# Lesson 8: Rotation Patterns

## Goals

- Draw and label rotations of 180 degrees of a line segment from centers of the midpoint, a point on the segment, and a point not on the segment.
- Generalize (orally and in writing) the outcome when rotating a line segment 180 degrees.
- Identify(orally and in writing) the rigid transformations that can build a diagram from one starting figure.

## Learning Targets

- I can describe how to move one part of a figure to another using a rigid transformation.

## Lesson Narrative

In this lesson, rigid transformations are applied to line segments and triangles. For line segments, students examine the impact of a 180 degree rotation. This is important preparatory work for studying parallel lines and rigid transformations, the topic of the next lesson. For triangles students look at a variety of transformations where rotations of 90 degrees and 180 degrees are again a focus. This work and the patterns that students build will be important later when they study the Pythagorean Theorem.

Throughout the lesson, students use the properties of rigid transformations (they do not change distances or angles) in order to make conclusions about the objects they are transforming (MP7).

### Alignments

### Addressing

- 8.G.A.1.a: Lines are taken to lines, and line segments to line segments of the same length.
- 8.G.A.1.b: Angles are taken to angles of the same measure.

### Building Towards

- 8.G.A.1.c: Parallel lines are taken to parallel lines.

### Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

### Required Materials

**Geometry toolkits**
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

### Student Learning Goals

Let’s rotate figures in a plane.

# 8.1 Building a Quadrilateral

**Warm Up: 5 minutes**
Students rotate a copy of a right isosceles triangle four times to build a quadrilateral. It turns out that the quadrilateral is a square. Students are not asked or expected to justify this but it can be addressed in the discussion. The fourth question about rotational symmetry of the quadrilateral will help students conclude that it is a square.

There are many more opportunities to build figures using rigid transformations in other lessons.

### Addressing

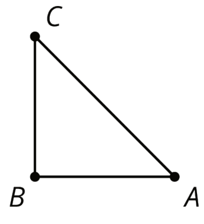
- 8.G.A.1.a
- 8.G.A.1.b

### Launch

Provide access to geometry toolkits, particularly tracing paper.

### Student Task Statement

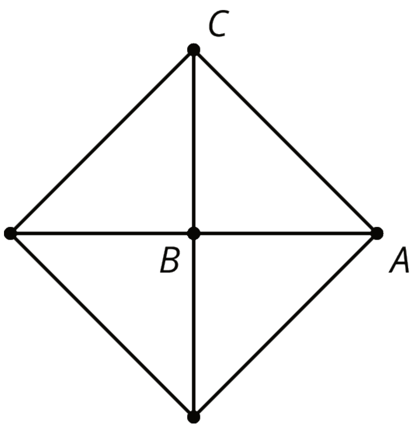
Here is a right isosceles triangle:



1. Rotate triangle $ABC$ 90 degrees clockwise around $B$.
2. Rotate triangle $ABC$ 180 degrees clockwise round $B$.
3. Rotate triangle $ABC$ 270 degrees clockwise around $B$.
4. What would it look like when you rotate the four triangles 90 degrees clockwise around $B$? 180 degrees? 270 degrees clockwise?

### Student Response

1–3.




4. The overall figure would look the same. These rotations just interchange the 4 triangles.

### Activity Synthesis

Ask students what they notice and wonder about the quadrilateral that they have built. Likely responses include:

- It looks like a square.
- Rotating it 90 degrees clockwise or counterclockwise interchanges the 4 copies of triangle $ABC$.
- Continuing the pattern of rotations, the next one will put $ABC$ back in its original position.

Ask the students how they know the four triangles fit together without gaps or overlaps to make a quadrilateral. Here the key point is that the triangle is isosceles, so the rotations match up these sides perfectly. The four right angles make a complete 360 degrees, so the shape really is a quadrilateral. The fact that the quadrilateral is a square can be deduced from the fact that it is mapped to itself by a 90 degree rotation, but this does not need to be stressed or addressed.

