Lesson 13: Expressions with Exponents

Goals

- Critique (orally and in writing) arguments that claim two different numerical expressions are equal.
- Justify (orally and in writing) whether numerical expressions involving whole-number exponents are equal.

Learning Targets

- I can decide if expressions with exponents are equal by evaluating the expressions or by understanding what exponents mean.

Lesson Narrative

In this lesson, students analyze the structure of expressions (MP7) to apply their understanding of exponents. While they practice using the notation of expressions with exponents, students recall and apply their prior understanding of operations and connect those understandings to the meaning of exponents. They write, interpret, and evaluate expressions with exponent notation where the exponents are whole numbers and the bases may be whole numbers, fractions, or decimals. Students also apply their new understanding from earlier in the unit about determining whether equations are true or false.

Alignments

Addressing

- 6.EE.A.1: Write and evaluate numerical expressions involving whole-number exponents.

Instructional Routines

- MLR8: Discussion Supports
- Which One Doesn’t Belong?

> **Student Learning Goals**
>
> Let's use the meaning of exponents to decide if equations are true.

13.1 Which One Doesn’t Belong: Twos

Warm Up: 5 minutes

This warm-up prompts students to compare expressions. It encourages students to explain their reasoning, hold mathematical conversations, and gives you the opportunity to hear how they use terminology and talk about characteristics of the expressions in comparison to one another.

Addressing

- 6.EE.A.1

Instructional Routines

- Which One Doesn’t Belong?

Launch

Arrange students in groups of 2–4. Display the questions for all to see. Ask students to indicate when they have noticed one expression that doesn’t belong and can explain why. Give students 1 minute of quiet think time and then time to share their thinking with their small group. In their small groups, tell each student to share their reasoning why a particular question doesn’t belong and together find at least one reason each question doesn't belong.

Student Task Statement

Which one doesn’t belong?

$2 \cdot 2 \cdot 2 \cdot 2$	2^4
16	$4 \cdot 2$

Student Response

Answers vary. Sample responses:

$2 \cdot 2 \cdot 2 \cdot 2$ doesn’t belong because it is the only expression that shows 4 repeated factors being multiplied.

16 doesn’t belong because it is the only one that is just a number.

2^4 doesn’t belong because it is the only expression that uses exponents.

$4 \cdot 2$ doesn’t belong because it is the only expression that is not equal to 16.

Activity Synthesis

Ask each group to share one reason why a particular expression does not belong. Record and display the responses for all to see. After each response, ask the class if they agree or disagree. Since there is no single correct answer to the question of which one does not belong, attend to students’ explanations and ensure the reasons given are correct.

During the discussion, ask students to explain the meaning of any terminology they use, such as “exponents.” Also, press students on unsubstantiated claims.

13.2 Is the Equation True?

15 minutes

The purpose of this task is to give students experience working with exponential expressions and to promote making use of structure (MP7) to compare exponential expressions. To this end, encourage students to rewrite expressions in a different form rather than evaluate them to a single number.

For students who are accustomed to viewing the equal sign as a directive that means "perform an operation," tasks like these are essential to shifting their understanding of the meaning of the equal sign to one that supports work in algebra, namely, "The expressions on either side have the same value."

Addressing

- 6.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Before students start working, it may be helpful to demonstrate how someone would figure out whether or not an equation is true without evaluating each expression. For example:

Is $4^2 = 2^3$ true? Well, let's see. We can rewrite each side like this:

$$4 \cdot 4 = 2 \cdot 2 \cdot 2$$

Then we can replace one of those $2 \cdot 2$'s with a 4, like this:

$$4 \cdot 4 = 4 \cdot 2$$

Now we can tell this equation is not true.

These problems can also be worked by directly evaluating expressions, which is fine, as it serves as practice evaluating exponential expressions.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin with the demonstration as described in the launch to support connections between new situations involving evaluating exponential expressions and prior understandings. Use color or annotations to highlight what changes and what stays the same at each step.
Supports accessibility for: Conceptual processing; Visual-spatial processing

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Student Task Statement

Decide whether each equation is true or false, and explain how you know.

