Lesson 4: Making the Moves

Goals

- Comprehend that a “transformation” is a translation, rotation, reflection, or a combination of these.
- Draw a transformation of a figure using information given orally.
- Explain (orally) the “sequence of transformations” that “takes” one figure to its image.
- Identify (orally and in writing) the features that determine a translation, rotation, or reflection.

Learning Targets

- I can use the terms translation, rotation, and reflection to precisely describe transformations.

Lesson Narrative

In the previous lesson, students were introduced to the terms “translation,” “rotation,” and “reflection.” In this lesson, students understand that:

- A translation is determined by two points that specify the distance and direction of the translation.
- A rotation is determined by a point, angle of rotation, and a direction of rotation.
- A reflection is determined by a line.

These moves are called **transformations** for the first time and students draw images of figures under these transformations. They also study where shapes go under sequences of these transformations and identify the steps in a **sequence of transformations** that takes one figure to another. Note the subtle shift in language. In the previous lesson, one shape “moves” to the other shape—it is as if the original shape has agency and does the moving. In this lesson, the transformation “takes” one shape to the other shape—this language choice centers the transformation itself as an object of study.

Students using the print version may make use of tracing paper to experiment moving shapes. Students using the digital version have access to geogebra applets with which to perform transformations. Whenever students choose to make use of an appropriate tool, they are engaging in MP5. Students are also likely starting to begin thinking strategically about which transformations will take one figure to another, identifying properties of the shapes that indicate whether a translation, rotation, reflection or sequence of these will achieve this goal (MP7).

Alignments

Addressing

- 8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:

Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR2: Collect and Display
- MLR7: Compare and Connect

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed cards, cut from copies of the blackline master

Required Preparation

Print and cut up cards from the Make that Move blackline master. Prepare 1 set of cards for every 4 students.

Student Learning Goals

Let's draw and describe translations, rotations, and reflections.

4.1 Reflection Quick Image

Warm Up: 5 minutes (there is a digital version of this activity)
In this warm-up, students are asked to sketch a reflection of a given triangle and explain the strategies they used. The goal is to prompt students to notice and articulate that they can use the location of a single point and the fact that the image is a reflection of the triangle to sketch the image. To encourage students to use what they know about reflections and not count every grid line, this image is flashed for a few seconds and then hidden. It is flashed once more for students to check their thinking.

Addressing

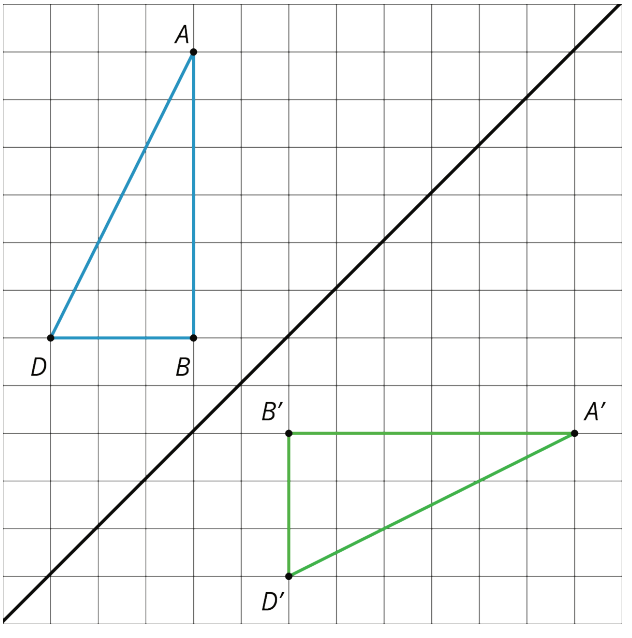
- 8.G.A.1

Launch

Before beginning, make sure students have their books or devices open to the correct page. Tell students you will show them an image of a reflection of triangle ABD for 3 seconds. Their job is to draw the image and explain any strategies they used.

For classes using the digital version of the materials, display the applet and demonstrate the use of the various tools. Give students a minute or two to test them out. Ask them to reset the applet before starting the activity.

Display the completed image for 3 seconds and then hide it. Do this twice. Give students 1 minute of quiet work time after each flash of the image. Encourage students to think about any shortcuts they used to draw the reflected image.

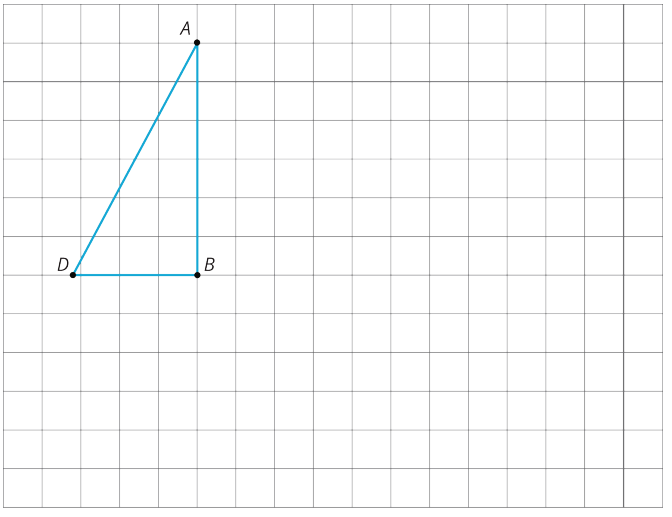


Anticipated Misconceptions

Students may struggle drawing the image under transformation from the quick flashes of the image because they are trying to count the number of spaces each vertex moves. Encourage these students to use the line in the image to help them reflect the image.