# 8.2 Rotating a Segment

**15 minutes (there is a digital version of this activity)**

The purpose of this activity is to allow students to explore special cases of rotating a line segment $180^\circ$. In general, rotating a segment $180^\circ$ produces a parallel segment the same length as the original. This activity also treats two special cases:

- When the center of rotation is the midpoint, the rotated segment is the same segment as the original, except the vertices are switched.
- When the center of rotation is an endpoint, the segment together with its image form a segment twice as long as the original.

As students look to make general statements about what happens when a line segment is rotated $180^\circ$ they engage in MP8. They are experimenting with a particular line segment but the conclusions that they make, especially in the last problem, are for *any* line segment.

Watch for how students explain that the $180^\circ$ rotation of segment $CD$ in the second part of the question is parallel to $CD$. Some students may say that they "look parallel" while others might try to reason using the structure of the grid. Tell them that they will investigate this further in the next lesson.

### Addressing

- 8.G.A.1.a

### Building Towards

- 8.G.A.1.c

### Instructional Routines

- MLR8: Discussion Supports
- Think Pair Share

### Launch

Arrange students in groups of 2. Provide access to geometry toolkits. Give 3 minutes of quiet work time, followed by sharing with a partner and a whole-class discussion.

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**Access for Students with Disabilities**

*Representation: Internalize Comprehension.* Begin the activity with concrete or familiar contexts. As in previous lessons, use tracing paper or digital software to rotate a segment 180 degrees around a point. Lead the class in a think aloud considering the point of rotation to be the midpoint or endpoint of the segment as an entry point for this activity.
*Supports accessibility for: Conceptual processing; Memory*

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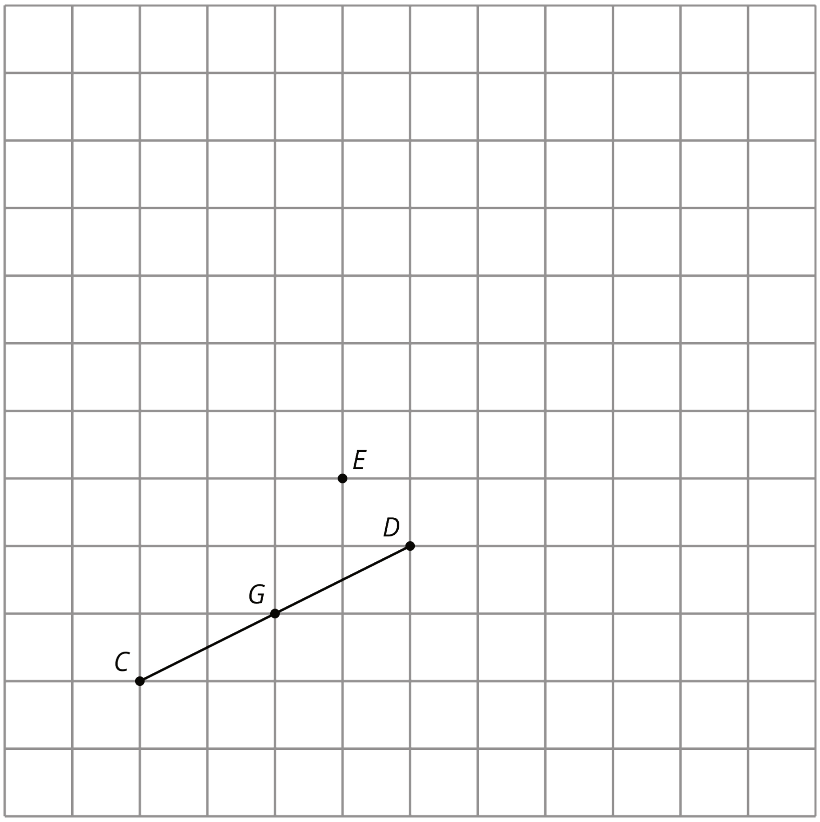
### Anticipated Misconceptions

Students may be confused when rotating around the midpoint because they think the image cannot be the same segment as the original. Assure students this can occur and highlight that point in the discussion.

