Lesson 10: What Are Percentages?

Goals

- Comprehend the word “percentage” (in written and spoken language) and the symbol “%” (in written language) to mean a rate per 100.
- Draw and label a double number line diagram to represent percentages of a dollar and to find corresponding monetary values or percentages.

Learning Targets

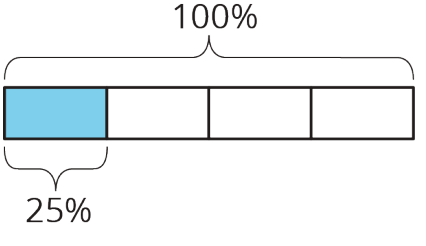
- I can create a double number line with percentages on one line and dollar amounts on the other line.
- I can explain the meaning of percentages using dollars and cents as an example.

Lesson Narrative

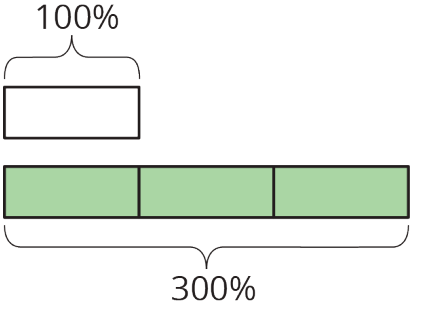
This lesson is the first of two that introduce students to **percentages** as a rate per 100 (MP6) and the ways they are used to describe different types of situations.

Percentages are commonly used in two ways:

1. To describe a part of a whole. For example, “Jada drank 25% of the bottle of water.” In this case, the percentage expressing the amount consumed is not bigger than 100% because it refers to a part of a whole, as shown in the diagram below.



2. To describe the size of one quantity as a percentage of another quantity. For example, “Jada drank 300% as much water as Diego did.” In this case, there is no restriction on the size of the percentage, because the percentage is describing a multiplicative comparison between two quantities, as shown below.



In the first usage there is a single quantity and we are describing a part of it; in the second usage we are comparing two quantities. Students may have prior exposure to percentages, but are likely to have only encountered the first usage and might not be able to make sense of percentages above 100% or those used in comparative contexts. This lesson exposes students to both applications of percentages.

Money is the main context for exploring percentages in this lesson and the warm up asks students to convert between dollars and cents providing an opportunity for the teacher to assess students' current abilities.

For the first several lessons exploring percentages, double number lines are the primary representation presented to students. This choice is intended to strongly communicate that we are working with percent *rates*, and that students can and should use all of the reasoning they have developed to deal with equivalent ratios and rates when dealing with rates per 100. That said, if students prefer to reason using tables or by multiplying or dividing by unit rates, they should not be discouraged from doing so.

Alignments

Building On

- 2.MD.C.8: Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately. Example: If you have 2 dimes and 3 pennies, how many cents do you have?
- 5.NBT.A.3: Read, write, and compare decimals to thousandths.
- 6.RP.A: Understand ratio concepts and use ratio reasoning to solve problems.

Addressing

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Building Towards

- 6.RP.A.3.c: Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

Instructional Routines

- MLR3: Clarify, Critique, Correct
- MLR7: Compare and Connect
- Think Pair Share

Student Learning Goals

Let's learn about percentages.

10.1 Dollars and Cents

Warm Up: 5 minutes
This warm-up prompts students to reason in monetary terms, preparing them for subsequent tasks in the lesson. It also provides insight into students' understanding of dollars and cents as well as their ability to reason mentally.

Building On

- 2.MD.C.8
- 5.NBT.A.3

Building Towards

- 6.RP.A.3.c

Launch

Display questions for all to see. Ask students to solve them mentally.

Anticipated Misconceptions

In response to "how many dollars are in one cent," students might say there are no dollars at all in one cent. Ask them what fraction of a dollar one cent represents.

Student Task Statement

Find each answer mentally.

1. A sticker costs 25 cents. How many dollars is that?
2. A pen costs 1.50 dollars. How many cents is that?
3. How many cents are in one dollar?
4. How many dollars are in one cent?

