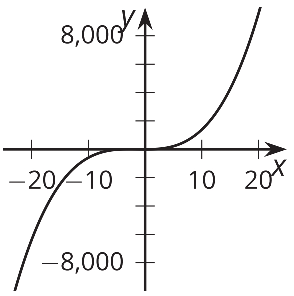
Lesson 8: End Behavior (Part 1)

- Let's investigate the shape of polynomials.

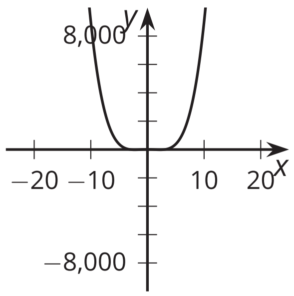
8.1: Notice and Wonder: A Different View

What do you notice? What do you wonder?

$y = x^3 + 4x^2 - x - 4$



$y = x^4 - 10x^2 + 9$



8.2: Polynomial End Behavior

1. For your assigned polynomial, complete the column for the different values of x. Discuss with your group what you notice.

x	$y = x^2 + 1$	$y = x^3 + 1$	$y = x^4 + 1$	$y = x^5 + 1$
-1000				
-100				
-10				
-1				
1				
10				
100				
1000				

2. Sketch what you think the **end behavior** of your polynomial looks like, then check your work using graphing technology.

Are you ready for more?

Mai is studying the function $p(x) = -\frac{1}{100}x^3 + 25{,}422x^2 + 8x + 26$. She makes a table of values for p with $x = \pm 1, \pm 5, \pm 10, \pm 20$ and thinks that this function has large positive output values in both directions on the x-axis. Do you agree with Mai? Explain your reasoning.

8.3: Two Polynomial Equations

Consider the polynomial $y = 2x^5 - 5x^4 - 30x^3 + 5x^2 + 88x + 60$.

1. Identify the degree of the polynomial.

2. Which of the 6 terms, $2x^5$, $5x^4$, $30x^3$, $5x^2$, $88x$, or 60, is greatest when:
    a. $x = 0$

    b. $x = 1$

    c. $x = 3$

    d. $x = 5$

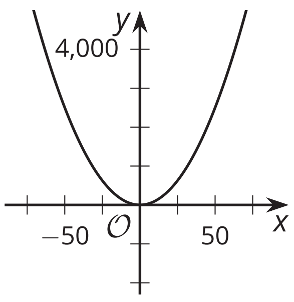
3. Describe the end behavior of the polynomial.

Lesson 8 Summary

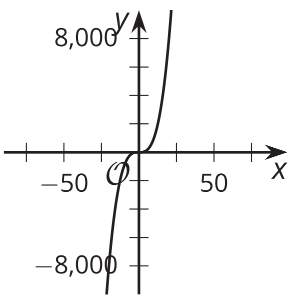
We know that if the expression for a polynomial function f written in factored form has the factor $(x - a)$, then a is a zero of f (that is, $f(a) = 0$) and the point $(a, 0)$ is on the graph of the function. But what about other values of x? In particular, as we consider values of x that get larger and larger in either the negative or positive direction, what happens to the values of $f(x)$?

The answer to this question depends on the degree of the polynomial, because any negative real number raised to an even power results in a positive number. For example, if we graph $y = x^2$, $y = x^3$ and $y = x^4$ and zoom out, we see the following:

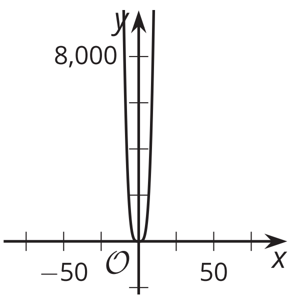
$y = x^2$



$y = x^3$



$y = x^4$



For both $y = x^2$ and $y = x^4$, large positive values of x or large negative values of x each result in large positive values of y. But for $y = x^3$, large positive values of x result in large positive values of y, while large negative values of x result in large negative values of y.

Consider the polynomial $P(x) = x^4 - 30x^3 - 20x^2 + 1000$. The leading term, x^4, almost seems smaller than the other 3 terms. For certain values of x, this is even true. But, for values of x far away from zero, the leading term will always have the greatest value. Can you see why?

x	x^4	$-30x^3$	$-20x^2$	1000	$P(x)$
-500	62,500,000,000	3,750,000,000	-5,000,000	1,000	66,245,001,000
-100	100,000,000	30,000,000	-200,000	1,000	129,801,000
-10	10,000	30,000	-2,000	1,000	39,000
0	0	0	0	1,000	1000
10	10,000	-30,000	-2,000	1,000	-21,000
100	100,000,000	-30,000,000	-200,000	1,000	69,801,000
500	62,500,000,000	-3,750,000,000	-5,000,000	1,000	58,745,001,000

The value of the leading term x^4 determines the **end behavior** of the function, that is, how the outputs of the function change as we look at input values farther and farther from 0. In the case of $P(x)$, as x gets larger and larger in the positive and negative directions, the output of the function gets larger and larger in the positive direction.