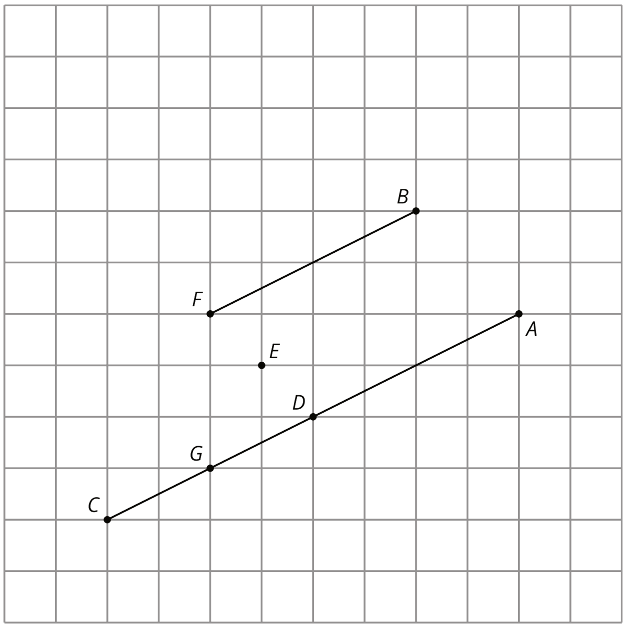
### Student Task Statement




1. Rotate segment $CD$ 180 degrees around point $D$. Draw its image and label the image of $C$ as $A$.
2. Rotate segment $CD$ 180 degrees around point $E$. Draw its image and label the image of $C$ as $B$ and the image of $D$ as $F$.
3. Rotate segment $CD$ 180 degrees around its midpoint, $G$. What is the image of $C$?
4. What happens when you rotate a segment 180 degrees around a point?

### Student Response

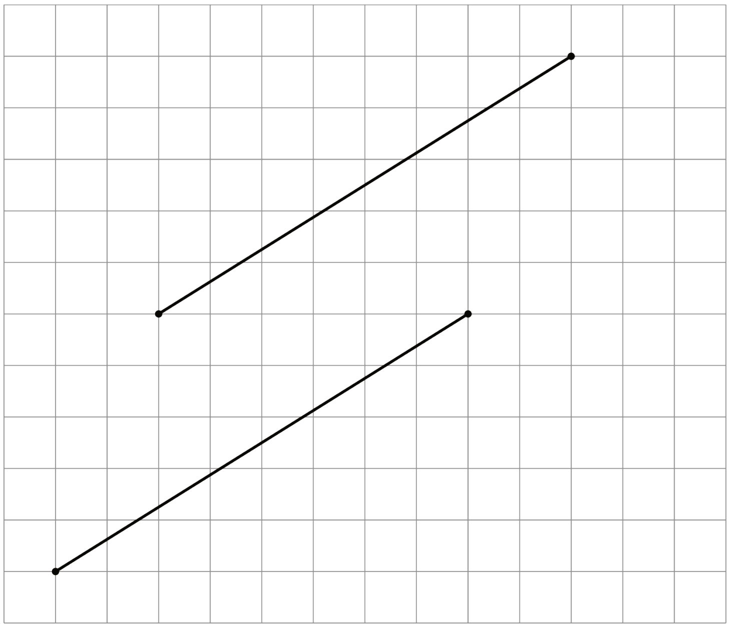
1.




2. The image of the segment lines up with itself, but the endpoints are switched. $D$ is now where $C$ was and $C$ is where $D$ was.

3. The new segment may change its location, but it remains the same length. The new segment is parallel to the original segment. When the point of rotation is the midpoint of the segment, then the rotated segment is the same as the original (the endpoints trade places) and when the point of rotation is an end point of the segment, the image connects to the original to form a segment twice as long.

