Lesson 3: Grid Moves

Goals

- Describe (orally) the moves needed to perform a transformation.
- Draw and label the image and “corresponding points” of figures that result from translations, rotations, and reflections.
- Draw the “image” of a figure that results from a translation, rotation, and reflection in square and isometric grids and justify (orally) that the image is a transformation of the original figure.

Learning Targets

- I can decide which type of transformations will work to move one figure to another.
- I can use grids to carry out transformations of figures.

Lesson Narrative

Prior to this lesson, students have learned the names for the basic moves (translation, rotation, and reflection) and have learned how to identify them in pictures. In this lesson, they apply translations, rotations, and reflections to figures. They also label the image of a point P as P'. While not essential, this practice helps show the structural relationship (MP7) between a figure and its image.

Students also encounter the isometric grid (one made of equilateral triangles with 6 meeting at each vertex). They perform translations, rotations, and reflections both on a square grid and on an isometric grid. Expect a variety of approaches, mainly making use of tracing paper (MP5) but students may also begin to notice how the structure of the different grids helps draw images resulting from certain moves (MP7).

For classrooms using the digital version of the materials: This is the lesson where students learn to use the transformation tools in Geogebra.

Alignments

Building On

- 7.G.A: Draw, construct, and describe geometrical figures and describe the relationships between them.

Addressing

- 8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:

Building Towards

- 8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:

Instructional Routines

- MLR8: Discussion Supports
- Notice and Wonder

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Required Preparation

Make sure students have access to items in their geometry toolkits: tracing paper, graph paper, colored pencils, scissors, ruler, protractor, and an index card to use as a straightedge or to mark right angles.

For classrooms using the print version of the materials: Access to tracing paper is particularly important. Each student will need about 10 small sheets of tracing paper (commercially available "patty paper" is ideal). If using large sheets of tracing paper, such as 8.5 inches by 11 inches, cut each sheet into fourths.

For classrooms using the digital version of the materials: If you have access to extra help from a tech-savvy person, this would be a good day to request their presence in your class.

> **Student Learning Goals**
>
> Let’s transform some figures on grids.

3.1 Notice and Wonder: The Isometric Grid

Warm Up: 10 minutes
The purpose of this warm-up is to familiarize students with an isometric grid. While students may notice and wonder many things, characteristics such as the measures of the angles in the grid and the diagonal parallel lines will be important properties for students to notice in their future work performing transformations on the isometric grid. Students are not expected to know each angle in an equilateral triangle is 60 degrees, but after previous experience with supplementary angles, circles and rotations, they may be able to explain why each smaller angle is 60 degrees. Many things they notice may be in comparison to the square grid paper which is likely more familiar.

Building On

- 7.G.A

Building Towards

- 8.G.A.1

Instructional Routines

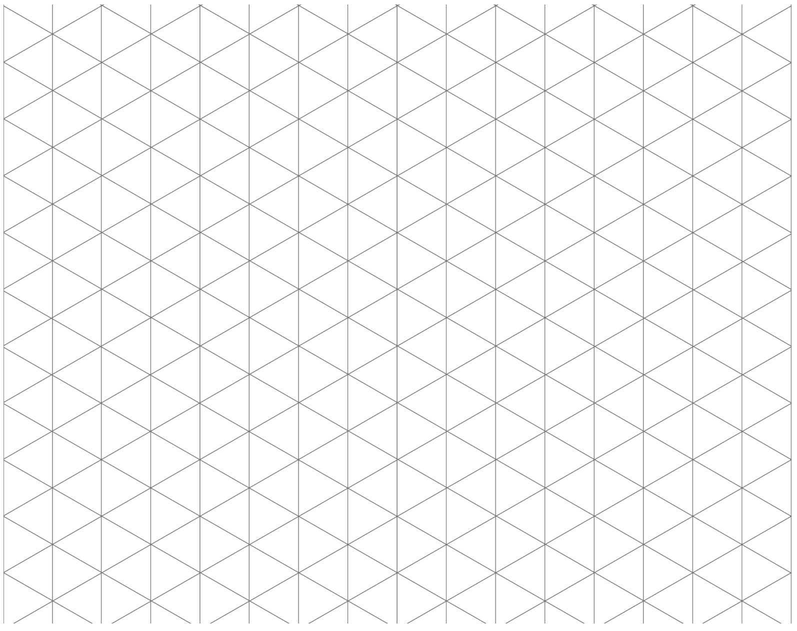
- Notice and Wonder

Launch

Arrange students in groups of 2. Tell students that they will look at an image. Their job is to think of at least one thing they notice and at least one thing they wonder. Display the image for all to see. Ask students to give a signal when they have noticed or wondered about something. Give students 1 minute of quiet think time, and then 1 minute to discuss the things they notice with their partner, followed by a whole-class discussion.

Student Task Statement

What do you notice? What do you wonder?



Student Response

Things students may notice:

- There are three sets of parallel grid lines.
- The line segments form equilateral triangles.
- The individual angles in the equilateral triangles are 60 degrees.
- There are vertical lines but no horizontal lines.
- There are no 90 degree angles made by the grid lines.

