## Lesson 14: Coordinate Proof

* Let’s use coordinates to prove theorems and to compute perimeter and area.

### 14.1: Which One Doesn’t Belong: Coordinate Quadrilaterals

Which one doesn’t belong?

A



B



C



D



### 14.2: Name This Quadrilateral

A quadrilateral has vertices $(0,0),(4,3),(13,-9),$ and $(9,-12)$.

1. What type of quadrilateral is it? Explain or show your reasoning.
2. Find the perimeter of this quadrilateral.
3. Find the area of this quadrilateral.

#### Are you ready for more?

1. A parallelogram has vertices $(0,0),(5,1),(2,3)$, and $(7,4)$. Find the area of this parallelogram.
2. Consider a general parallelogram with vertices $(0,0),(a,b),(c,d),$ and $(a+c,b+d),$ where $a,b,c,$ and $d$ are positive. Write an expression for its area in terms of $a,b,c,$ and $d$.

### 14.3: Circular Logic

The image shows a circle with several points plotted on the circle.



1. What kind of segment is $BC$ in reference to the circle?
2. Choose one of the plotted points on the circle and call it $D$. Each student in the group should choose a different point. Draw segments $BD$ and $DC$. What does the measure of angle $BDC$ appear to be?
3. Calculate the slopes of segments $BD$ and $DC$. What do your results tell you?
4. Compare your results to those of others in your group. What did they find?
5. Based on your group’s results, write a conjecture that captures what you are seeing.

### Lesson 14 Summary



What kind of shape is quadrilateral $ABCD$? It looks like it might be a rhombus. To check, we can calculate the length of each side. Using the Pythagorean Theorem, we find that the lengths of segments $AB$ and $CD$ are $\sqrt{45}$ units, and the lengths of segments $BC$ and $DA$ are $\sqrt{37}$ units. All side lengths are between 6 and 7 units long, but they are not exactly the same. So our calculations show that $ABCD$ is not really a rhombus, even though at first glance we might think it is.

We did just show that two pairs of opposite sides of $ABCD$ are congruent. This means that $ABCD$ must be a parallelogram. Checking slopes confirms this. Sides $AB$ and $CD$ each have slope $\frac{1}{2}$. Sides $BC$ and $DA$ each have slope 6.

Can we find the area of triangle $EFG$? That seems tricky, because we don’t know the height of the triangle using $EG$ as the base. However, angle $EFG$ seems like it could be a right angle. In that case, we could use sides $EF$ and $FG$ as the base and height.

To see if $EFG$ is a right angle, we can calculate slopes. The slope of $EF$ is $\frac{8}{6}$ or $\frac{4}{3}$, and the slope of $FG$ is $-\frac{3}{4}$. Since the slopes are opposite reciprocals, the segments are perpendicular and angle $EFG$ is indeed a right angle. This means that we can think of $EF$ as the base and $FG$ as the height. The length of $EF$ is 10 units and the length of $FG$ is 5 units. So the area of triangle $EFG$ is 25 square units because $\frac{1}{2}⋅10⋅5=25$.



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