Lesson 16: Solving Quadratics

- Let's solve quadratic equations.

16.1: Find the Perfect Squares

The expression $x^2 + 8x + 16$ is equivalent to $(x + 4)^2$. Which expressions are equivalent to $(x + n)^2$ for some number n?

1. $x^2 + 10x + 25$
2. $x^2 + 10x + 29$
3. $x^2 - 6x + 8$
4. $x^2 - 6x + 9$

16.2: Different Ways to Solve It

Elena and Han solved the equation $x^2 - 6x + 7 = 0$ in different ways.

Elena said, "First I added 2 to each side:

$$x^2 - 6x + 7 + 2 = 2$$

So that tells me:

$$(x - 3)^2 = 2$$

I can find the square roots of both sides:

$$x - 3 = \pm\sqrt{2}$$

Which is the same as:

$$x = 3 \pm \sqrt{2}$$

So the two solutions are $x = 3 + \sqrt{2}$ and $x = 3 - \sqrt{2}$."

Han said, “I used the quadratic formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

Since $x^2 - 6x + 7 = 0$, that means $a = 1$, $b =$ -6, and $c = 7$. I know:

$$x = \frac{6 \pm \sqrt{36 - 4 \cdot 1 \cdot 7}}{2 \cdot 1}$$

or

$$x = \frac{6 \pm \sqrt{8}}{2}$$

So:

$$x = 3 \pm \frac{\sqrt{8}}{2}$$

I think the solutions are $x = 3 + \frac{\sqrt{8}}{2}$ and $x = 3 - \frac{\sqrt{8}}{2}$.”

Do you agree with either of them? Explain your reasoning.

Are you ready for more?

Under what circumstances would solving an equation of the form $x^2 + bx + c = 0$ lead to a solution that doesn’t involve fractions?

16.3: Solve These Ones

Solve each quadratic equation with the method of your choice. Be prepared to compare your approach with a partner's.

1. $x^2 = 100$

2. $x^2 = 38$

3. $x^2 - 10x + 25 = 0$

4. $x^2 + 14x + 40 = 0$

5. $x^2 + 14x + 39 = 0$

6. $3x^2 - 5x - 11 = 0$

Lesson 16 Summary

Consider the quadratic equation:

$$x^2 - 5x = 25$$

It is often most efficient to solve equations like this by completing the square. To complete the square, note that the perfect square $(x+n)^2$ is equal to $x^2 + (2n)x + n^2$. Compare the coefficients of x in $x^2 + (2n)x + n^2$ to our expression $x^2 - 5x$ to see that we want $2n = \text{-}5$, or just $n = \text{-}\frac{5}{2}$. This means the perfect square $\left(x - \frac{5}{2}\right)^2$ is equal to $x^2 - 5x + \frac{25}{4}$, so adding $\frac{25}{4}$ to each side of our equation will give us a perfect square.

$$\begin{aligned} x^2 - 5x &= 25 \\ x^2 - 5x + \frac{25}{4} &= 25 + \frac{25}{4} \\ \left(x - \frac{5}{2}\right)^2 &= \frac{100}{4} + \frac{25}{4} \\ \left(x - \frac{5}{2}\right)^2 &= \frac{125}{4} \end{aligned}$$

The two numbers that square to make $\frac{125}{4}$ are $\frac{\sqrt{125}}{2}$ and -$\frac{\sqrt{125}}{2}$, so:

$$x - \frac{5}{2} = \pm\frac{\sqrt{125}}{2}$$

which means the two solutions are:

$$x = \frac{5}{2} \pm \frac{\sqrt{125}}{2}$$

Other times, it is most efficient to use the quadratic formula. Look at the quadratic equation:

$$3x^2 - 2x = 0.8$$

We could divide each side by 3 and then complete the square like before, but the equation would get even messier and the chance of making a mistake might be higher. With messier equations like this, it is often most efficient to use the quadratic formula:

$$x = \frac{\text{-}b \pm \sqrt{b^2 - 4ac}}{2a}$$

To use this formula, we first need to put the equation in standard form and identify a, b, and c. Rearranging, we get:

$$3x^2 - 2x - 0.8 = 0$$

so $a = 3$, $b = \text{-}2$, and $c = \text{-}0.8$. We have to be careful to pay attention to the negative signs. Using the quadratic formula, we get:

$$x = \frac{\text{-}(\text{-}2) \pm \sqrt{(\text{-}2)^2 - 4(3)(\text{-}0.8)}}{2(3)}$$

$$x = \frac{2 \pm \sqrt{4 + (12)(0.8)}}{6}$$

Evaluating these solutions with a calculator gives decimal approximations -0.281 and 0.948.