Lesson 4: Positive Rational Exponents

- Let's use roots to write exponents that are fractions.

4.1: Math Talk: Regrouping Fractions

Find the value of each expression mentally.

$\frac{1}{2} \cdot 5 \cdot 4$

$\frac{5}{2} \cdot 4$

$\frac{2}{3} \cdot 7 \cdot \frac{3}{2}$

$7 \cdot \frac{5}{3} \cdot \frac{3}{7}$

4.2: You Can Use Any Fraction As an Exponent

1. Use exponent rules to explain why these expressions are equal to each other:

   $\left(5^{\frac{1}{3}}\right)^2 \quad \left(5^2\right)^{\frac{1}{3}}$

2. Write $5^{\frac{2}{3}}$ using radicals.

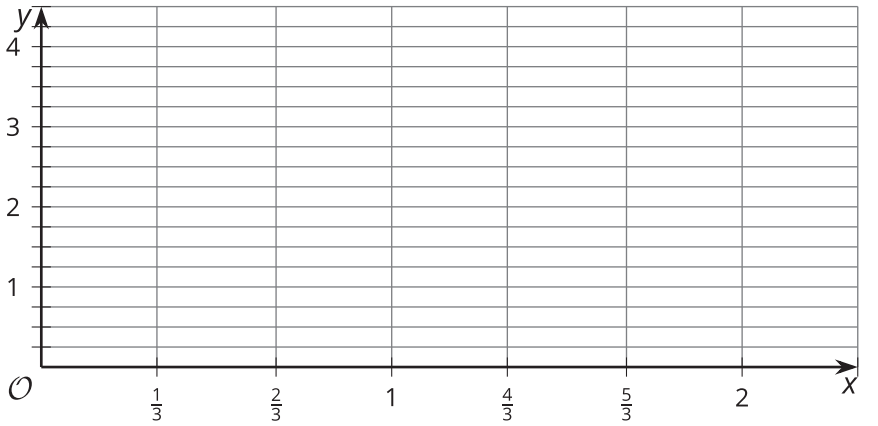
3. Write $5^{\frac{4}{3}}$ using radicals. Show your reasoning using exponent rules.

4.3: Fractional Powers Are Just Numbers

1. Complete the table as much as you can without using a calculator. (You should be able to fill in three spaces.)

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
2^x **(using exponents)**	2^0	$2^{\frac{1}{3}}$	$2^{\frac{2}{3}}$	2^1	$2^{\frac{4}{3}}$	$2^{\frac{5}{3}}$	2^2
2^x **(decimal approximation)**							

a. Plot the points that you filled in.



b. Connect the points as smoothly as you can.

c. Use this graph of $y = 2^x$ to estimate the value of the other powers in the table, and write your estimates in the table.

2. Let's investigate $2^{\frac{1}{3}}$:

a. Write $2^{\frac{1}{3}}$ using radical notation.

b. What is $\left(2^{\frac{1}{3}}\right)^3$?

c. Raise your estimate from the table of $2^{\frac{1}{3}}$ to the third power. What should it be? How close did you get?

3. Let's investigate $2^{\frac{2}{3}}$:

a. Write $2^{\frac{2}{3}}$ using radical notation.

b. What is the value of $\left(2^{\frac{2}{3}}\right)^3$?

c. Raise your estimate from the table of $2^{\frac{2}{3}}$ to the third power. What should it be? How close did you get?

Are you ready for more?

Answer these questions using the fact that $(1.26)^3 = 2.000376$.

1. Explain why $\sqrt[3]{2}$ is very close to 1.26. Is it larger or smaller than 1.26?

2. Is it possible to write $\sqrt[3]{2}$ exactly with a finite decimal expansion? Explain how you know.

Lesson 4 Summary

Using exponent rules, we know $3^{\frac{1}{4}}$ is the same as $\sqrt[4]{3}$ because $\left(3^{\frac{1}{4}}\right)^4 = 3$. But what about $3^{\frac{5}{4}}$?

Using exponent rules,

$$3^{\frac{5}{4}} = \left(3^5\right)^{\frac{1}{4}}$$

which means that

$$3^{\frac{5}{4}} = \sqrt[4]{3^5}$$

Since $3^5 = 243$, we could just write $3^{\frac{5}{4}} = \sqrt[4]{243}$.

Alternatively, we could express the fraction $\frac{5}{4}$ as $\frac{1}{4} \cdot 5$ instead. Using exponent rules, we get

$$3^{\frac{5}{4}} = \left(3^{\frac{1}{4}}\right)^5 = \left(\sqrt[4]{3}\right)^5$$

Here are more examples of exponents that are fractions and their equivalents:

x	0	$\frac{1}{3}$	$\frac{2}{3}$	1	$\frac{4}{3}$	$\frac{5}{3}$	2
5^x **(using exponents)**	5^0	$5^{\frac{1}{3}}$	$5^{\frac{2}{3}}$	5^1	$5^{\frac{4}{3}}$	$5^{\frac{5}{3}}$	5^2
5^x **(equivalent expression)**	1	$\sqrt[3]{5}$	$\sqrt[3]{5^2}$ or $\sqrt[3]{25}$	5	$\sqrt[3]{5^4}$ or $\sqrt[3]{625}$	$\sqrt[3]{5^5}$ or $\sqrt[3]{3125}$	25