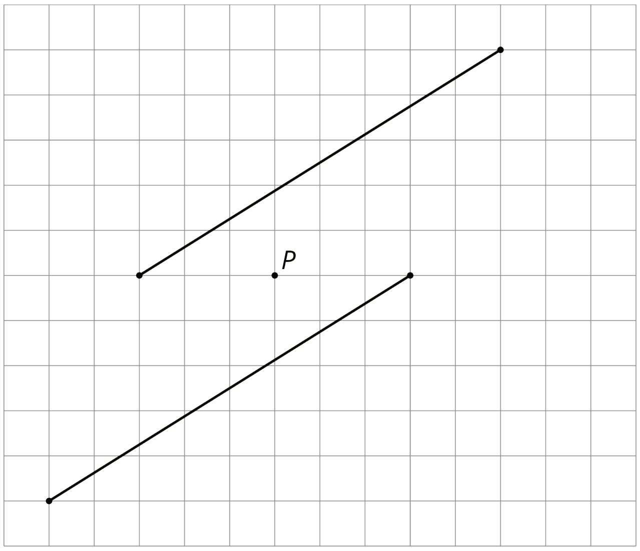
**Are You Ready for More?**



Here are two line segments. Is it possible to rotate one line segment to the other? If so, find the center of such a rotation. If not, explain why not.

### Student Response

Yes




### Activity Synthesis

Ask students why it is not necessary to specify the direction of a 180 degree rotation (because a 180 degree clockwise rotation around point $P$ has the same effect as a 180 degree counterclockwise rotation around $P$). Invite groups to share their responses. Ask the class if they agree or disagree with each response. When there is a disagreement, have students discuss possible reasons for the differences.

Three important ideas that emerge in the discussion are:

- Rotating a segment $180^\circ$ around a point that is not on the original line segment produces a parallel segment the same length as the original.
- When the center of rotation is the midpoint, the rotated segment is the same segment as the original, except the vertices are switched.
- When the center of rotation is an endpoint, the segment together with its image form a segment twice as long.

If any of the ideas above are not brought up by the students during the class discussion, be sure to make them known.

All of these ideas can be emphasized dynamically by carrying out a specified rotation in the applet and then moving the center of rotation or an endpoint of the original line segment. Even if students are not using the digital version of the activity, you may want to display and demonstrate with the applet.

### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Use this routine to support whole-class discussion when students discuss whether it is necessary to specify the direction of a 180 degree rotation. After a student speaks, call on students to restate and/or revoice what was shared using mathematical language (e.g., rotation, line segment, midpoint, etc.). This will provide more students with an opportunity to produce language as they explore special cases of rotating a line segment $180^\circ$.
*Design Principle(s): Support sense-making; Maximize meta-awareness*

# 8.3 A Pattern of Four Triangles

**10 minutes (there is a digital version of this activity)**
In this activity, students use rotations to build a pattern of triangles. In the previous lesson, students examined a right triangle and a rigid transformation of the triangle. In this activity, several rigid transformations of the triangle form an interesting pattern.

Triangle $ABC$ can be mapped to each of the three other triangles in the pattern with a single rotation. As students work on the first three questions, watch for any students who see that a single rotation can take triangle $ABC$ to $CDE$. The center for the rotation is not drawn in the diagram: it is the intersection of segment $AE$ and segment $CG$. For students who finish early, guide them to look for a *single* transformation taking $ABC$ to each of the other triangles.

This pattern will play an important role later when students use this shape to understand a proof of the Pythagorean Theorem.

Identify students who notice that they have already solved the first question in an earlier activity. Watch for students who think that $CAGE$ is a square and tell them that this will be addressed in a future lesson. However, encourage them to think about what they conclude about $CAGE$ now. Also watch for students who repeat the same steps to show that $ABC$ can be mapped to each of the other three triangles.