- Each vertex has 6 line segments coming from it.
- The grid is made out of equilateral triangles instead of squares.

Things students may wonder:

- Why are there no 90 degree angles?
- Why are there no squares?
- Are we going to use this kind of grid?
- Why would we use this grid instead of the square grid?
- Why are there no horizontal lines?

Activity Synthesis

Ask students to share the things they noticed and wondered. Record and display their responses for all to see. If possible, record the relevant reasoning on or near the image, and show where each of the features students notice is located on the actual grid itself, such as triangles, angles, and line segments. After each response, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to the images each time. If angle measures do not come up during the conversation, ask them to think about how they could figure out the measure of each angle. Some may measure with a protractor, and some may argue that since 6 angles share a vertex where each angle is identical, each angle measures 60° because $360 \div 6 = 60$. Establish that each angle measures 60°.

3.2 Transformation Information

25 minutes (there is a digital version of this activity)
The purpose of this activity is for students to interpret the information needed to perform a transformation and draw an image resulting from the transformation.

For digital classrooms, an additional purpose of this activity is for students to learn how to use the transformation tools available in geogebra. These tools will be used throughout the unit. As they become familiar with the dynamic tools, they see that geogebra places the image of a figure based on the instructions given by the user.

Through hands-on experience with transformations, students prepare for the more precise definitions they will learn in later grades. This activity is the first time students start to use A', B', etc. to denote points in the image that correspond to A, B, etc. in the original figure. This is also a good activity to use the word "image" to describe the transformed figure—this can happen before, as, or after students work.

If students exploit the mathematical properties of the grid lines to draw transformed figures, they are making use of structure (MP7). In order to draw the transformed figures correctly, students must attend to the details of the given information (MP6).

Watch for students who use tracing paper and those who use properties of the grids to help decide where to place the transformed figures. Tracing paper may be particularly useful for the isometric grid which may be unfamiliar to some students.

Addressing

- 8.G.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Point out A' in the first question. Tell students we call point A' "A prime" and that, after a transformation, it corresponds to A in the original.

For students using print materials: Optionally, before students start working, demonstrate the mechanics of performing each type of transformation using tracing paper. Distribute about 10 small sheets of tracing paper to each student (or ensure they can find it in their geometry toolkits). Give students about 10 minutes of quiet work time followed by whole-class discussion.

For students using digital materials: Depending on the needs of your class, either demonstrate how each transformation tool works in the applet, or instruct students to read and follow the instructions for working the applets. It would work well to demonstrate the first, third, and fourth items and allow students to complete the other items independently.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with a physical demonstration of using tracing paper to perform each type of transformation to support connections between new situations and prior understandings. Ask students, "What does this demonstration have in common with previous activities where both images were given?" or "How does the point A' correspond to the point A?"
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Access for English Language Learners

Listening, Representing: MLR8 Discussion Supports. To help develop students' meta-awareness and understanding of the task expectations, think aloud as you transform the quadrilateral in the question about rotating quadrilateral $ABCD$ 60° counterclockwise using center B. As you talk, model mathematical language use and highlight the relationship between quadrilateral $ABCD$, the image (i.e., quadrilateral $A'B'C'D'$), and the steps taken to rotate quadrilateral $ABCD$.
Design Principle(s): Maximize meta-awareness

Anticipated Misconceptions

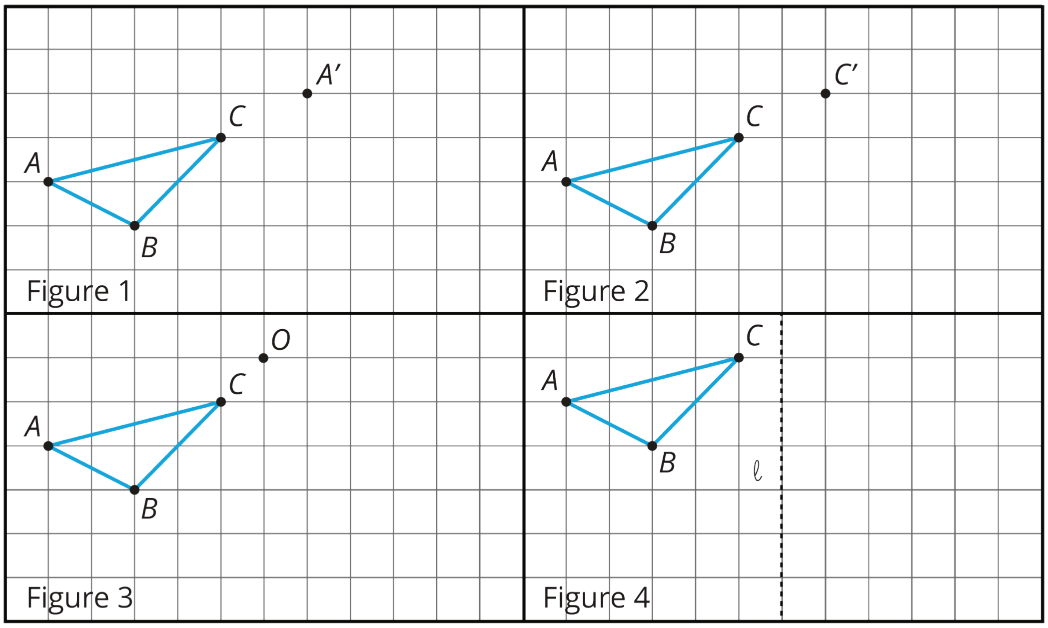
Students may struggle to understand the descriptions of the transformations to carry out. For these students, explain the transformations using the words they used in earlier activities, such as "slide," "turn," and "mirror image" to help them get started. Students may also struggle with reflections that are not over horizontal or vertical lines.

