Lesson 6: Describing Transformations

Goals

- Create a drawing on a coordinate grid of a transformed object using verbal descriptions.
- Identify what information is needed to transform a polygon. Ask questions to elicit that information.

Learning Targets

- I can apply transformations to a polygon on a grid if I know the coordinates of its vertices.

Lesson Narrative

Prior to this lesson, students have studied and classified different types of transformations (translations, rotations, reflections). They have practiced applying individual transformations and sequences of transformations to figures both on and off of a coordinate grid. In this lesson, they focus on communicating precisely the information needed to apply a sequence of transformations to a polygon on the coordinate grid. They must think carefully about what information they need (MP1) and request this information from their partner in a clear, precise way. They also explain *why* they need each piece of information (MP3). The coordinate grid plays a key role in this work, allowing students to communicate precisely about the locations of polygons and how they are transformed.

Alignments

Addressing

- 8.G.A.1: Verify experimentally the properties of rotations, reflections, and translations:
- 8.G.A.3: Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

Instructional Routines

- MLR4: Information Gap Cards
- Think Pair Share

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Pre-printed slips, cut from copies of the blackline master

Required Preparation

Print 1 copy of the blackline master for every 2 students. Cut them up ahead of time.

From the geometry toolkits, graph paper and tracing paper are especially helpful.

Student Learning Goals

Let's transform some polygons in the coordinate plane.

6.1 Finding a Center of Rotation

Warm Up: 5 minutes
Sometimes it is easy to forget to communicate all of the vital information about a transformation. In this case, the center of a rotation is left unspecified. Students do not need to develop a general method for finding the center of rotation, given a polygon and its rotated image. They identify the center in one situation and this can be done via geometric intuition and a little trial and error.

Addressing

- 8.G.A.1

Instructional Routines

- Think Pair Share

Launch

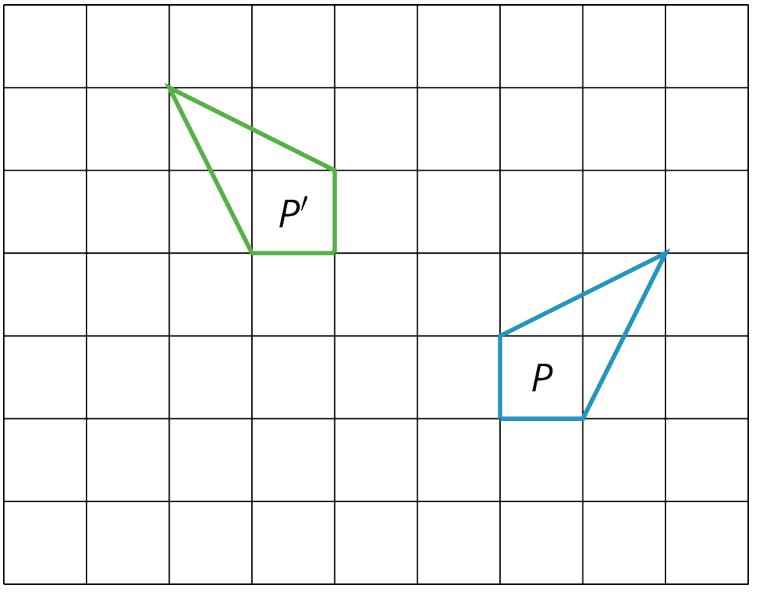
Arrange students in groups of 2 and provide access to geometry toolkits. Tell students that they have a diagram of a figure and its rotated image and that they need to identify the center of rotation. Give them 2 minutes of quiet work time and an opportunity to share with a partner, followed by a whole-class discussion.

Anticipated Misconceptions

Students may have trouble getting started. Suggest that they trace P on to tracing paper and try rotating it 90°. How must they rotate it to get it to land on P'? Where is the center of the "spin"?

Student Task Statement

Andre performs a 90-degree counterclockwise rotation of Polygon P and gets Polygon P′, but he does not say what the center of the rotation is. Can you find the center?



Student Response

The rotation takes the horizontal side of P to the vertical side of P', and the center of rotation is the intersection of the grid lines containing these two sides.

Activity Synthesis

Emphasize that it is important to communicate clearly. When we perform a transformation, we should provide the information necessary for others to understand what we have done. For a rotation, this means communicating:

- The center of the rotation.
- The direction of the rotation (clockwise or counterclockwise).
- The angle of rotation.