Student Task Statement

Here is an incomplete image. Your teacher will display the completed image twice, for a few seconds each time. Your job is to complete the image on your copy.



Student Response

Answers vary. Possible response: The first flash showed where to put B' and the second flash where to put A'. Once these were in place, there was only one place D' could go, underneath the segment $A'B'$, so that $\triangle A'B'D'$ is a reflected image of $\triangle ABD$.

Activity Synthesis

Select a few students to share strategies they used in sketching their figure. Consider asking some of the following questions:

- "What was important in creating your sketch (what did you need)?"
- "What did you look for in the first flash? The second?"
- "What stayed the same and what is different in the shape and its image?"
- "How did you decide where to place the vertices of the image?"
- "How did you decide how long to make the sides?"

4.2 Make That Move

15 minutes (there is a digital version of this activity)
The purpose of this activity is for students to give precise descriptions of translations, rotations, and reflections. By the end of the previous lesson, students have identified and sketched these transformations from written directions, however they have not used this more precise language themselves to give descriptions of the three motions. The images in this activity are given on grids to allow and to encourage students to describe the transformation in terms of specific points, lines or angles. Describing the moves precisely and clearly will require students to engage in MP6.

There are four different transformation cards students use in this activity: 1A, 1B, 2A, and 2B. Each card has the original image and the image under a transformation. Students are arranged in groups of 2 and each one gets a different transformation card: some pairs are given cards 1A and 1B while

other pairs get 2A and 2B. Each A card is a translation, while the B cards show either a rotation or reflection.

As students are describing their transformations and sketching their images under transformation, monitor for students using precise descriptions to their partner in terms of specific points, lines, or angles.

Addressing

- 8.G.A.1

Instructional Routines

- MLR2: Collect and Display

Launch

Introduce the word **transformation:** This is a word for a move (like the reflection in the warm-up) or for a sequence of moves. It's a mathematical word which will be used in place of "move," which has been used up to this point.

Arrange students in groups of 2. Display the original image that each student also has in front of them. Give each student one of the four transformation cards and tracing paper—make sure students know *not* to show their card to their partner! Tell students they will sketch a transformation based on directions given to them by a partner.

Partners have Cards 1A and 1B or Cards 2A and 2B. Tell students in the first round, those with the A cards give a precise description of the transformation displayed on their card to their partner. Their partners may use tracing paper to produce the image under transformation on the grid with the original image. In digital classrooms, students have access to applets they can use to transform the figure. When the sketch is complete, the student describing the transformation reveals their card, and together, students decide if the sketch is correct. In the second round, the roles are reversed. The students with the B cards describe their transformation while their partner sketches.

The student describing the transformation is allowed to repeat, revise, or add any important information as their partner sketches, however, they are not allowed to tell them to fix anything until they are finished. The student sketching should not speak, just sketch. (This is to encourage the describer to use mathematical language.) Remind students to use the geometric language for describing reflections, rotations, and translations that was used in the previous lesson.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Display or provide charts with symbols and meanings. For example, display the terms translation, rotation, and reflection and their respective meanings “slide,” “turn,” and “mirror,” accompanied by a visual example depicting the transformation. Remind students that they can refer to the display when giving directions to their partner.
Supports accessibility for: Conceptual processing; Memory

Access for English Language Learners

Listening, Speaking: Math Language Routine 2 Collect and Display. This is the first time Math Language Routine 2 is suggested as a support in this course. In this routine, the teacher circulates and listens to student talk while jotting down words, phrases, drawings, or writing students use. The language collected is displayed visually for the whole class to use throughout the lesson and unit. Generally, the display contains different examples of students using features of the disciplinary language functions, such as interpreting, justifying, or comparing. The purpose of this routine is to capture a variety of students' words and phrases in a display that students can refer to, build on, or make connections with during future discussions, and to increase students' awareness of language used in mathematics conversations.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

How It Happens:

1. As students describe the transformation of triangle ABC to their partner, listen for and collect vocabulary and phrases students use to describe the moves. Focus on capturing students using geometric language for describing reflections, rotations, and translations.

   If the speaker is stuck, consider asking these questions: "How did point A transform to A'?", "Choose one of the points, lines, or angles and describe how it changed.", and "Overall, does it look like the new triangle is a translation, rotation, or reflection of the original?"

   If using the applet, check for precision and labels as students place the new image under the transformation.

2. Write students' words on a visual display. Divide the display into 3 sections. Group language about Cards 1A and 2A on the left side of the display, language about Card 1B in the middle, and language about Card 2B on the right side.

   Record all language (whether precise, ambiguous, correct, or incorrect) in the appropriate column. as described by the students.

3. Arrange students in groups of 2, and invite partners to discuss which words or phrases stand out to them. Prompt students by asking, "Are there any words or phrases that stand out to you or don't belong in a specific column? Why?" Again, circulate around the room, collecting any additional words, phrases, and sketches onto the display.