## Addressing

- 8.G.A.1.a
- 8.G.A.1.b

## Instructional Routines

- MLR8: Discussion Supports

## Launch

Arrange students in groups of 2–4. Provide access to geometry toolkits.

If using the digital activity, give students individual work time before allowing them to converse with a partner.

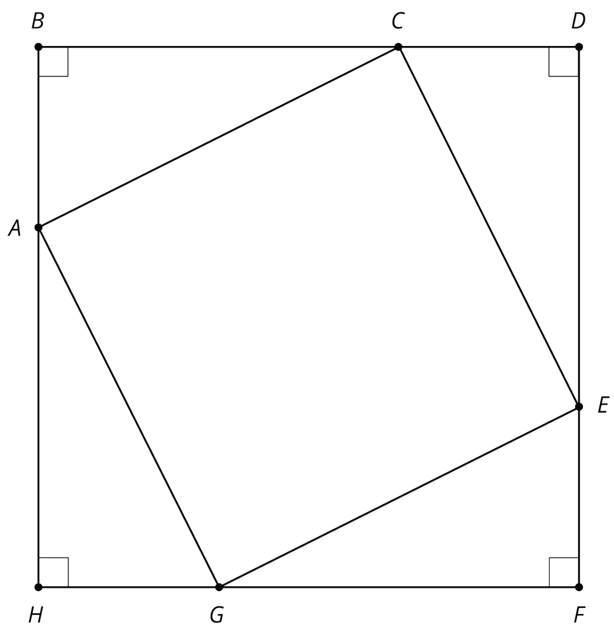
### Access for Students with Disabilities

*Representation: Internalize Comprehension.* Demonstrate and encourage students to use color coding and annotations to draw students' attention to the triangles in the diagram and the corresponding question. For example, highlight and isolate triangles $ABC$ and $CDE$ to help make connections to work in the previous activity.
*Supports accessibility for: Visual-spatial processing*

## Anticipated Misconceptions

Some students might not recognize how this work is similar to the previous activity. For these students, ask them to step back and consider only triangles $ABC$ and $CDE$, perhaps covering the bottom half of the diagram.

## Student Task Statement




You can use rigid transformations of a figure to make patterns. Here is a diagram built with three different transformations of triangle $ABC$.

1. Describe a rigid transformation that takes triangle $ABC$ to triangle $CDE$.
2. Describe a rigid transformation that takes triangle $ABC$ to triangle $EFG$.
3. Describe a rigid transformation that takes triangle $ABC$ to triangle $GHA$.
4. Do segments $AC$, $CE$, $EG$, and $GA$ all have the same length? Explain your reasoning.

### Student Response

1. Answers vary. Sample responses:

- Translate point $B$ to point $D$, then rotate 90 degrees clockwise using $D$ as center.
- Rotate counterclockwise using $C$ as center until segment $CA$ matches up perfectly with segment $CE$, then rotate 180 degrees using the midpoint of segment $CE$ as center.

1. Answers vary. Sample responses:

- Translate $B$ to $F$ and then rotate 180 degrees with center $F$.
- Translate so segment $AC$ matches up with segment $GE$ and then rotate 180 degrees with the midpoint of segment $GE$ as center of rotation.

1. Answers vary. Sample responses:

- Translate $B$ to $H$ and then rotate 90 degrees counterclockwise with center $H$.
- Rotate with center $A$ so that segment $AC$ matches up with segment $AG$ and then rotate 180 degrees with the midpoint of segment $AG$ as center.

1. Yes, because the size and shape of triangle $ABC$ did not change under the rigid transformation. Segment $AC$ can be matched up exactly with segments $CE$, $EG$, and $GA$ so the lengths of these segments are all the same.

### Activity Synthesis

Select a student previously identified who noticed how the first question relates to a previous activity to share their observation. Discuss here how previous work can be helpful in new work, since students may not be actively looking for these connections. The next questions are like the first, but the triangles have a different orientation and different transformations are needed.

