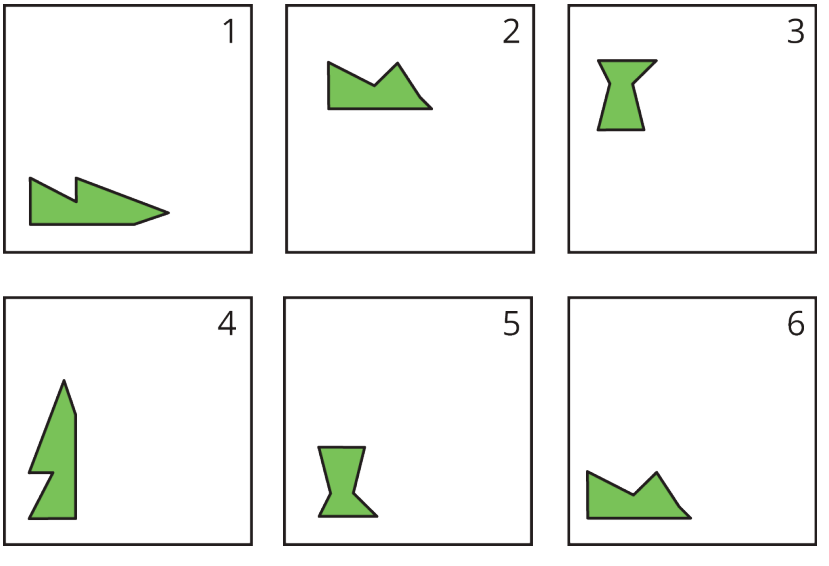
- clockwise
- counterclockwise
- reflection
- rotation
- translation

Lesson 2 Practice Problems

Problem 1

Statement

Each of the six cards shows a shape.



a. Which pair of cards shows a shape and its image after a rotation?

b. Which pair of cards shows a shape and its image after a reflection?

Solution

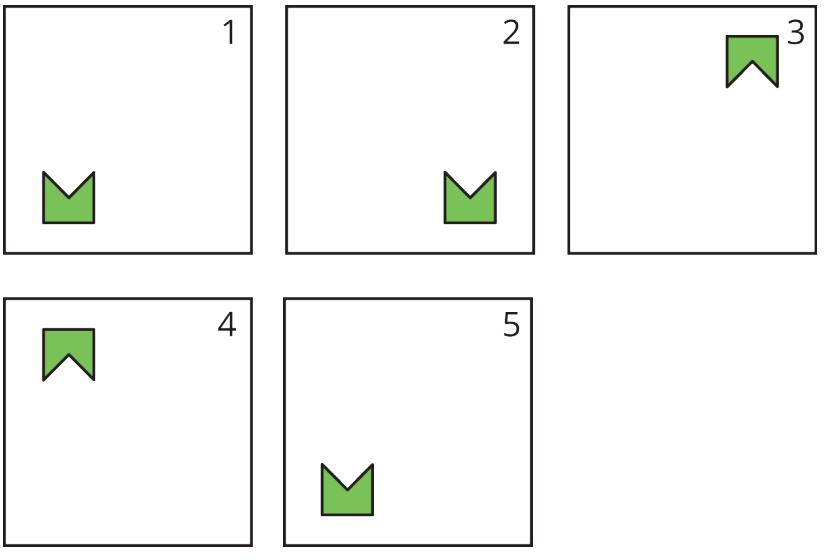
a. Cards 1 and 4

b. Cards 3 and 5

Problem 2

Statement

The five frames show a shape's different positions.



Describe how the shape moves to get from its position in each frame to the next.

Solution

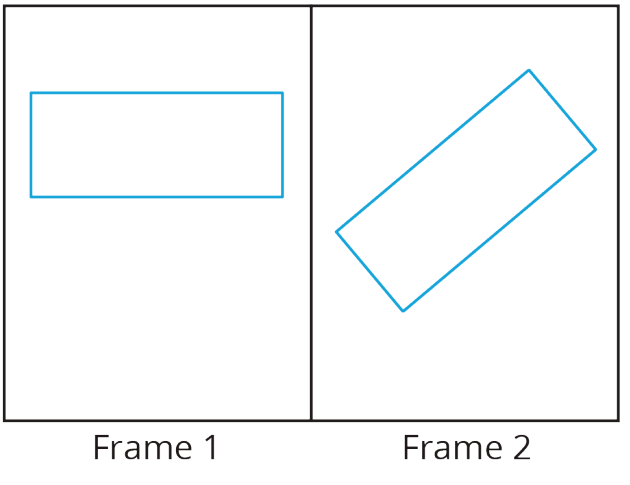
To get from Position 1 to Position 2, the shape moves to the right. To get from Position 2 to Position 3, the shape flips over a horizontal line. To get from Position 3 to Position 4, the shape moves to the left. To get from Position 4 to Position 5, the shape flips over a horizontal line again. The shape has then returned to its original position in Position 1.

Alternatively, to get from Position 1 to Position 2 or from Position 3 to Position 4, the shape may flip over a vertical line. Since the shape is symmetric, a flip looks the same as a shift here. To get from Position 2 to Position 3 or from Position 4 to Position 5, the shape may be rotated 180 degrees about a point not on the polygon.

Problem 3

Statement

The rectangle seen in Frame 1 is rotated to a new position, seen in Frame 2.



Select **all** the ways the rectangle could have been rotated to get from Frame 1 to Frame 2.

A. 40 degrees clockwise

B. 40 degrees counterclockwise

C. 90 degrees clockwise

D. 90 degrees counterclockwise

E. 140 degrees clockwise

F. 140 degrees counterclockwise

Solution

["B", "E"]

(From Unit 1, Lesson 1.)