   Students should notice that the left side consists of language describing translations, the middle consists of language describing reflections, and the right side consists of language describing rotations.

4. Select 3–4 groups to share their ideas with the class. Invite students to demonstrate their reasoning with the applet or tracing paper and be sure to modify the display accordingly.

Use this discussion to clarify, revise, and improve how ideas are communicated and represented. If students are still using vague words (e.g., move, flip, mirror image, etc.), reinforce the precise geometric terms (e.g., transformation, translation, rotation, reflection, etc.). Ask students, "Is there another way we can say this?" or "Can someone help clarify this language?"

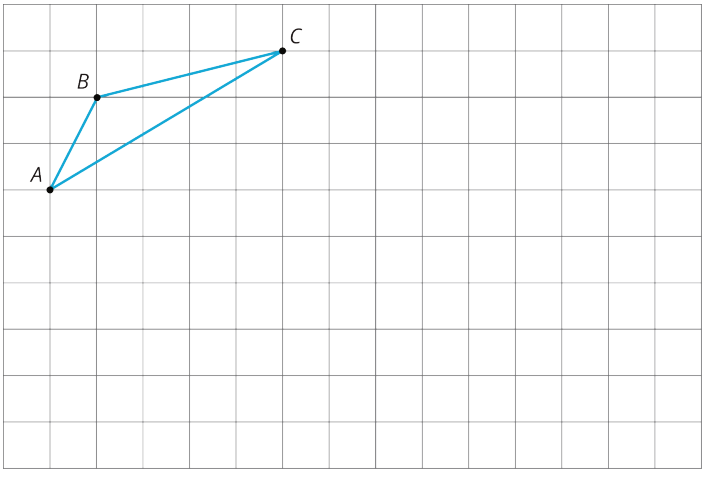
5. Close this conversation by posting the display in the front of the classroom for students to reference for the remainder of the lesson, and be sure to update the display throughout the remainder of the lesson.

Anticipated Misconceptions

Students may get stuck thinking they need to use the precise terms for the transformation in their description. Encourage these students to describe it in a way that makes sense to them and to look for things they know about the specific points, lines, or angles on their card to help them.

Student Task Statement

Your partner will describe the image of this triangle after a certain **transformation**. Sketch it here.



Student Response

The correct transformations are shown on the cards.

Activity Synthesis

Display the following questions for all to see and give groups 2 minutes to discuss:

- What pieces of your partner's description were helpful when you were sketching?
- What pieces did you find difficult to explain to your partner? Point to specific examples on your cards.

- When you were sketching, what questions would have been helpful to be able to ask the describer?

Ask selected students who were observed using precise descriptions and sketching based on those descriptions to explain why they used the information they did and how it was helpful in sketching. Focus on:

- The direction and distance of a translation.
- The center and the measure of a rotation.
- The line of a reflection.

If there is time, ask students who were both using and not using tracing paper to explain their process.

Reinforce the term **transformation** as a term that encompasses translations, rotations, and reflections. Tell them that there are other types of transformations, but for now, we will focus on these three types.

4.3 A to B to C

15 minutes (there is a digital version of this activity)
Students have seen images showing a sequence of transformations in the first lesson of this unit, however they have not heard the term sequence of transformations. They have also not been asked to describe the moves in the sequence using precise language. The launch of this activity introduces this term and gives students an opportunity to describe the sequence of more than one transformation.

For the second problem, encourage students to find a different sequence of transformations than the one shown in the image. Each time a reflection is mentioned, ask students where the line of reflection is located and when a rotation is mentioned, ask for the center of the rotation and the number of degrees. Monitor for students who apply different transformations (or apply transformations in a different order).