Some students may need to see an actual mirror to understand what reflections do, and the role of the reflection line. If you have access to rectangular plastic mirrors, you may want to have students check their work by placing the mirror along the proposed mirror line.

Working with the isometric grid may be challenging, especially rotations and reflections across lines that are not horizontal or vertical. For the rotations, you may want to ask students what they know about the angle measures in an equilateral triangle. For reflections, the approach of using a mirror can work or students can look at individual triangles in the grid, especially those with a side on the line of reflection, and see what happens to them. After checking several triangles, they develop a sense of how these reflections behave.

Student Task Statement

Your teacher will give you tracing paper to carry out the moves specified. Use A', B', C', and D' to indicate vertices in the new figure that correspond to the points A, B, C, and D in the original figure.

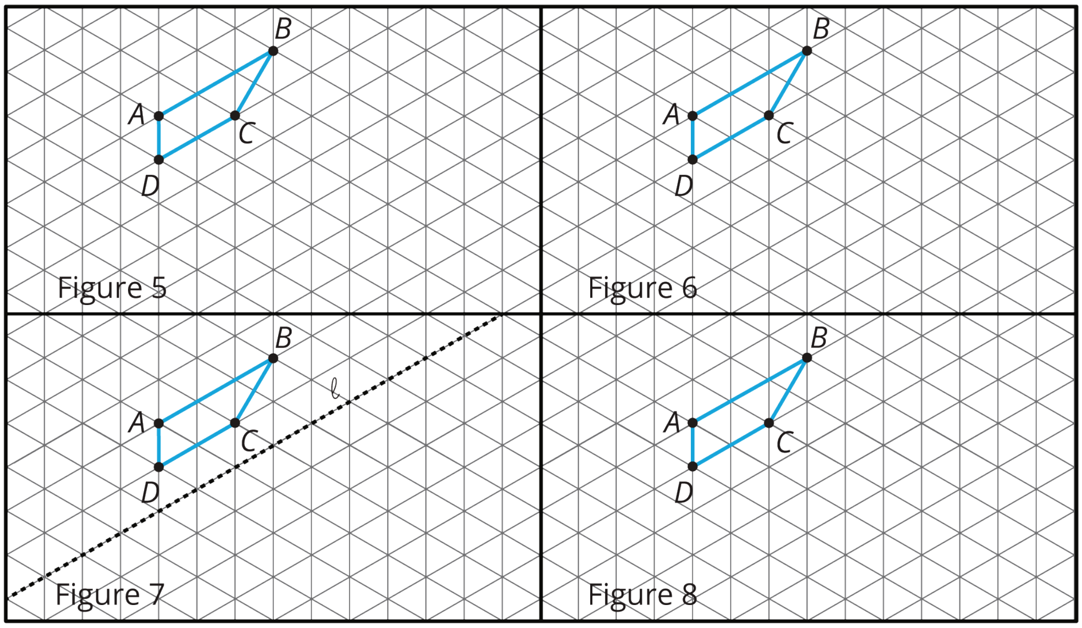


1. In Figure 1, translate triangle ABC so that A goes to A'.

2. In Figure 2, translate triangle ABC so that C goes to C'.

3. In Figure 3, rotate triangle ABC 90° counterclockwise using center O.

4. In Figure 4, reflect triangle ABC using line ℓ.



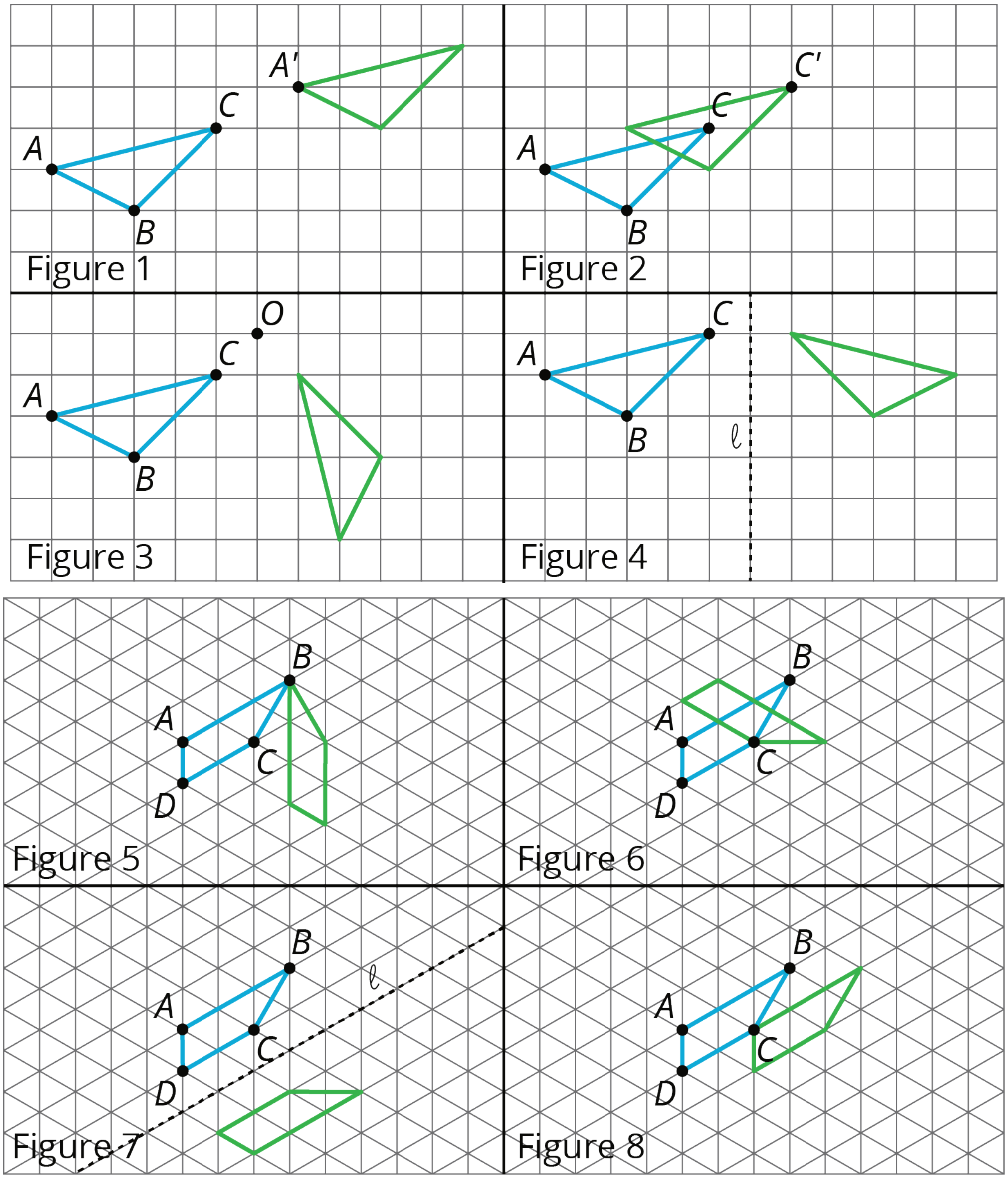
5. In Figure 5, rotate quadrilateral $ABCD$ 60° counterclockwise using center B.

6. In Figure 6, rotate quadrilateral $ABCD$ 60° clockwise using center C.

7. In Figure 7, reflect quadrilateral $ABCD$ using line ℓ.

8. In Figure 8, translate quadrilateral $ABCD$ so that A goes to C.

Student Response



Sample strategy:

- Trace the figure onto tracing paper, and then move the tracing paper according to the description of each move. Observe where the tracing paper ends up, and draw a copy of the figure at that location.