Student Response

1. $\frac{1}{4}$ (or 0.25) of a dollar. There are four quarters in a dollar, and a quarter is 25 cents.
2. 150 cents. There are 100 cents in 1 dollar, so 1.50 dollars is multiplied by 100 to find the number of cents.
3. There are 100 cents in 1 dollar.
4. $\frac{1}{100}$ (or 0.01) of a dollar. Since there are 100 cents in 1 dollar, 1 dollar is divided by 100.

Activity Synthesis

After students solved all problems mentally, for each problem, ask 1–2 students to share their thinking. Pause between problems to give everyone time to reflect on the shared answers.

10.2 Coins

15 minutes
In this activity, students learn the definition of a **percentage** as a rate per 100 and apply this definition in the context of money. They label various coin amounts as percentages of 100 cents or 1 dollar.

Students are likely able to name the values of each coin and their individual percentages (in the first two questions) fairly quickly. Assigning a percentage to a group of coins (in the last two questions) adds complexity and should be the focus of the activity as students may use a variety of strategies. One possible strategy is to reason in terms of ratios. For example, a student may think that if a dime is 10% of a dollar, then 6 dimes is 60% of a dollar. This type of ratio thinking is a robust way for dealing with percent problems and should be encouraged early.

As students work, notice the strategies being used to solve the two problems and identify those with effective approaches so they can share later.

Building Towards

- 6.RP.A.3.c

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Remind students that previously they have learned that a “rate per 1” tells us the amount of one quantity for 1 of another quantity. Explain that in this task, they will explore “rates per 100.”

Solicit a couple of ideas on what “rates per 100” might mean. Students are likely to suggest a description along the lines of “the amount of something for 100 of something else.” Tell students that a rate per 100 is called a percentage and that they will explore percentages in the context of money. Point out the half-dollar and dollar coins in the task, as some students may not be familiar with them.

Arrange students in groups of 2. Give students 3 minutes of quiet think time to begin work on the task. After that time, ask students to share their responses with a partner and complete the remaining questions together.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their ideas. For example, "If _____ then _____ because..." or "How do you know...?"
Supports accessibility for: Language; Organization

Anticipated Misconceptions

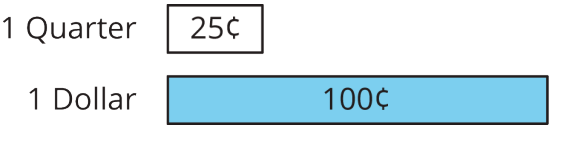
Students may notice a pattern particular to this activity—that the percent value is the same as that for cents—and carry that assumption forward and apply it incorrectly to situations in which 100% does not correspond to 100. This conversation is addressed in the Activity Synthesis.

Student Task Statement

1. Complete the table to show the values of these U.S. coins.

coin	penny	nickel	dime	quarter	half dollar	dollar
value (cents)						

The value of a quarter is 25% of the value of a dollar because there are 25 cents for every 100 cents.



2. Write the name of the coin that matches each expression.

- ○ 25% of a dollar
- ○ 5% of a dollar
- ○ 1% of a dollar
- ○ 100% of a dollar
- ○ 10% of a dollar
- ○ 50% of a dollar

3. The value of 6 dimes is what **percent** of the value of a dollar?

4. The value of 6 quarters is what percent of the value of a dollar?

Student Response

1. From left to right in the table: 1, 5, 10, 25, 50, 100
2. a. Quarter

   b. Nickel

   c. Penny

   d. Dollar

   e. Dime

   f. Half-dollar
3. The value of 6 dimes are 60 cents and a dollar is 100 cents, so 6 dimes are 60% of the value of 1 dollar.
4. The value of 6 quarters is 150 cents and a dollar is 100 cents, so 6 quarters is 150% of the value of 1 dollar.

Are You Ready for More?

Find two different sets of coins that each make 120% of a dollar, where no type of coin is in both sets.