Addressing

- 8.G.A.1

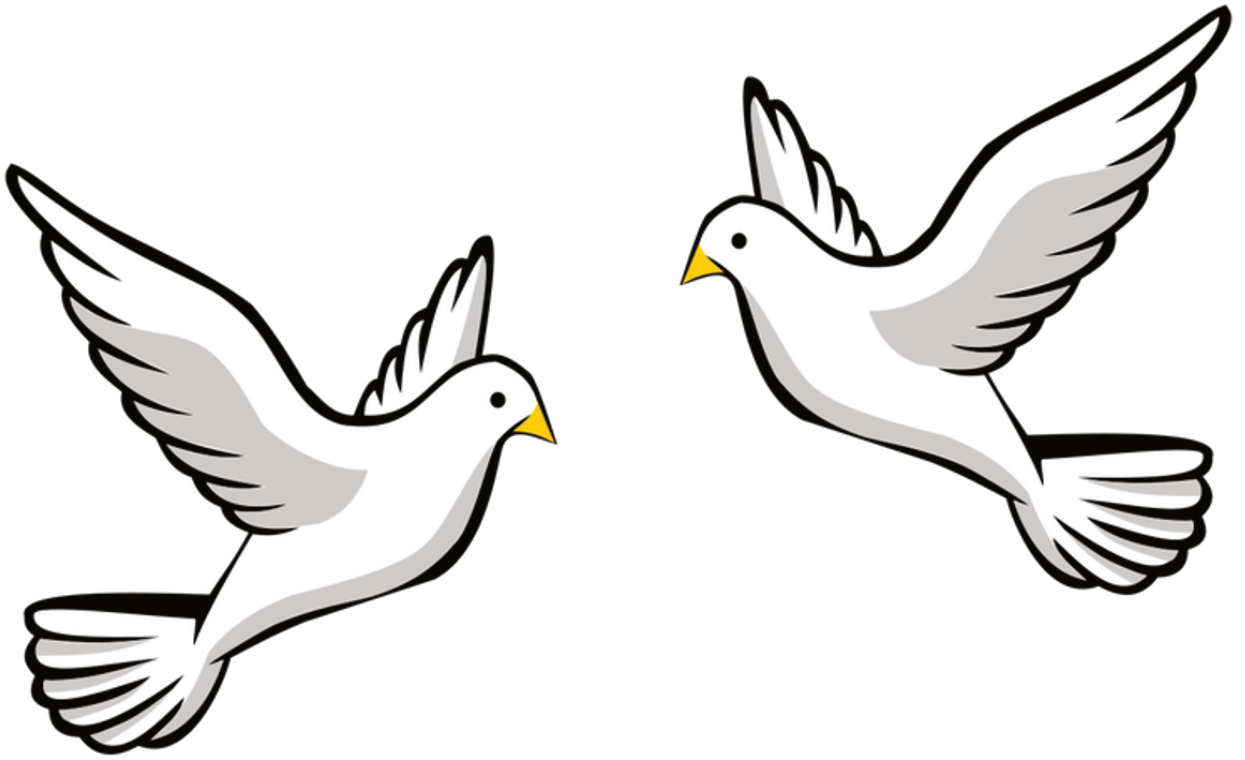
Instructional Routines

- Anticipate, Monitor, Select, Sequence, Connect
- MLR7: Compare and Connect

Launch

Arrange students in groups of 2, and provide access to their geometry toolkits. Display the image for all to see. Ask students if they can imagine a single translation, rotation, or reflection that would take one bird to another? After a minute, verify that this is not possible.

Ask students to describe how we could use translations, rotations, and reflections to take one bird to another. Collect a few different responses. (One way would be to take the bird on the left, translate it up, and then reflect it over a vertical line.) Tell students when we do one or more transformations in a row to take one figure to another, it is called a **sequence of transformations**.



If using the digital activity, you may want to review the transformation tools in the applet. (The instructions are repeated in the activity for students' reference.)

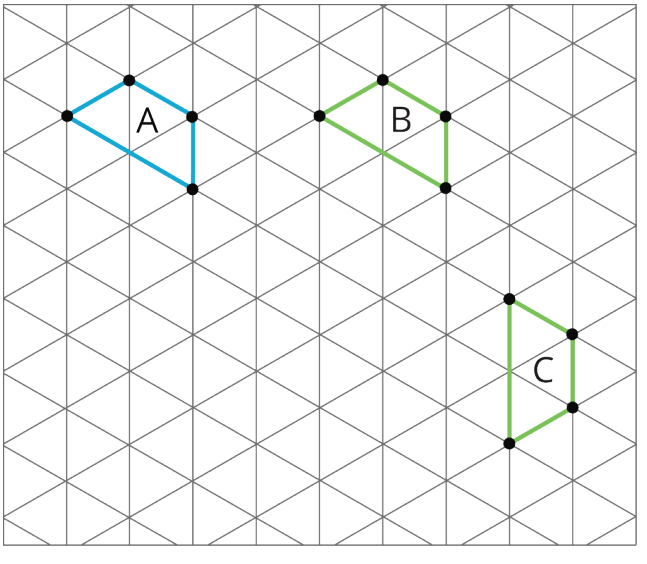
Give students 2 minutes of quiet work time to engage in the task followed by 3 minutes to discuss their responses with a partner and complete any unfinished questions. Follow with a whole-class discussion.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Prior to the whole-class discussion, invite students to share their work with a partner. Display sentence frames to support student conversation such as "To take Figure A to Figure B, I _____ because…", "I noticed _____ so I…", "Why did you…?", or "I agree/disagree because…."
Supports accessibility for: Language; Social-emotional skills

Student Task Statement

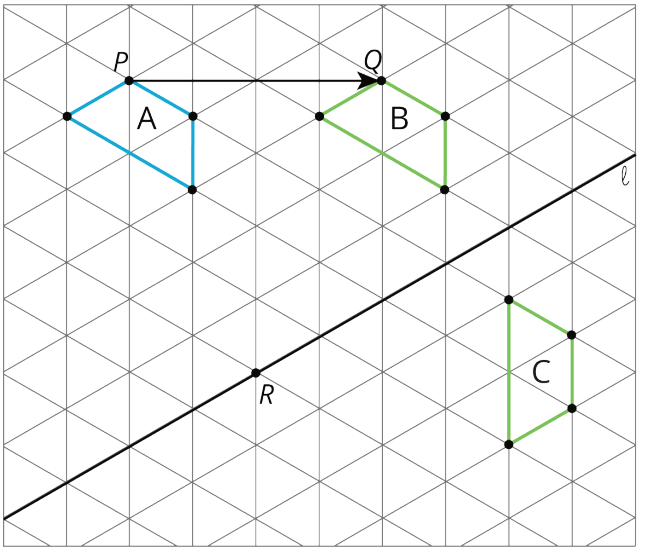
Here are some figures on an isometric grid.