Discuss rigid transformations. Focus especially on the question about lengths. A key concept in this section is the idea that lengths and angle measures are preserved under rigid transformations.

Some students may claim $CAGE$ is a square. If this comes up, leave it as an open question for now. This question will be revisited at the end of this unit, once the angle sum in a triangle is known. The last question establishes that $CAGE$ is a rhombus.

#### Access for English Language Learners

*Speaking: MLR8 Discussion Supports.* Give students additional time to make sure that everyone in their group can explain whether the segments AC, CE, EG, and GA all have the same lengths. Then, vary who is called on to represent the ideas of each group. This routine will prepare students for the role of group representative and to support each other to take on that role.
*Design Principle(s): Optimize output (for explanation)*

## Lesson Synthesis

Ask students to describe the possible outcomes when a line segment $AB$ is rotated 180 degrees.

- $AB$ is mapped to itself, when the center of rotation is the midpoint of the segment
- $AB$ is mapped to another segment collinear with the first, when the center of rotation is $A$ or $B$ (or any other point on segment $AB$)
- $AB$ is mapped to a parallel segment, when the center of rotation is not on line $AB$.

# 8.4 Is it a rotation?

**Cool Down: 5 minutes**

Having studied rotations in detail throughout this lesson, students look at a triangle and its image after a rigid transformation. They decide whether or not one is a rotation of the other. It turns out to be a reflection rather than a rotation. Students can use tracing paper to verify their conjectures, but at this point they should start to have an intuition for the effects of a rotation versus a reflection.

### Addressing

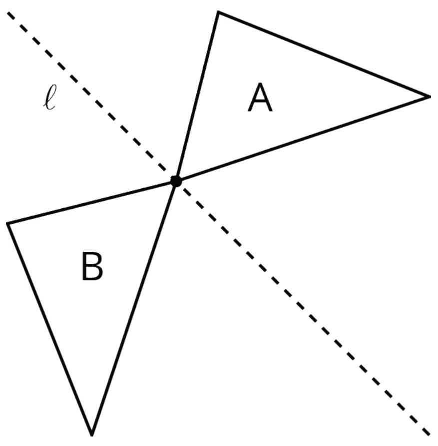
- 8.G.A.1.a
- 8.G.A.1.b

### Launch

Make tracing paper available.

### Student Task Statement

Here are two triangles.




Is Triangle B a rotation of Triangle A? Explain your reasoning.

**Student Response**

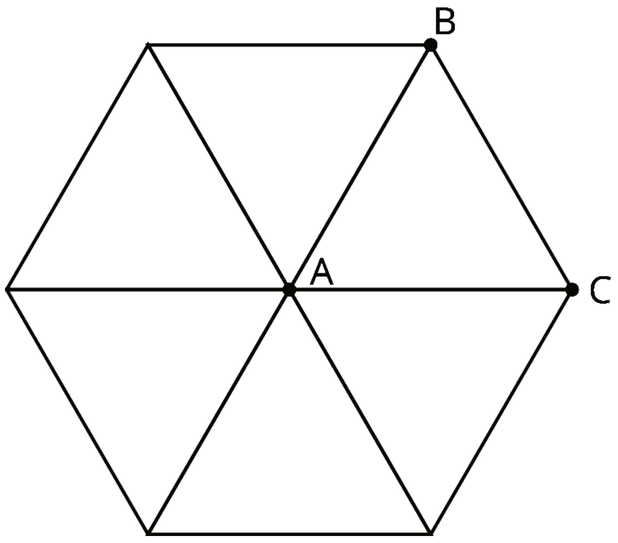
No, Triangle $B$ is a reflection of Triangle $A$ over line $\ell$. A rotation can be used to match two sides of the triangles but will not match one up perfectly with the other.

## Student Lesson Summary

When we apply a 180-degree rotation to a line segment, there are several possible outcomes:

- The segment maps to itself (if the center of rotation is the midpoint of the segment).
- The image of the segment overlaps with the segment and lies on the same line (if the center of rotation is a point on the segment).
- The image of the segment does not overlap with the segment (if the center of rotation is *not* on the segment).