Student Response

Answers vary. Sample response: A dollar and two dimes, four quarters and four nickels

Activity Synthesis

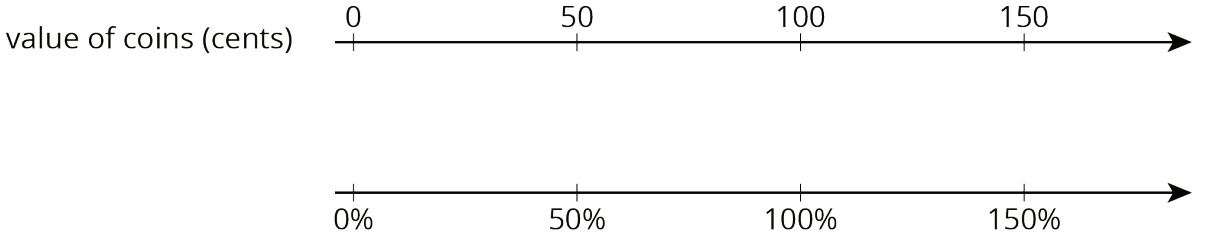
Focus the discussion on the ways students approached the last two questions and on precise use of language and notation (MP6). For example, in the first two problems students can write only a number or matched a coin to a pre-written phrase. In the last two problems, however, expressing a percentage with only a number and without the % symbol should be considered an incomplete answer.

Select students with successful strategies to share their thinking with the class. Display a concise version of their reasoning for all to see. Invite others to express support, disagreement, or questions (MP3).

If no one reasoned about percentages in terms of ratios (e.g., If a quarter is 25% of a dollar, 6 quarters are 150% of a dollar), illustrate it.

Many students may reason by noticing a pattern—that the number of cents in an amount matches its percentage of a dollar (e.g., 60 cents is 60%)—rather by thinking in terms of ratio or scaling. Since the pattern only holds up in the context of percentages of 100 of a quantity, students will need to

be prompted to look more closely at the meaning of "rate per 100." Conclude the discussion by displaying the following double number line with 100 at the 100%:



Point out that we were finding **percentages** of 100, so in the double number line, we line up 100**%** and 100 because 100% of 100 is 100.

Access for English Language Learners

Reading, Writing, Speaking: MLR3 Clarify, Critique, Correct. Before students share their explanations for the final question, present an incorrect answer and explanation. For example, "The value of 6 quarters is 50% of the value of a dollar because the value of 6 quarters is 150 cents, which is 50 cents greater than 100 cents. This means that the value of 6 quarters is 50% of the value of a dollar." Ask students to identify the error, critique the reasoning, and write a correct explanation. As students discuss in pairs, listen for students who identify and clarify the ambiguous language in the statement. For example, the author probably meant to say that 6 quarters is 50% greater than the value of a dollar, or that 6 quarters is 150% of the value of a dollar. This will help students understand how to use percentages to describe the size of one quantity as a percentage of another quantity.
Design Principle(s): Optimize output (for explanation); Maximize meta-awareness

10.3 Coins on a Number Line

10 minutes
Previously, students found percentages of 100 cents. In this activity, they reason about percentages of 1 dollar.

One important question to think about here is how students know or decide how the numbers on the double number line diagram should be aligned. Students build on their extensive work on equivalent ratios and double number lines to make sense of percentages and "per 100" reasoning.

Building On

- 6.RP.A

Addressing

- 6.RP.A.3.c

Instructional Routines

- MLR7: Compare and Connect
- Think Pair Share

Launch

Recap that in the previous activity students found percentages of 100 cents. Tell students they will now find percentages of 1 dollar. Draw their attention to the fact that, on the double number line, the 1 dollar and 100% are lined up vertically to reflect this.

Keep students in the same groups. Give students 2–3 minutes of quiet think time, and then ask them to share their responses with their partner. Display and read aloud the following questions. Ask partners to use them to guide their discussion.