- Use the structure of the grid to move each vertex of the original figure according to the description of each move. For example in Figure 1, point A moves up 2 and right 6 to A'. A translation is a slide, so each vertex makes this same move along the grid from its original location.

Are You Ready for More?

The effects of each move can be "undone" by using another move. For example, to undo the effect of translating 3 units to the right, we could translate 3 units to the left. What move undoes each of the following moves?

1. Translate 3 units up
2. Translate 1 unit up and 1 unit to the left
3. Rotate 30 degrees clockwise around a point P
4. Reflect across a line ℓ

Student Response

1. Translate 3 units down.
2. Translate 1 unit down and 1 unit to the right.
3. Rotate 30 degrees counterclockwise around P.
4. Reflect again across ℓ.

Activity Synthesis

Ask students to share how they found the images, and highlight the information they needed in each to perform the transformation. Invite students who used tracing paper to share how they found the images and also ask students what mathematical patterns they found. For example, for the reflection in Figure 4, ask where some intersections of grid lines go (they stay on the same horizontal line and go to the other side of ℓ, the same distance away). How can this be used to identify the image of $\triangle ABC$?

Ask students how working on the isometric grid is similar to working on a regular grid and how it is different. Possible responses include:

- Translations work the same way, identifying how far and in which direction to move the shape
- Rotations also work the same way but the isometric grid works well for multiples of 60 degrees (with center at a grid point), while the regular grid works well for multiples of 90 degrees (also with center at a grid point).
- Reflections on the isometric grid require looking carefully at the triangular pattern to place the reflection in the right place. Like for the regular grid, these reflections are difficult to visualize if the line of reflection is not a grid line.