The grid provides extra structure that helps to identify these three parts of the rotation. Invite students to share how they identified the center of rotation. Methods may include:

- Experimenting with tracing paper.
- Understanding that the rotation does not change the distance between the center of rotation and each vertex, so the center should be the same distance from each vertex and its image.

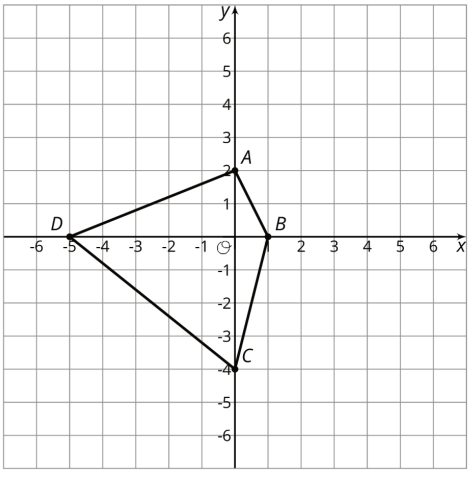
6.2 Info Gap: Transformation Information

30 minutes

This info gap activity gives students an opportunity to determine and request the information needed to perform a transformation in the coordinate plane. A sample pair of cards looks as follows:

Info Gap: Transformation Information

Problem Card 1



Polygon *A'B'C'D'* is the image of *ABCD* after some transformations.

Find *A'B'C'D'* .

Info Gap: Transformation Information

Data Card 1

Translation: 2 units up and 3 units to the right

Rotation: none

Reflection: over x-axis

Order of Transformations: Translation first and then reflection

Students likely need several rounds to determine the information they need.

- They need to know which transformations were applied (i.e., translation, rotation, or reflection)
- They need to determine the order in which the transformations were applied.
- They need to remember what information is needed to describe a translation, rotation, or reflection.

Monitor for students who successfully determine or remember each of these three important pieces of information as well as students who have partially but not completely solved the problem. Students may not realize that the order in which the transformations are applied is important, and this should be addressed in the Synthesis.

The info gap structure requires students to make sense of problems by determining what information is necessary, and then to ask for information they need to solve it. This may take several rounds of discussion if their first requests do not yield the information they need (MP1). It also allows them to refine the language they use and ask increasingly more precise questions until they get the information they need (MP6).

Addressing

- 8.G.A.1

- 8.G.A.3

Instructional Routines

- MLR4: Information Gap Cards

Launch

Tell students they will continue to describe transformations using coordinates. Explain the info gap structure, and consider demonstrating the protocol if students are unfamiliar with it.

Arrange students in groups of 2. Provide access to graph paper. In each group, distribute a problem card to one student and a data card to the other student. After you review their work on the first problem, give them the cards for a second problem and instruct them to switch roles.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Display or provide students with a physical copy of the written directions. Check for understanding by inviting students to rephrase directions in their own words. Keep the display of directions visible throughout the activity.
Supports accessibility for: Memory; Organization

Access for English Language Learners

Conversing: This activity uses MLR4 Information Gap to give students a purpose for discussing information necessary to transform a polygon. Display questions or question starters for students who need a starting point such as: "Can you tell me . . . (specific piece of information)", and "Why do you need to know . . . (that piece of information)?"
Design Principle(s): Cultivate Conversation

Anticipated Misconceptions

Students may struggle to ask their partner for all of the information they need or may ask a question that is not sufficiently precise, such as, "What are the transformations?" Ask these students what kinds of transformations they have worked with. What information is needed to perform a translation? What about a rotation or reflection? Encourage them to find out *which* transformations they need to perform (Is there a translation? Is there a rotation?) and then find out the information they need for each transformation.

Student Task Statement

Your teacher will give you either a *problem card* or a *data card*. Do not show or read your card to your partner.

If your teacher gives you the *problem card*:

1. Silently read your card and think about what information you need to be able to answer the question.
2. Ask your partner for the specific information that you need.
3. Explain how you are using the information to solve the problem.

   Continue to ask questions until you have enough information to solve the problem.
4. Share the *problem card* and solve the problem independently.
5. Read the *data card* and discuss your reasoning.

