## Lesson 7: Expressing Transformations of Functions Algebraically

* Let’s express transformed functions algebraically.

### 7.1: Describing Translations

Let $g(x)=\sqrt{x}$. Complete the table. Be prepared to explain your reasoning.

|  |  |  |
| --- | --- | --- |
| words (the graph of $y=g(x)$ is...) | function notation | expression |
| translated left 5 units | $g(x+5)$ |   |
| translated left 5 units and down 3 units |   | $\sqrt{x+5}−3$ |
|   | $g(-x)$ | $\sqrt{-x}$ |
| translated left 5 units, then down 3 units,then reflected across the $y$-axis |   |   |

### 7.2: Translating Vertex Form

Let $f$ be the function given by $f(x)=x^{2}$.

1. Write an equation for the function $g$ whose graph is the graph of $f$ translated 3 units left and up 5 units.
2. What is the vertex of the graph of $g$? Explain how you know.
3. Write an equation for a quadratic function $h$ whose graph has a vertex at $(1.5,2.6)$.
4. Write an equation for a quadratic function $k$ whose graph opens downward and has a vertex at $(3.2,-4.7)$.

### 7.3: An Even Better Fit

In an earlier lesson, we looked at the temperature $T$, in degrees Fahrenheit, of a bottle of soda water left outside for $h$ hours. Let’s model this data with a function. This time, we will start with the function $f(h)=33(0.6)^{h}$. This graph has a shape that fits the data well.





1. Describe a translation of this graph that fits the data.
2. Write an equation defining a function $g$ that models the data.
3. What does your function tell you about the temperature outside?

#### Are you ready for more?

Han tried the following steps to model the soda water temperature. First he shifts the given graph left by one hour, then he applies a vertical shift.

1. What vertical shift does Han need to apply to model the 45 degree Fahrenheit temperature in the refrigerator?
2. How does Han’s model compare to yours?

### Lesson 7 Summary

You can use the equation of a function to write an equation for its transformation. For example, let $f(x)=x^{2}$. Take the graph of $f$, reflect it across the $x$-axis, translate it up 10 units, and translate it left 3 units. What is an equation for this new function? The new function $g$ is related to $f$ by $g(x)=-f(x+3)+10$, since



Which means $g(x)=-(x+3)^{2}+10$.

Sometimes you can recognize from the expression for a function that it is the transformation of a simpler function. For example, consider:

$H(t)=10−(1.2)^{t+5}$

One way to obtain the expression for $H$ from $1.2^{t}$ is:

* adding 5 to the input to get $(1.2)^{t+5}$
* multiplying the output by -1 to get $-(1.2)^{t+5}$
* adding 10 to the output to get $10−(1.2)^{t+5}$

So the graph of $H$ is obtained from the graph of $f(t)=1.2^{t}$ by translating left 5 units, reflecting across the $x$-axis, and translating up 10 units. Consider the point $(0,1)$ on the graph of $f$. After translating, reflecting, and translating again, it becomes the point $(-5,9)$ on the graph of $H$.





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