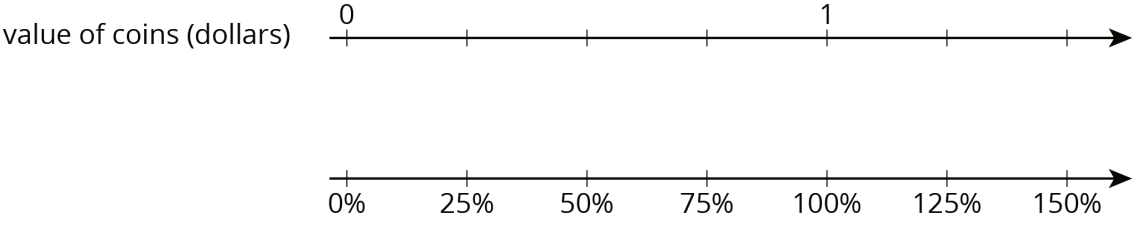
- How did each of you arrive at your answers for the first two questions?
- Where do your answers fall on the double number line diagram? How do you know?
- Are your answers the same for the third question? If they are not, can they both be correct? If they are, can you think of another answer that would also be correct?

Anticipated Misconceptions

Based on previous work with labeling number lines less than 1, students may label the tick marks with fractions instead of the decimal value of the coins. This may not be helpful for answering the first two questions, but provides an opportunity to discuss alternative ways to label the number line given the context of the problem. Consider prompting them to write fractional values as cents or to rewrite the cents as dollar values.

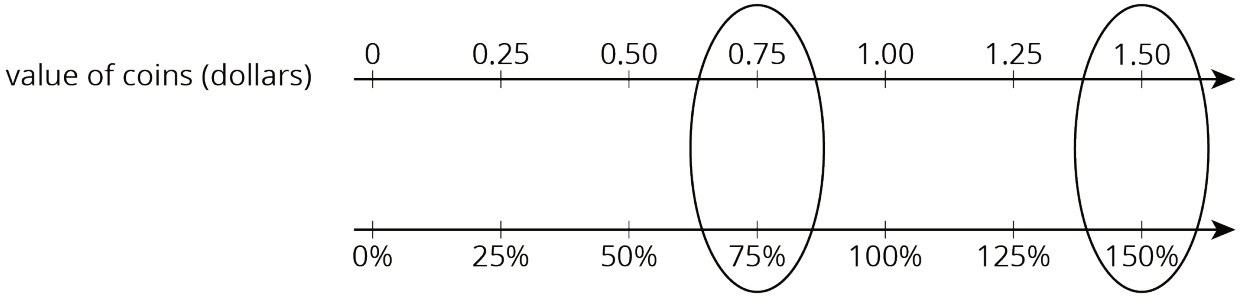
Student Task Statement

A $1 coin is worth 100% of the value of a dollar. Here is a double number line that shows this.



1. The coins in Jada's pocket are worth 75% of a dollar. How much are they worth (in dollars)?
2. The coins in Diego's pocket are worth 150% of a dollar. How much are they worth (in dollars)?
3. Elena has 3 quarters and 5 dimes. What percentage of a dollar does she have?

Student Response



1. $0.75
2. $1.50
3. 125%

Activity Synthesis

Select students who used the provided double number line to share their reasoning. This is an opportunity to refresh students' number line reasoning. Some students may see the four equally spaced tick marks from 0 to 1 and conclude that each is worth 0.25, or $\frac{1}{4}$. Others may fill in the 0.50 first, as it is half of 1, then the 0.25 for half of 0.50, and then use additive thinking to fill in the other tick mark values along the top.

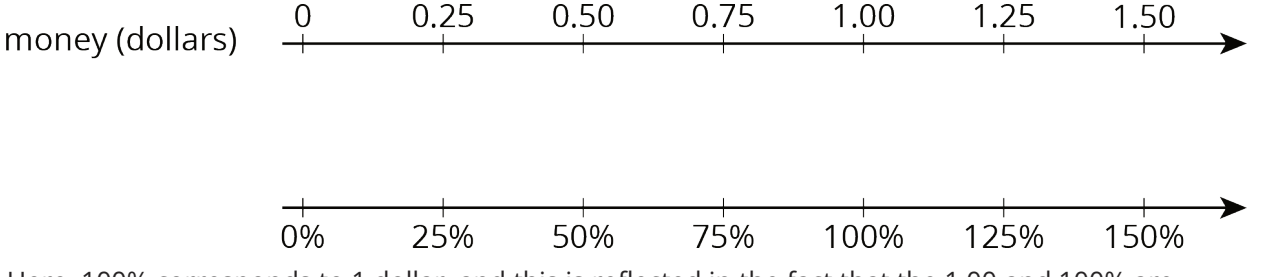
Some students may reason in terms of equivalent ratios and say, for example, that since 100 divided by 4 is 25, then $1 \div 4 = 0.25$ must be 25% of 1. They would then assign the 0.25 value to the first tick mark and use additive thinking to conclude that 0.75 is 75% of a dollar. Ask students who used such an approach to present last to emphasize that the familiar ratio thinking applies to percentage problems as well, even though the % symbol may be unfamiliar. If no students took this approach, illustrate it to make this point.

