## Lesson 8: Equivalent Quadratic Expressions

### 8.1: Diagrams of Products



1. Explain why the diagram shows that $6(3+4)=6⋅3+6⋅4$.
2. Draw a diagram to show that $5(x+2)=5x+10$.

### 8.2: Drawing Diagrams to Represent More Products

Applying the distributive property to multiply out the factors of, or expand, $4(x+2)$ gives us $4x+8$, so we know the two expressions are equivalent. We can use a rectangle with side lengths $(x+2)$ and 4 to illustrate the multiplication.



1. Draw a diagram to show that $n(2n+5)$ and $2n^{2}+5n$ are equivalent expressions.
2. For each expression, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.
* a. $6\left(\frac{1}{3}n+2\right)$
* b. $p(4p+9)$
* c. $5r\left(r+\frac{3}{5}\right)$
* d. $(0.5w+7)w$

### 8.3: Using Diagrams to Find Equivalent Quadratic Expressions

1. Here is a diagram of a rectangle with side lengths $x+1$ and $x+3$. Use this diagram to show that $(x+1)(x+3)$ and $x^{2}+4x+3$ are equivalent expressions.
* 
1. Draw diagrams to help you write an equivalent expression for each of the following:
	1. $(x+5)^{2}$
	2. $2x(x+4)$
	3. $(2x+1)(x+3)$
	4. $(x+m)(x+n)$
2. Write an equivalent expression for each expression without drawing a diagram:
	1. $(x+2)(x+6)$
	2. $(x+5)(2x+10)$

#### Are you ready for more?



1. Is it possible to arrange an $x$ by $x$ square, five $x$ by 1 rectangles and six 1 by 1 squares into a single large rectangle?  Explain or show your reasoning.
2. What does this tell you about an equivalent expression for $x^{2}+5x+6$?
3. Is there a different non-zero number of 1 by 1 squares that we could have used instead that would allow us to arrange the combined figures into a single large rectangle?

### Lesson 8 Summary

A quadratic function can often be defined by many different but equivalent expressions. For example, we saw earlier that the predicted revenue, in thousands of dollars, from selling a downloadable movie at $x$ dollars can be expressed with $x(18−x)$, which can also be written as $18x−x^{2}$. The former is a product of $x$ and $18−x$, and the latter is a difference of $18x$ and $x^{2}$, but both expressions represent the same function.

Sometimes a quadratic expression is a product of two factors that are each a linear expression, for example $(x+2)(x+3)$. We can write an equivalent expression by thinking about each factor, the $(x+2)$ and $(x+3)$, as the side lengths of a rectangle, and each side length decomposed into a variable expression and a number.



Multiplying $(x+2)$ and $(x+3)$ gives the area of the rectangle. Adding the areas of the four sub-rectangles also gives the area of the rectangle. This means that $(x+2)(x+3)$ is equivalent to $x^{2}+2x+3x+6$, or to $x^{2}+5x+6$.

Notice that the diagram illustrates the distributive property being applied. Each term of one factor (say, the $x$ and the 2 in $x+2$) is multiplied by every term in the other factor (the $x$ and the 3 in $x+3$).



In general, when a quadratic expression is written in the form of $(x+p)(x+q)$, we can apply the distributive property to rewrite it as $x^{2}+px+qx+pq$ or $x^{2}+(p+q)x+pq$.



© CC BY 2019 by Illustrative Mathematics