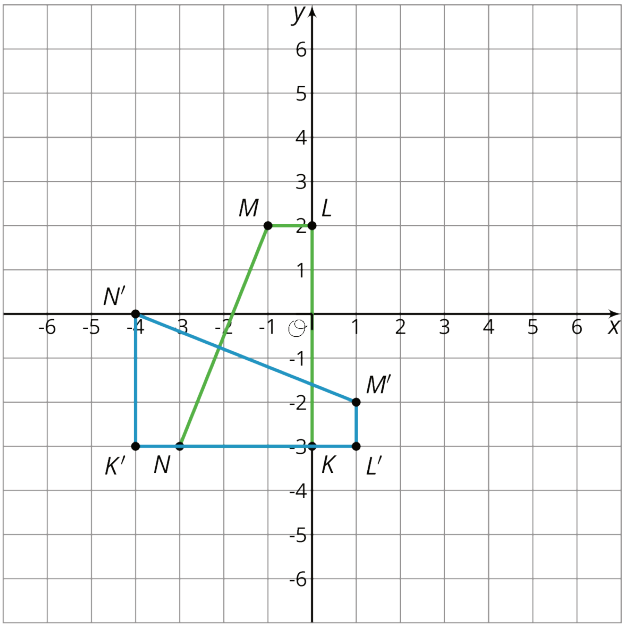
If your teacher gives you the *data card*:

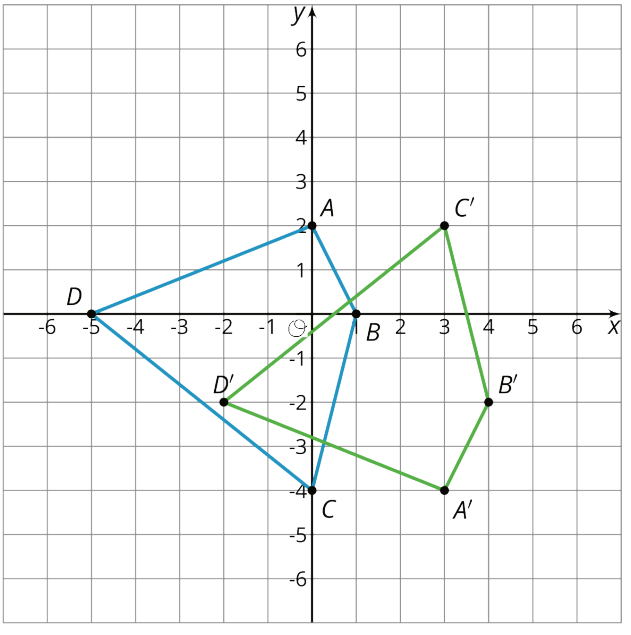
1. Silently read your card.
2. Ask your partner *"What specific information do you need?"* and wait for them to *ask* for information.

   If your partner asks for information that is not on the card, do not do the calculations for them. Tell them you don't have that information.
3. Before sharing the information, ask "*Why do you need that information?*" Listen to your partner's reasoning and ask clarifying questions.
4. Read the *problem card* and solve the problem independently.
5. Share the *data card* and discuss your reasoning.

Pause here so your teacher can review your work. Ask your teacher for a new set of cards and repeat the activity, trading roles with your partner.

Student Response





Are You Ready for More?

Sometimes two transformations, one performed after the other, have a nice description as a single transformation. For example, instead of translating 2 units up followed by translating 3 units up, we could simply translate 5 units up. Instead of rotating 20 degrees counterclockwise around the origin followed by rotating 80 degrees clockwise around the origin, we could simply rotate 60 degrees clockwise around the origin.

Can you find a simple description of reflecting across the x-axis followed by reflecting across the y-axis?

Student Response

Reflecting across the x-axis followed by reflecting across the y-axis is the same as rotating 180 degrees (in either direction) around the origin.

Activity Synthesis

After students have completed their work, share the correct answers and ask students to discuss the process of solving the problems. Some guiding questions:

- "How did using coordinates help in talking about the problem?"
- "Was the order in which the transformations were applied important? Why?"
- "If this same problem were a picture on a grid without coordinates, how would you talk about the points?"