Access for English Language Learners

Speaking, Listening: MLR7 Compare and Connect. As students prepare a visual display of how they made sense of the problem, look for students who labeled the tick marks on the double number line with fractions or cents instead of dollar values. This may result in answers such as ¾, or 75 cents is 75% of a dollar rather than 0.75 is 75% of a dollar. Although 75 cents is 75% of a dollar, the number line should be labeled with the decimal value of the coins in dollars. As students investigate each other's work, ask students to share what worked or did not work well in the way they labeled the double number line. Is there a particular advantage to using decimals instead of fractions to label the double number line? Emphasize that although there are several ways to label the double number line given the context of the problem, certain methods are more helpful for answering the question. This will foster students' meta-awareness and support constructive conversations as they compare the various ways to label a double number line given a context.
Design Principles(s): Cultivate conversation; Maximize meta-awareness

Lesson Synthesis

Remind students that a **percentage** is a "rate per 100." We saw that the value of a quarter is 25**%** of the value of a dollar, because a quarter is worth 25 cents and a dollar is worth 100 cents. Reiterate that we found percentages of the value of a dollar using a double number line as shown here:



Here, 100% corresponds to 1 dollar, and this is reflected in the fact that the 1.00 and 100% are aligned in the double number line.

10.4 Eight Dimes

Cool Down: 5 minutes
The purpose of this activity is to see how students make sense of the percentage as a rate per 100.

Addressing

- 6.RP.A.3.c

Anticipated Misconceptions

In the first question, students may write that 8 dimes is 8% of the value of a dollar because they account for the number of coins but not account their value. In the second question, students may

put 130 cents as the answer, not differentiating between combination of coins and the value of the coins.

Student Task Statement

1. Fill in the blank: The value of 8 dimes is _____% of the value of a dollar.

2. Name a combination of coins that is 130% of the value of a dollar.

Student Response

1. 80%. 8 dimes are 80 cents, and a dollar is 100 cents, so 8 dimes are 80% the value of 1 dollar.

2. Answers vary. Sample responses:
   a. 1 dollar and 3 dimes

   b. 5 quarters and 1 nickel

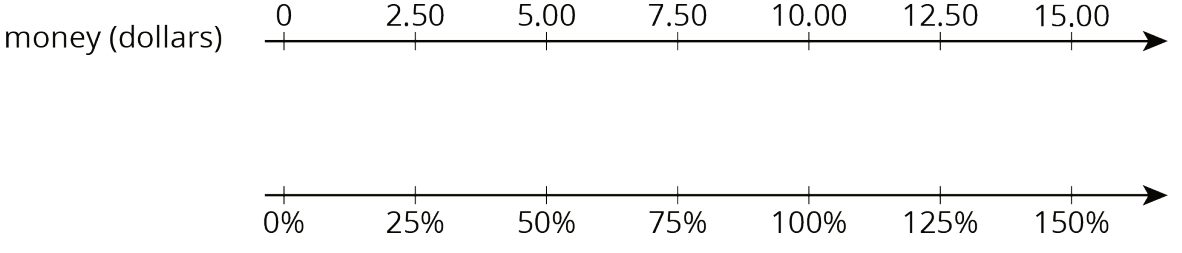
   c. 13 dimes

   d. 26 nickels

Student Lesson Summary

A **percentage** is a *rate per 100*.