1. Name a transformation that takes Figure A to Figure B. Name a transformation that takes Figure B to Figure C.

2. What is one **sequence of transformations** that takes Figure A to Figure C? Explain how you know.

Student Response



1. There are many of ways to describe the translation that takes A to B: Any pair of corresponding points works. In the figure, two corresponding points P and Q are shown. There are two ways to take B to C with a single transformation. One is a reflection with line of reflection ℓ (shown). The other is a rotation 60° clockwise around point R on line ℓ.

2. Answers vary. Sample response: Using the transformations from problem 1, first apply a translation so that A goes to B and then a reflection taking B to C.

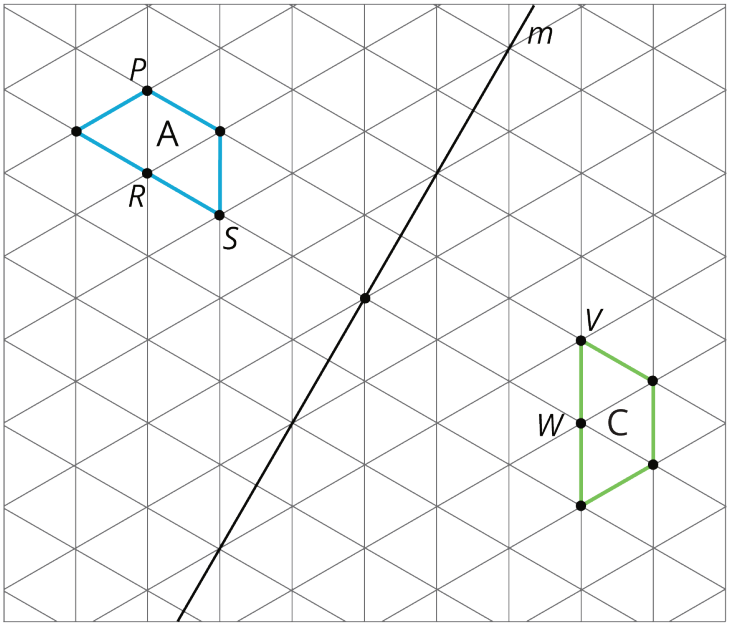
Are You Ready for More?

Experiment with some other ways to take Figure A to Figure C. For example, can you do it with. . .

- No rotations?
- No reflections?
- No translations?

Student Response

Answers vary. Sample response with no reflection: First translate P to S and then rotate 60 degrees clockwise.



Sample response with no rotation: Translate S to V and then reflect over a gridline to make the figures align.

Sample response with no reflection: Translate R to W and then rotate around W to make the figures align.

Sample response with no translation: Reflect across the line m, taking R to W, and then rotate around W to make the figures align.

Activity Synthesis

Select students with different correct responses to show their solutions. Be sure to highlight at least one rotation. If no students mention that, demonstrate a way to take A to C that involves a rotation. Whether or not students use the geogebra applet, it may be helpful to display the applet to facilitate discussion: ggbm.at/jqvTEgsj

- Emphasize that there are many ways to describe the translation that takes figure A to figure B. All one needs is to identify a pair of corresponding points and name them in the correct order (and to use the word “translate”).
- For students who used a reflection to take B to C, emphasize that reflections are determined by lines and we should name the line when we want to communicate about it.
- After a student or the teacher uses a rotation, emphasize that a rotation is defined by a center point and an angle (with a direction). The center point needs to be named and the angle measure or an angle with the correct measure needs to be named as well (as does the direction). Reinforce to students that when we do more than one transformation in a row, we call this a sequence of transformations.

Access for English Language Learners

Speaking: Math Language Routine 7 Compare and Connect. This is the first time Math Language Routine 7 is suggested as a support in this course. In this routine, students are given a problem that can be approached using multiple strategies or representations and are asked to prepare a visual display of their method. Students then engage in investigating the strategies (by means of a teacher-led gallery walk, partner exchange, group presentation, etc.), comparing approaches, and identifying correspondences between different representations. A typical discussion prompt is: "What is the same and what is different?" regarding their own strategy and that of the others. The purpose of this routine is to allow students to make sense of mathematical strategies by identifying, comparing, contrasting, and connecting other approaches to their own, and to develop students' awareness of the language used through constructive conversations.
Design Principle(s): Maximize meta-awareness: Support sense-making

How It Happens:

1. Use this routine to compare and contrast different strategies for transforming Figure A to Figure C. Invite students to create a visual display showing how they made sense of the problem and why their solution makes sense for transforming Figure A to Figure C.

   Students should include these features on their display:
   - a sketch of the figures (not necessary if using the applet)
   - a sketch of the figures after each transformation (not necessary if using the applet)
   - a written sequence of transformations with an explanation

2. Before selecting students to show their solutions to the class, first give students an opportunity to do this in a group of 3–4. Ask students to exchange and investigate each other's work. Allow 1 minute for each display and signal when it is time to switch.