Lesson Synthesis

Display one example each of a translation, a rotation, and a reflection. Choose one of each from the activity, or create new examples. Ask students:

- “What are important things to keep in mind when we want to do a [translation, rotation, reflection]?”
- “What is something new that you learned today about [translations, rotations, reflections]?”
- “Describe the two different kinds of grids we saw. What was the same and what was different about them?”

3.3 Some are Translations and Some Aren’t

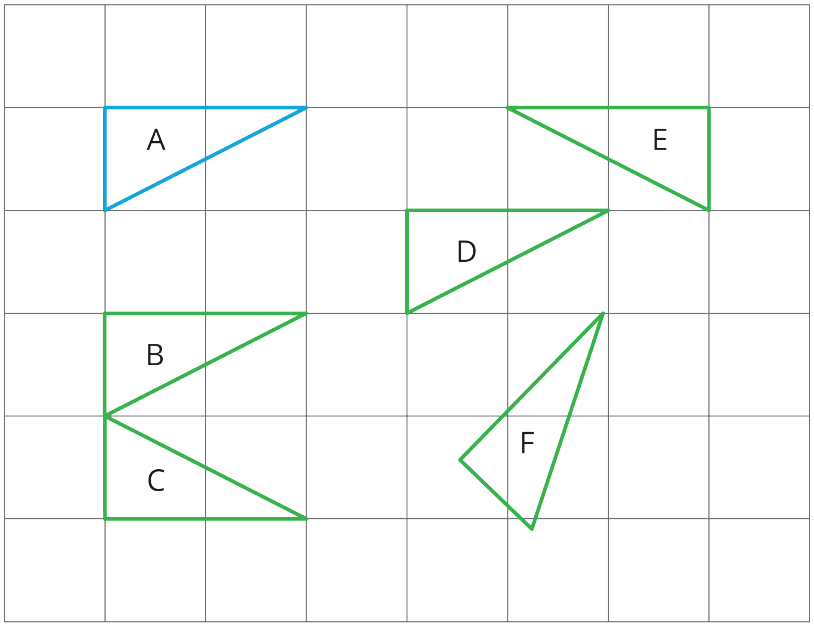
Cool Down: 5 minutes
This is a quick check to see if students can distinguish translations from rotations and reflections.

Addressing

- 8.G.A.1

Student Task Statement

Which of these triangles are translations of Triangle A? Select **all** that apply.

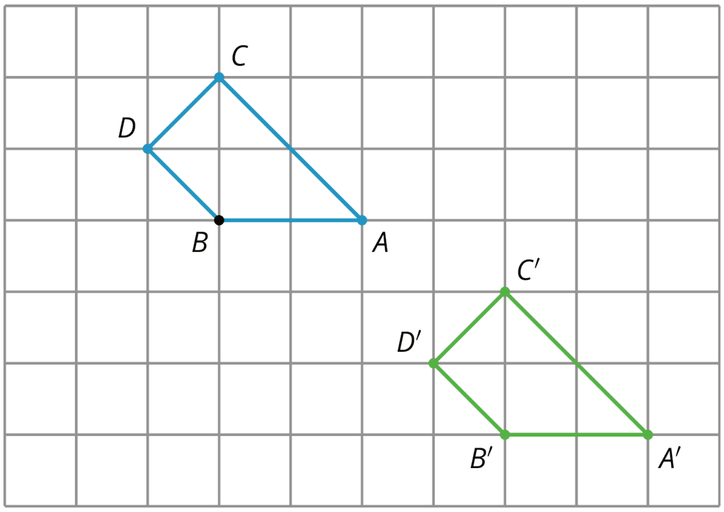


Student Response

Triangle B and Triangle D. Triangles C and E are reflections of Triangle A, while Triangle F is rotated.

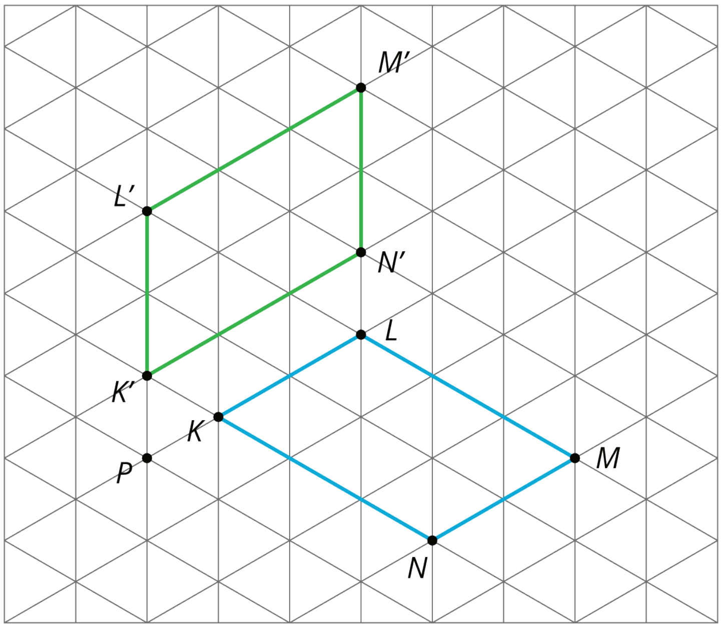
Student Lesson Summary

When a figure is on a grid, we can use the grid to describe a transformation. For example, here is a figure and an **image** of the figure after a move.