We can find percentages of $10 using a double number line where 10 and 100% are aligned, as shown here:



Looking at the double number line, we can see that $5.00 is 50% of $10.00 and that $12.50 is 125% of $10.00.

Glossary

- percent
- percentage

Lesson 10 Practice Problems

Problem 1

Statement

What percentage of a dollar is the value of each coin combination?

a. 4 dimes

b. 1 nickel and 3 pennies

c. 5 quarters and 1 dime

Solution

a. 40%

b. 8%

c. 135%

Problem 2

Statement

a. List three different combinations of coins, each with a value of 30% of a dollar.

b. List two different combinations of coins, each with a value of 140% of a dollar.

Solution

Answers vary. Sample response:

a. 30 pennies, 6 nickels, or 3 dimes

b. 140 pennies, 14 dimes, or 5 quarters and 3 nickels

Problem 3

Statement

The United States government used to make coins of many different values. For each coin, state its worth as a percentage of $1.

$\frac{1}{2}$ cent

3 cents

20 cents

$2\frac{1}{2}$

$5

Solution

a. $\frac{1}{2}\%$

b. 3%

c. 20%

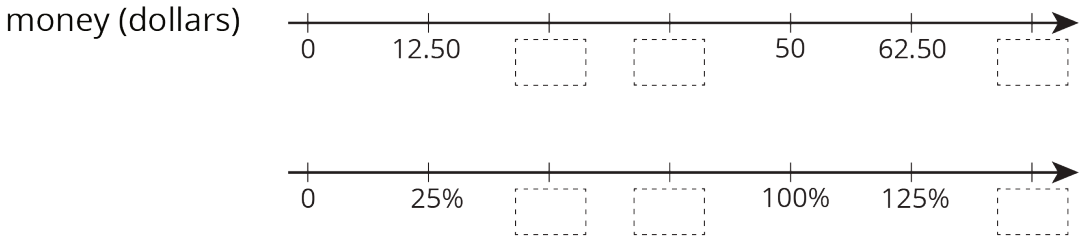
d. 250%

e. 500%

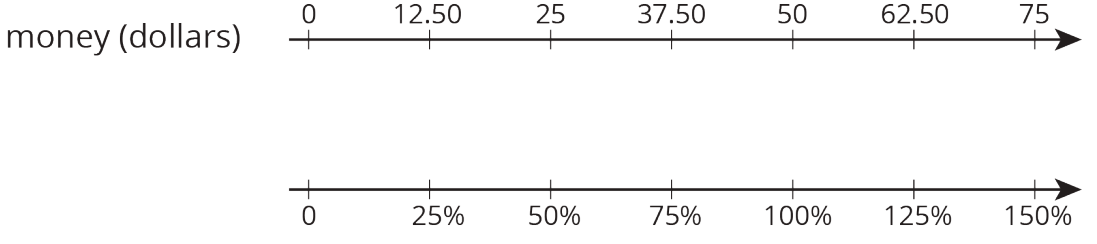
Problem 4

Statement

Complete the double number to line show percentages of $50.



Solution



Problem 5

Statement

Elena bought 8 tokens for $4.40. At this rate:

a. How many tokens could she buy with $6.05?

b. How much do 19 tokens cost?

Solution

a. 11 tokens

b. $10.45

(From Unit 3, Lesson 9.)

Problem 6

Statement

A snail travels 10 cm in 4 minutes. At this rate:

a. How long will it take the snail to travel 24 cm?

b. How far does the snail travel in 6 minutes?

Solution

a. 9.6 minutes (or equivalent)

b. 15 cm

(From Unit 3, Lesson 8.)

Problem 7

Statement

a. 3 tacos cost $18. Complete the table to show the cost of 4, 5, and 6 tacos at the same rate.

number of tacos	cost in dollars	rate in dollars per taco
3	18	
4		
5		
6		

b. If you buy t tacos for c dollars, what is the unit rate?

Solution

a.

number of tacos	cost in dollars	rate in dollars per taco
3	18	6
4	24	6
5	30	6
6	36	6

b. $\frac{c}{t}$ dollars per taco or $\frac{t}{c}$ tacos per dollar.

(From Unit 3, Lesson 7.)