   While investigating each other's work, ask students to consider what is the same and what is different about each approach. Next, give each student the opportunity to add detail to their own display for 1–2 minutes.

3. As groups are presenting, circulate the room and select 2–3 students to share their sequence of transformations taking Figure A to Figure C. Be sure to select a variety of approaches, including one that involves a rotation.

   Draw students' attention to the different ways the figures were transformed (e.g., rotations, reflections, and translations) and how the sequence of transformation is expressed in their explanation. Also, use the bullet points in the Activity Synthesis to emphasize specific features of translations, reflections, and rotations.

4. After the selected students have finished sharing with the whole class, lead a discussion comparing, contrasting, and connecting the different approaches.

   Consider using these prompts to amplify student language while comparing and contrasting the different approaches: “Why did the approaches lead to the same outcome?”, “What worked well in __’s approach? What did not work well?”, and “What would make __’s strategy more complete or easy to understand?”

   Consider using these prompts to amplify student language while connecting the different approaches: “What role does a translation play in each approach?” “Is it possible to use the all three types of transformations?”, and “What transformation do you see present in all the strategies?”

5. Close the discussion by inviting 3 students to revoice the strategies used in the presentations, and then transition back to the Lesson Synthesis and Cool-Down.

Lesson Synthesis

The goal for this lesson is for students to begin to identify the features that determine a translation, rotation, or reflection. Refer to the permanent display produced in a previous lesson as you discuss. To highlight the features specific to each type of transformation, consider asking the following questions:

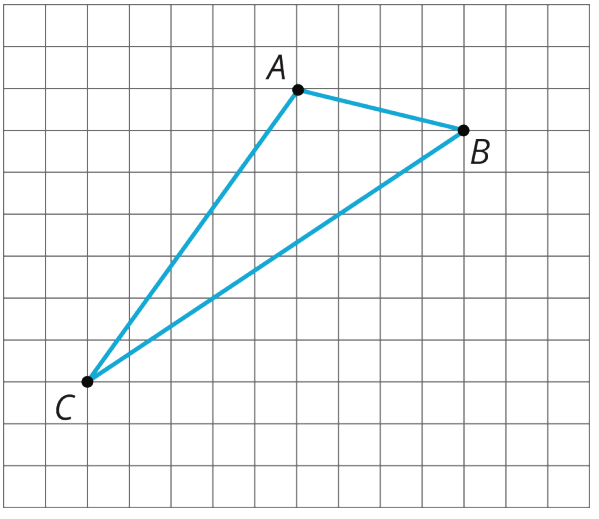
- "If you want to describe a translation, what important information do you need to include?" (A translation is determined by two points that specify the distance and direction of the translation.)
- "If you want to describe a rotation, what important information do you need to include?" (A rotation is determined by a center point and an angle with a direction.)
- "If you want to describe a reflection, what important information do you need to include?" (A reflection is determined by a line.)
- "What does the word *transformation* mean?" (Translations, rotations, and reflections, or any combination of these.)
- "What does *sequence of transformations* mean?" (More than one applied one after the other.)

4.4 What Does It Take?

Cool Down: 5 minutes
Addressing

- 8.G.A.1

Student Task Statement



1. If you were to describe a translation of triangle ABC, what information would you need to include in your description?
2. If you were to describe a rotation of triangle ABC, what information would you need to include in your description?
3. If you were to describe a reflection of triangle ABC, what information would you need to include in your description?

Student Response

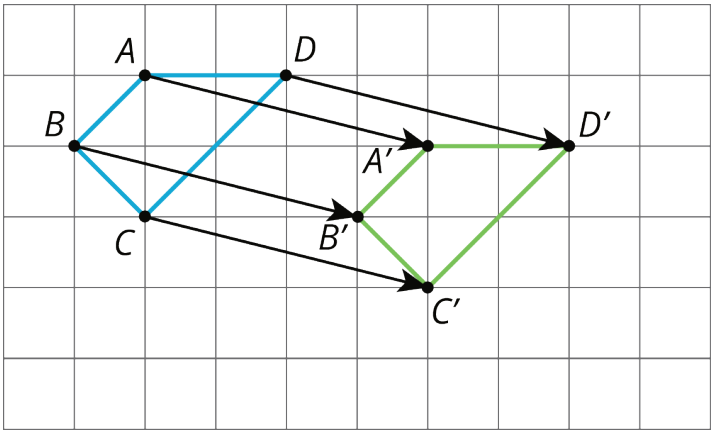
1. The distance and direction of the translation. One way to do this would be by picking a point on the triangle (A, for example) and then showing where this point goes (A' on the translated triangle).
2. A center point, an angle, and a direction (clockwise or counterclockwise).
3. A line.

Student Lesson Summary

A move, or combination of moves, is called a **transformation**. When we do one or more moves in a row, we often call that a **sequence of transformations**. To distinguish the original figure from its image, points in the image are sometimes labeled with the same letters as the original figure, but with the symbol $'$ attached, as in A' (pronounced "A prime").