Quadrilateral $ABCD$ is translated 4 units to the right and 3 units down to the position of quadrilateral $A'B'C'D'$.

A second type of grid is called an *isometric grid*. The isometric grid is made up of equilateral triangles. The angles in the triangles all measure 60 degrees, making the isometric grid convenient for showing rotations of 60 degrees.



Here is quadrilateral $KLMN$ and its image $K'L'M'N'$ after a 60-degree counterclockwise rotation around a point P.

Glossary

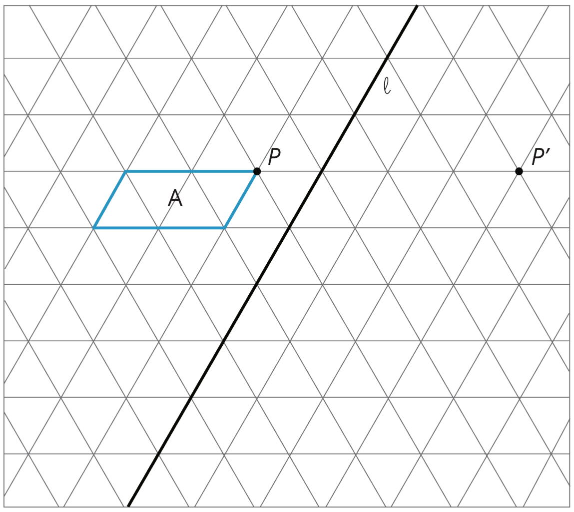
- image

Lesson 3 Practice Problems

Problem 1

Statement

Apply each transformation described to Figure A. If you get stuck, try using tracing paper.



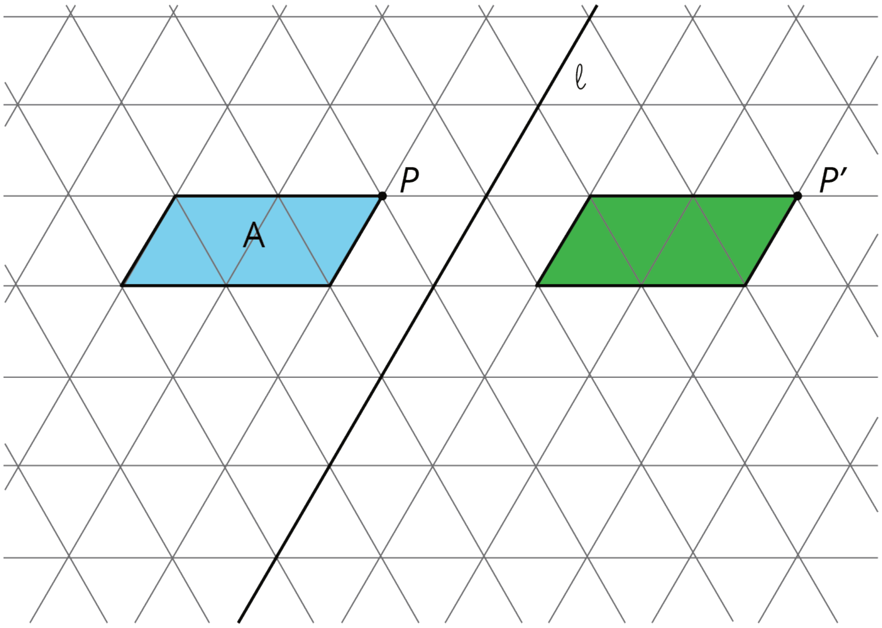
a. A translation which takes P to P'