We can also build patterns by rotating a shape. For example, triangle $ABC$ shown here has $m(\angle A) = 60$. If we rotate triangle $ABC$ 60 degrees, 120 degrees, 180 degrees, 240 degrees, and 300 degrees clockwise, we can build a hexagon.




# Lesson 8 Practice Problems

## Problem 1

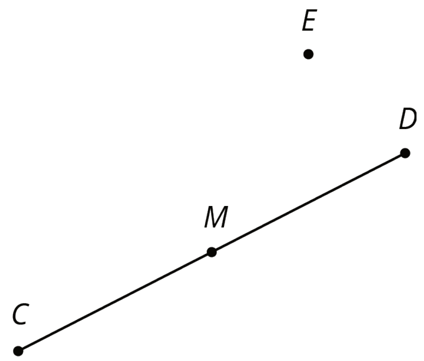
### Statement

For the figure shown here,

a. Rotate segment $CD$ $180^\circ$ around point $D$.

b. Rotate segment $CD$ $180^\circ$ around point $E$.

c. Rotate segment $CD$ $180^\circ$ around point $M$.




## Solution

a. The segment is attached at point $D$ and is an extension of segment $CD$.

b. The segment is above point $E$ and is parallel to segment $CD$.

c. The segment is identical to segment $CD$.

## Problem 2

### Statement

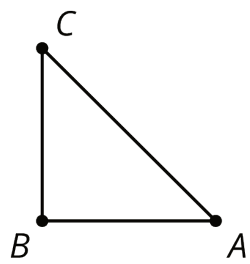
Here is an isosceles right triangle:

Draw these three rotations of triangle $ABC$ together.

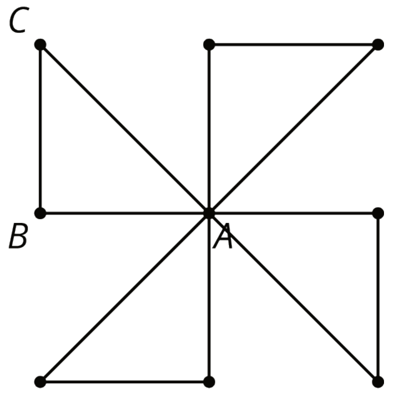
a. Rotate triangle $ABC$ 90 degrees clockwise around $A$.

b. Rotate triangle $ABC$ 180 degrees around $A$.

c. Rotate triangle $ABC$ 270 degrees clockwise around $A$.