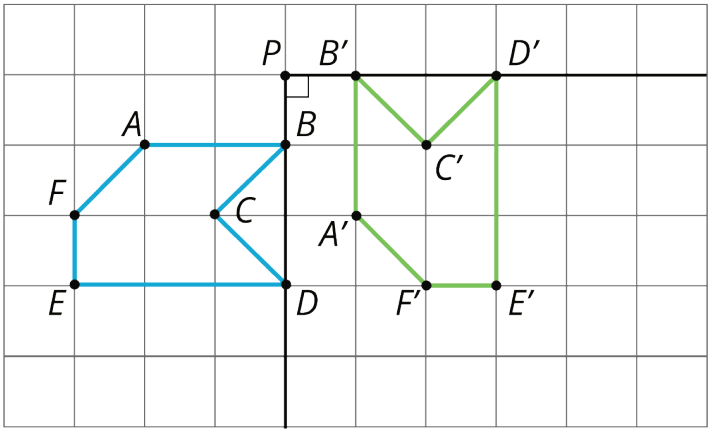
- A translation can be described by two points. If a translation moves point A to point A', it moves the entire figure the same distance and direction as the distance and direction from A to A'. The distance and direction of a translation can be shown by an arrow.

For example, here is a translation of quadrilateral $ABCD$ that moves A to A'.



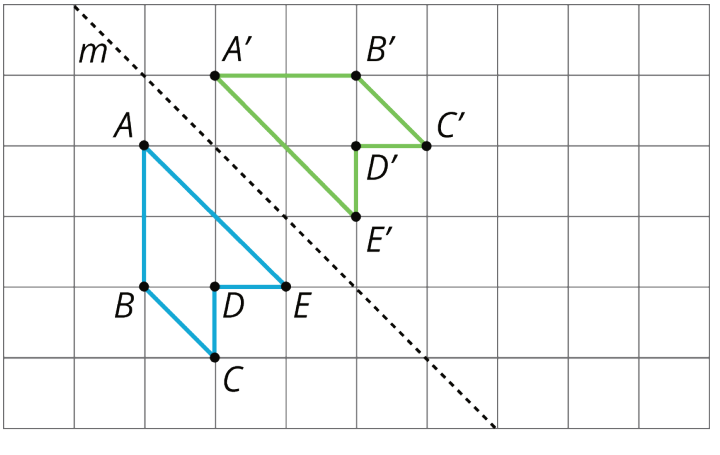
- A rotation can be described by an angle and a center. The direction of the angle can be clockwise or counterclockwise.

  For example, hexagon $ABCDEF$ is rotated 90° counterclockwise using center P.



- A reflection can be described by a line of reflection (the "mirror"). Each point is reflected directly across the line so that it is just as far from the mirror line, but is on the opposite side.

  For example, pentagon $ABCDE$ is reflected across line m.



Glossary

- sequence of transformations

- transformation

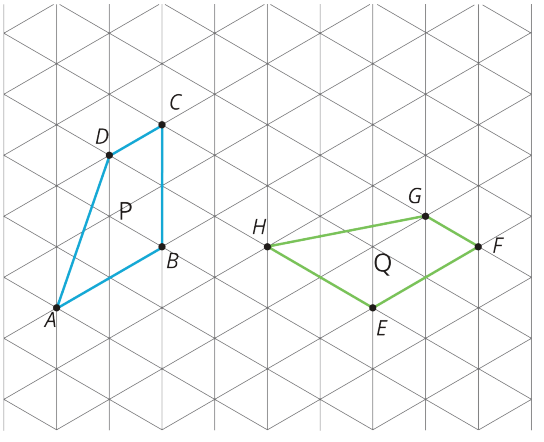
Lesson 4 Practice Problems

Problem 1

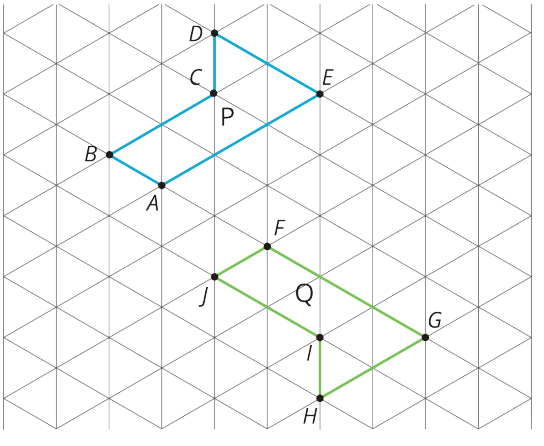
Statement

For each pair of polygons, describe a sequence of translations, rotations, and reflections that takes Polygon P to Polygon Q.

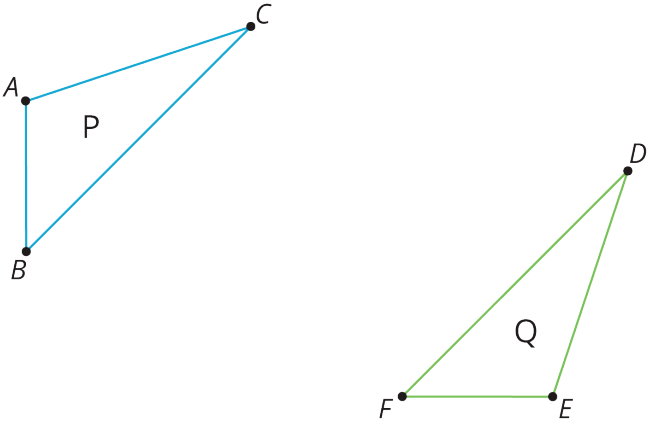
a.



b.



c.