Highlight for students that one advantage of the coordinate plane is that it allows us to communicate information about transformations precisely. Here is what is needed for each type of transformation (consider showing one example of each while going through the different transformations):

- For a translation, the distance of vertical and horizontal components
- For a rotation, the center of rotation, the direction of rotation, and the angle of rotation
- For a reflection, the line of reflection

Lesson Synthesis

Ask students to choose which of the three transformations they have studied so far (translation, reflection, rotation) is their favorite and give 2–3 minutes for students to write a few sentences explaining why. Have students first share their explanations with a partner and then invite students to share their favorite with the class.

6.3 Describing a Sequence of Transformations

Cool Down: 5 minutes
Students describe what information is required to perform a translation and what information is required to perform a reflection. They also need to think about the order in which the two transformations are applied as they have just seen that switching the order can impact the outcome.

Addressing

- 8.G.A.1
- 8.G.A.3

Student Task Statement

Jada applies two transformations to a polygon in the coordinate plane. One of the transformations is a translation and the other is a reflection. What information does Jada need to provide to communicate the transformations she has used?

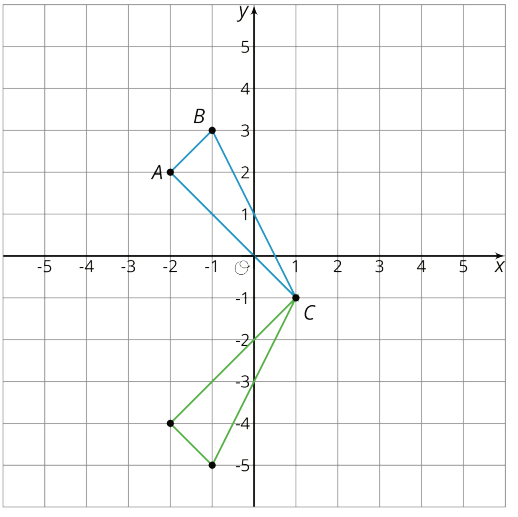
Student Response

For the translation, Jada needs to provide the distance and direction of the vertical displacement and the distance and direction of the horizontal displacement. For the reflection, Jada needs to give the line of reflection. It is also important for Jada to communicate the order in which the transformations are applied.

Student Lesson Summary

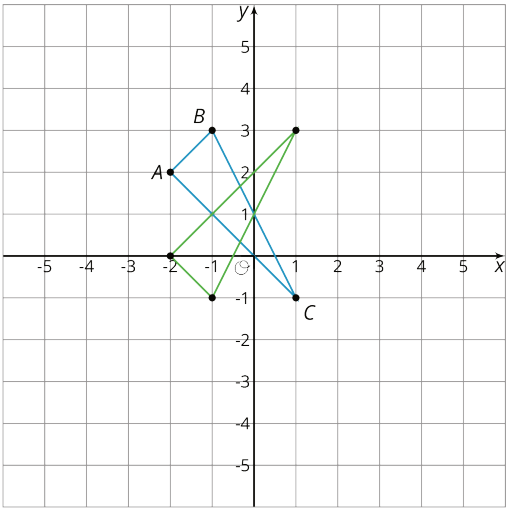
The center of a rotation for a figure doesn't have to be one of the points on the figure. To find a center of rotation, look for a point that is the same distance from two corresponding points. You will probably have to do this for a couple of different pairs of corresponding points to nail it down.

When we perform a sequence of transformations, the order of the transformations can be important. Here is triangle ABC translated up two units and then reflected over the x-axis.



Here is triangle ABC reflected over the x-axis and then translated up two units.

Triangle ABC ends up in different places when the transformations are applied in the opposite order!

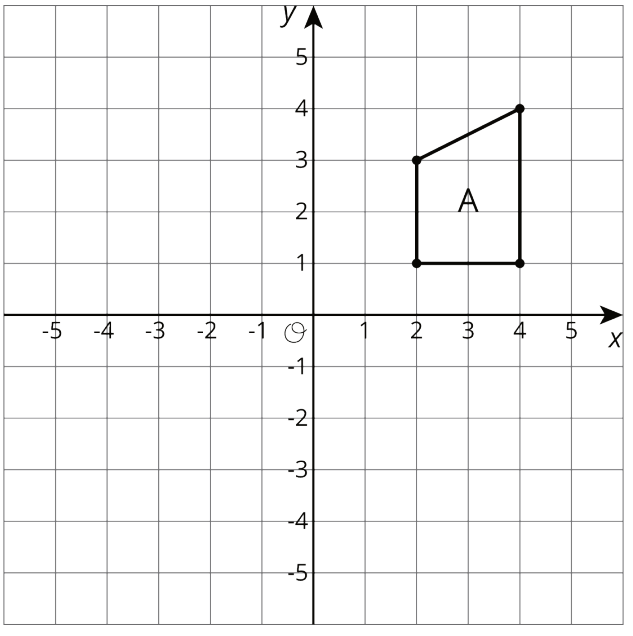


Lesson 6 Practice Problems

Problem 1

Statement

Here is Trapezoid A in the coordinate plane:

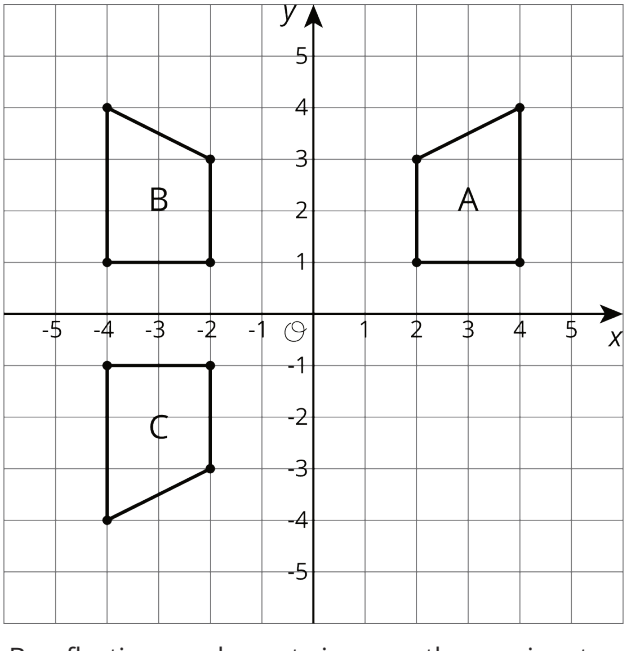


a. Draw Polygon B, the image of A, using the y-axis as the line of reflection.

b. Draw Polygon C, the image of B, using the x-axis as the line of reflection.

c. Draw Polygon D, the image of C, using the x-axis as the line of reflection.

Solution



Polygon D is the same as B: reflecting a polygon twice over the x-axis returns it to its original position.

Problem 2

Statement

The point $(-4, 1)$ is rotated 180 degrees counterclockwise using center $(-3, 0)$. What are the coordinates of the image?

A. $(-5, -2)$

B. $(-4, -1)$

C. $(-2, -1)$

D. $(4, -1)$

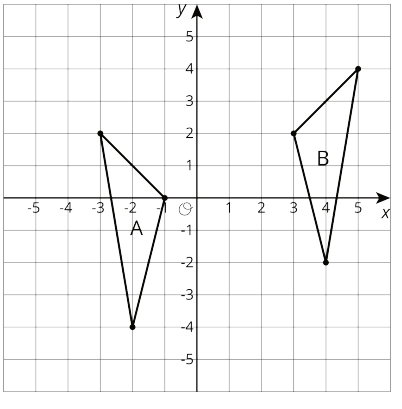
Solution

C

Problem 3

Statement

Describe a sequence of transformations for which Triangle B is the image of Triangle A.



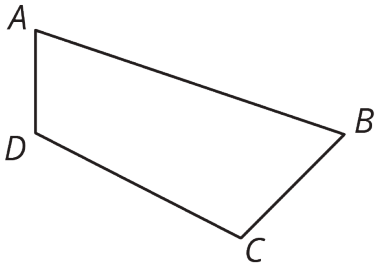
Solution

Answers vary. Sample response: B is the image of A under a reflection over the y-axis, then a translation 2 units to the right and 2 units up.

Problem 4

Statement

Here is quadrilateral $ABCD$.



Draw the image of quadrilateral $ABCD$ after each transformation.

a. The translation that takes B to D.

b. The reflection over segment BC.

c. The rotation about point A by angle DAB, counterclockwise.

Solution

a. Image of trapezoid moved to the left so that B lines up with D

b. Image of trapezoid sharing segment BC with $ABCD$

c. Image of trapezoid rotated so that the side corresponding to AD is now part of segment AB

(From Unit 1, Lesson 2.)