## Solution

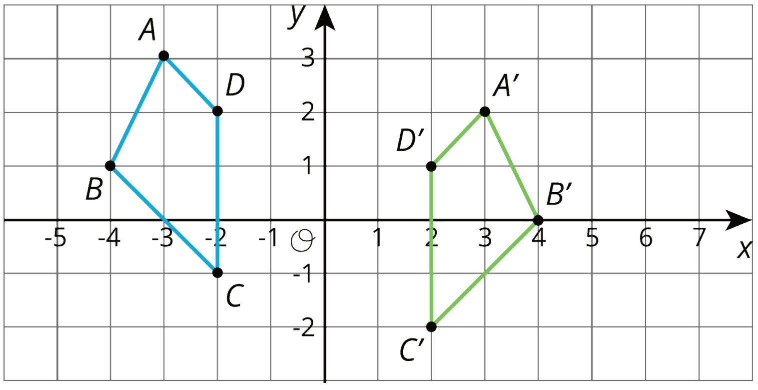



## Problem 3

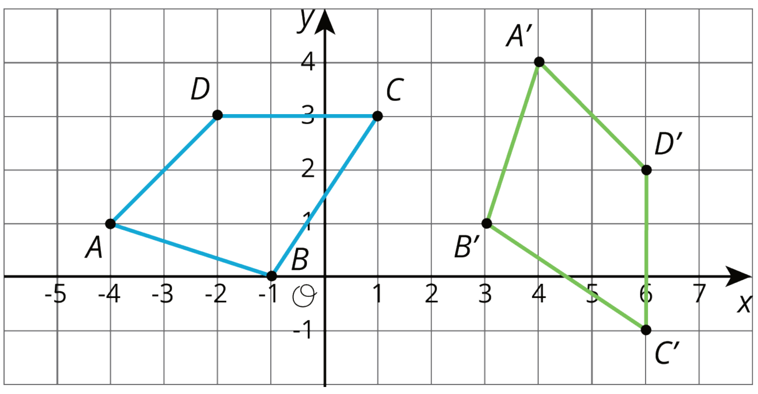
### Statement

Each graph shows two polygons $ABCD$ and $A'B'C'D'$. In each case, describe a sequence of transformations that takes $ABCD$ to $A'B'C'D'$.

a.




b.



## Solution

a. Reflect $ABCD$ over the $y$-axis, and then translate down 1.

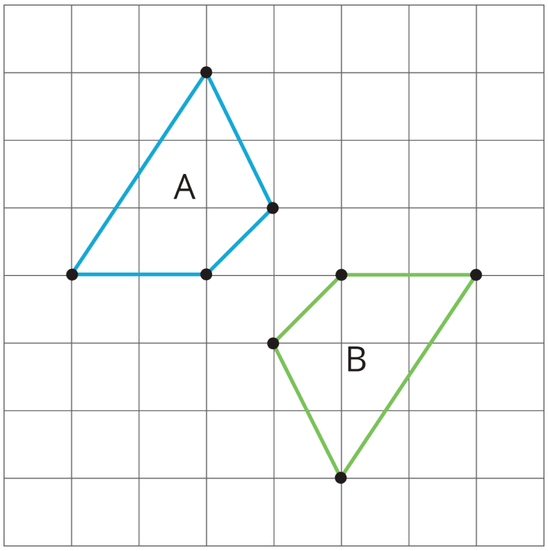
b. Rotate $ABCD$ 90 degrees clockwise with center $B = (-1, 0)$, and then translate $(-1, 0)$ to $(3, 1)$.

(From Unit 1, Lesson 5.)

# Problem 4

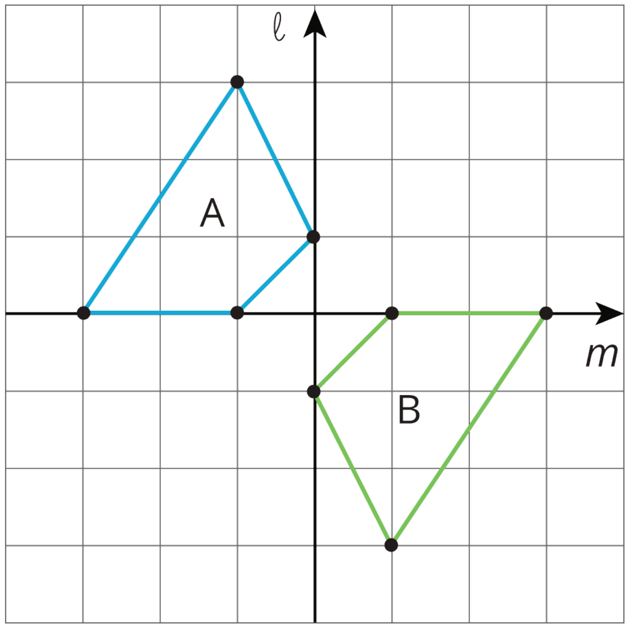
## Statement

Lin says that she can map Polygon A to Polygon B using *only* reflections. Do you agree with Lin? Explain your reasoning.




## Solution

I agree with Lin. If Polygon A is reflected first over the vertical line $\ell$ and then over the horizontal line $m$, this takes Polygon A to Polygon B.




(From Unit 1, Lesson 4.)