Solution

a. Sample response: A is translated to H, followed by a rotation 60 degrees clockwise with center H.

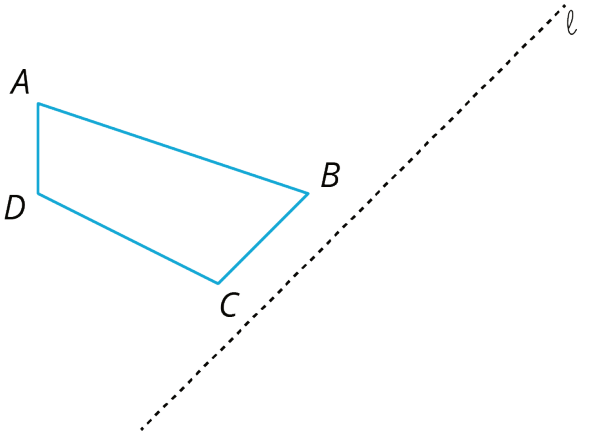
b. Sample response: Polygon $ABCDE$ is reflected over line AE. A is then translated to F and a rotation of 60 degrees clockwise with center F is applied.

c. Sample response: A is translated to E, then apply a rotation with center E so that B lands on top of F. Finally the polygon is reflected over line EF.

Problem 2

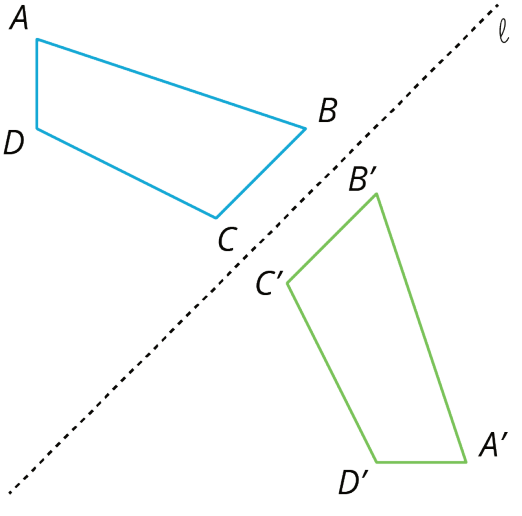
Statement

Here is quadrilateral $ABCD$ and line ℓ.



Draw the image of quadrilateral $ABCD$ after reflecting it across line ℓ.

Solution

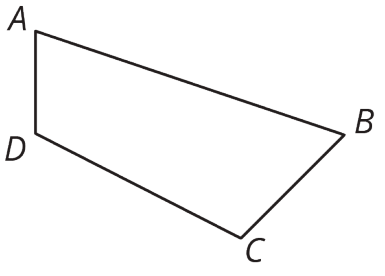


(From Unit 1, Lesson 2.)

Problem 3

Statement

Here is quadrilateral $ABCD$.



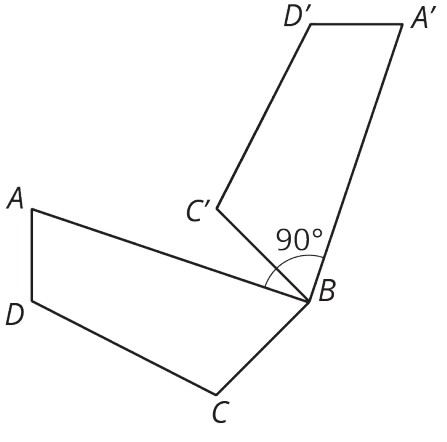
Draw the image of quadrilateral $ABCD$ after each rotation using B as center.

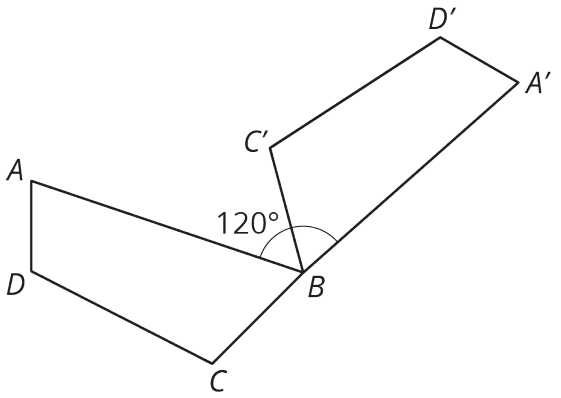
a. 90 degrees clockwise

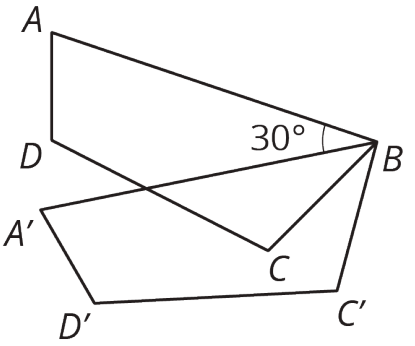
b. 120 degrees clockwise

c. 30 degrees counterclockwise

Solution







(From Unit 1, Lesson 2.)