1. $2^4 = 2 \cdot 4$
2. $3 + 3 + 3 + 3 + 3 = 3^5$
3. $5^3 = 5 \cdot 5 \cdot 5$
4. $2^3 = 3^2$
5. $16^1 = 8^2$
6. $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$
7. $\left(\frac{1}{2}\right)^4 = \frac{1}{8}$
8. $8^2 = 4^3$

Student Response

1. False, $2^4 = 2 \cdot 2 \cdot 2 \cdot 2$ and $2 \cdot 4 = 2 \cdot 2 \cdot 2$
2. False, $3 + 3 + 3 + 3 + 3 = 15$ and $3^5 = 243$
3. True, because $5 \cdot 5 \cdot 5$ is what 5^3 means
4. False, $2^3 = 8$ and $3^2 = 9$
5. False, $16^1 = 16$ and $8^2 = 64$
6. False, $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{16}$ and $4 \cdot \frac{1}{2} = 2$
7. False, $\frac{1}{8}$ is $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$
8. True, both sides of the equation equal 64

Activity Synthesis

Invite students who evaluated the expressions and students who used structure or the meaning of exponents and operations to present their work. Compare and connect the strategies by noting where the use of structure might prove more efficient, where evaluating might be simpler, where thinking about the meaning of exponents and operations can make the true or false determination simpler.

Some guiding questions to highlight the meaning of exponents:

- "Can we switch the order with exponents like we can with addition and multiplication—specifically, are a^b and b^a equivalent? How do you know?" (No, you can try different values of a and b or use the meaning of exponents to see that a multiplied b times is not always the same as b multiplied a times.)
- "What change can we make to the equation $3 + 3 + 3 + 3 + 3 = 3^5$ to make it true?" (Change addition to multiplication on the left, or change the exponent to multiplication on the right.)

- “Your friend claims the equation $\frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2} = 4 \cdot \frac{1}{2}$ is true. What do you think they are misunderstanding? How can you convince them it is false?” (Possibly by saying that the left side shows multiplying $\frac{1}{2}$ 4 times, but the right side shows multiplying 4 by $\frac{1}{2}$, which means *adding* 4 copies of $\frac{1}{2}$, not multiplying them. You can show similar examples with other numbers, but the best way to convince them is to talk about what exponents and multiplication mean.)
- “Can we show that $8^2 = 4^3$ is true without evaluating both sides? What understanding about the meaning of exponents and operations can help us?” (8^2 means $8 \cdot 8$ or $4 \cdot 2 \cdot 4 \cdot 2$, which also equals $4 \cdot 4 \cdot 2 \cdot 2$ or $4 \cdot 4 \cdot 4$. Another way to write $4 \cdot 4 \cdot 4$ is 4^3. We are using the understanding that we can multiply in any order, or the commutative and associative properties of multiplication.)

Access for English Language Learners

Speaking, Writing: MLR8 Discussion Supports. Revoice language and push for clarity in reasoning when students discuss their strategies for determining whether the equations are true or false. Provide a sentence frame such as “The equation is true (or false) because ______.” This will strengthen students’ mathematical language use and reasoning when discussing the meaning of exponents and operations that can make the equivalence of expressions true or false.
Design Principle(s): Maximize meta-awareness

13.3 What’s Your Reason?

15 minutes

In this activity, students search for numerical expressions that are equivalent. Students construct arguments and critique the reasoning of others (MP3) as they explain to their partner why they think two expressions are equivalent and respond to their partner’s arguments about equivalence.

Addressing

- 6.EE.A.1

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Partners work for 10 minutes, alternating every question which partner is explaining why a match is a match and listening to the explanation. If their partner disagrees, the partner explains why they don’t think the match is equivalent.

Encourage students to determine matches by looking for structure in the expressions and applying the meaning of exponents. It is not always necessary to evaluate the expressions in order to find equivalent expressions.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. To help get students started, display sentence frames such as "_____ and _____ are equivalent because . . . " and "I agree/disagree because . . ."
Supports accessibility for: Language; Organization

Student Task Statement

In each list, find expressions that are equivalent to each other and explain to your partner why they are equivalent. Your partner listens to your explanation. If you disagree, explain your reasoning until you agree. Switch roles for each list. (There may be more than two equivalent expressions in each list.)