b. A counterclockwise rotation of A, using center P, of 60 degrees

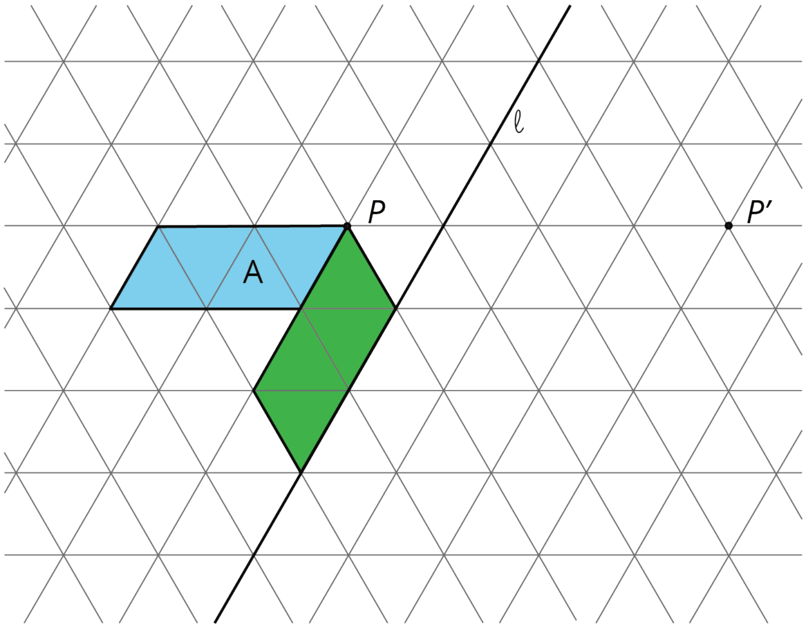
c. A reflection of A across line ℓ

Solution

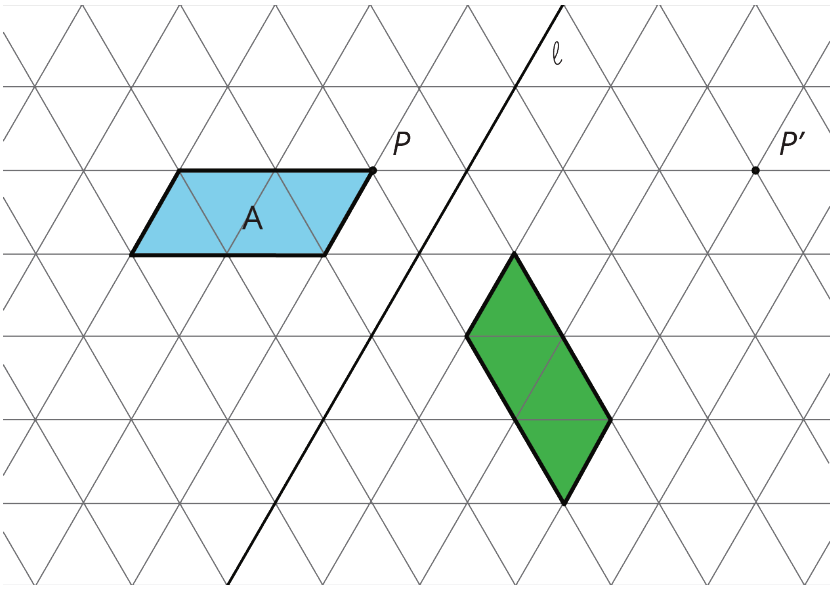
a.



b.



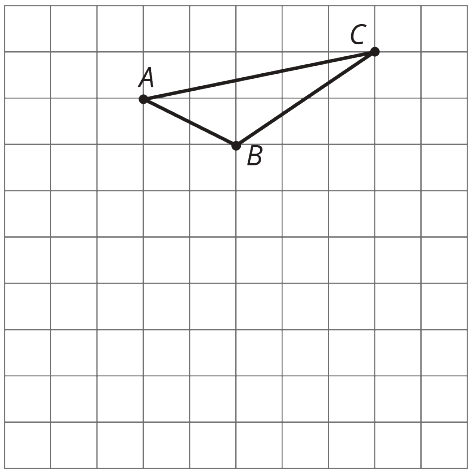
c.



Problem 2

Statement

Here is triangle ABC drawn on a grid.



On the grid, draw a rotation of triangle ABC, a translation of triangle ABC, and a reflection of triangle ABC. Describe clearly how each was done.

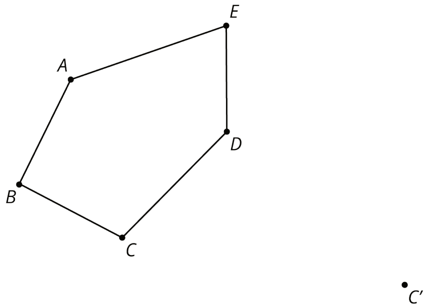
Solution

Answers vary. Sample answers: The rotation is a 90-degree counterclockwise rotation using center A. The translation is 4 units down and 3 to the left. The reflection is across a horizontal line through point B.

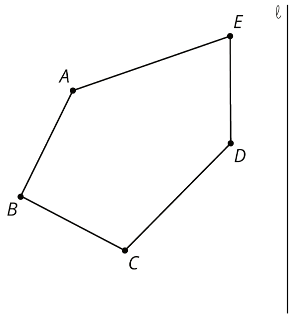
Problem 3

Statement

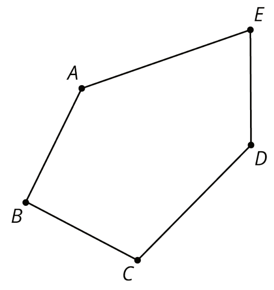
a. Draw the translated image of $ABCDE$ so that vertex C moves to C'. Tracing paper may be useful.



b. Draw the reflected image of Pentagon $ABCDE$ with line of reflection ℓ. Tracing paper may be useful.

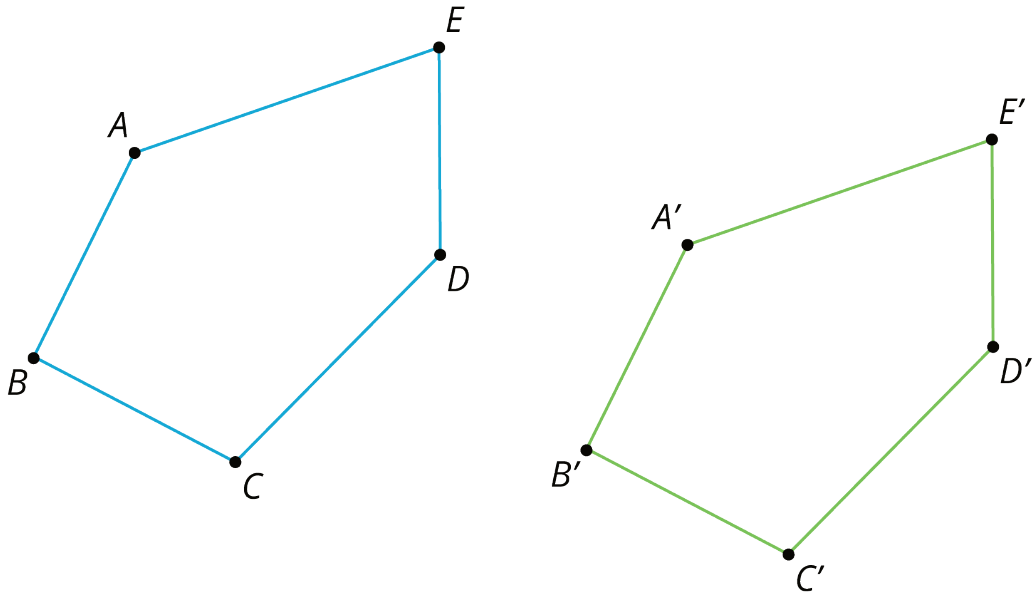


c. Draw the rotation of Pentagon $ABCDE$ around C clockwise by an angle of 150 degrees. Tracing paper and a protractor may be useful.

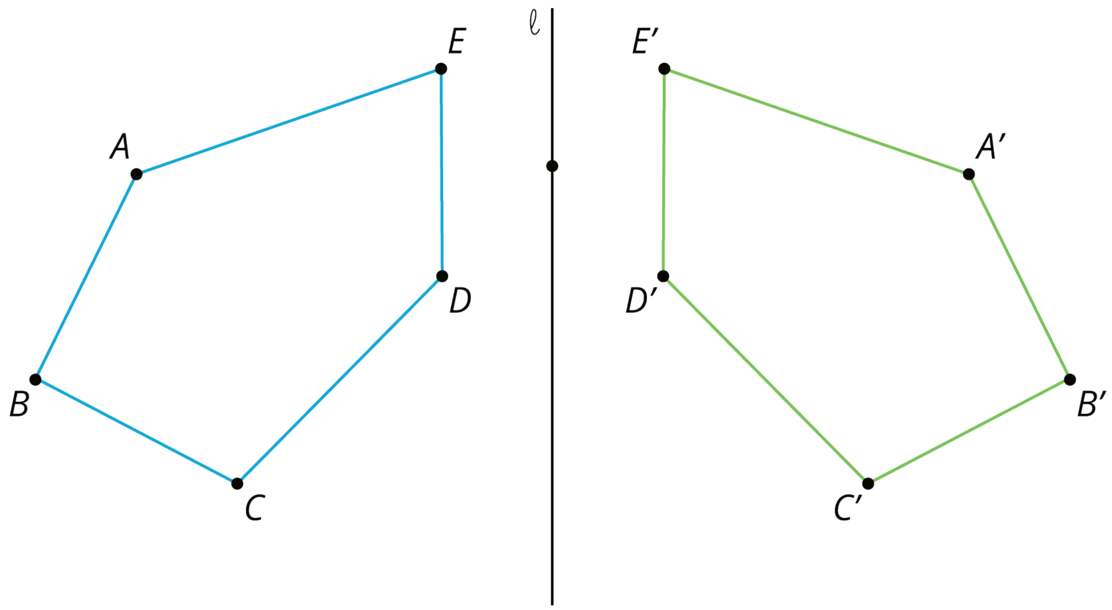


Solution

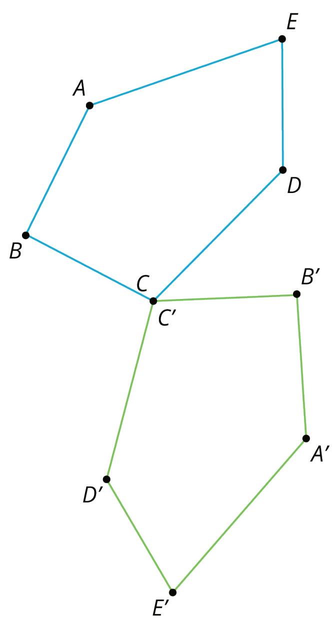
a.



b.



c.



In the picture, angle DCD' measures 150 degrees.

(From Unit 1, Lesson 2.)