1. a. $5 \cdot 5$

   b. 2^5

   c. 5^2

   d. $2 \cdot 5$

2. a. 4^3

   b. 3^4

   c. $4 \cdot 4 \cdot 4$

   d. $4 + 4 + 4$

3. a. $6 + 6 + 6$

   b. 6^3

   c. 3^6

   d. $3 \cdot 6$

4. a. 11^5

   b. $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$

   c. $11 \cdot 5$

   d. 5^{11}

5. a. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$

   b. $\left(\frac{1}{5}\right)^3$

   c. $\frac{1}{15}$

   d. $\frac{1}{125}$

6. a. $\left(\frac{5}{3}\right)^2$

   b. $\left(\frac{3}{5}\right)^2$

   c. $\frac{10}{6}$

   d. $\frac{25}{9}$

Student Response

1. $5 \cdot 5$ and 5^2
2. 4^3 and $4 \cdot 4 \cdot 4$
3. $6 + 6 + 6$ and $3 \cdot 6$
4. 11^5 and $11 \cdot 11 \cdot 11 \cdot 11 \cdot 11$
5. $\frac{1}{5} \cdot \frac{1}{5} \cdot \frac{1}{5}$ and $\left(\frac{1}{5}\right)^3$ and $\frac{1}{125}$
6. $\left(\frac{5}{3}\right)^2$ and $\frac{25}{9}$

Are You Ready for More?

What is the last digit of $3^{1,000}$? Show or explain your reasoning.

Student Response

The last digit is 1. Explanations vary. Sample response:

Some experimentation reveals a pattern—the first few powers of 3 are 3, 9, 27, 81, 243, 729, 2,187, 6,561, 19,683, etc. Specifically, the pattern of last digits goes 3, 9, 7, 1, 3, 9, 7, 1, 3, etc., repeating every four terms. So every exponent which is a multiple of 4, like $3^{1,000}$, evaluates to a number whose last digit is a 1.

Activity Synthesis

Invite students to describe their strategies for finding matches. As students respond, record the equivalent expressions using an equal sign.

Consider asking students:

- To share any expressions they had to work on to agree with their partner
- How they could find a match without evaluating every expression
- To describe some ways to recognize equivalence in expressions

Access for English Language Learners

Conversing: MLR8 Discussion Supports. To help students explain why two or more expressions in each list are equivalent to each other, provide sentence frames such as, "The expressions ________ and ________ are equivalent because ________." To help students explain why they agree or disagree with their partner's explanation, provide sentence frames such as, "I agree/disagree with your reasoning because ________." As students work on the task, listen for and amplify the language students use to explain the meaning of exponents. This routine will support rich and inclusive discussion about the meaning of exponents and the equivalence of numerical expressions.
Design Principle(s): Optimize output (for explanation); Cultivate conversation

Lesson Synthesis

Throughout the lesson, students saw many instances of typical errors and misconceptions when working with exponent notation. A possible activity for the end of the lesson is the creation of displays showing some of the errors that came up in the activity discussions. Students can work in groups to choose one error and create a visual display of why it is incorrect. For example, students might use a drawing similar to the dot picture in the warm-up of the last lesson to show the meaning of exponents while using an array to show the meaning of multiplication to illustrate that 3^5 is not the same number of dots as $3 \cdot 5$. Another display could show that the meaning of exponents is always the same, regardless of whether the number being repeatedly multiplied is a whole number, fraction, or decimal.

13.4 Coin Calculation

Cool Down: 5 minutes
Addressing

- 6.EE.A.1

Student Task Statement

Andre and Elena knew that after 28 days they would have 2^{28} coins, but they wanted to find out how many coins that actually is. Andre wrote:

$$2^{28} = 2 \cdot 28 = 56$$

Elena said, "No, exponents mean repeated multiplication. It should be $28 \cdot 28$, which works out to be 784." Do you agree with either of them? Explain your reasoning.

Student Response

I disagree with both Andre and Elena. Andre thinks exponents are just a different way of writing multiplication of two numbers. Elena calculates 28^2 rather than 2^{28}.

Student Lesson Summary

When working with exponents, the bases don't have to always be whole numbers. They can also be other kinds of numbers, like fractions, decimals, and even variables. For example, we can use exponents in each of the following ways:

$$\left(\frac{2}{3}\right)^4 = \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3}$$

$$(1.7)^3 = (1.7) \cdot (1.7) \cdot (1.7)$$

$$x^5 = x \cdot x \cdot x \cdot x \cdot x$$

Lesson 13 Practice Problems

Problem 1

Statement

Select **all** expressions that are equal to $3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$.

A. $3 \cdot 5$

B. 3^5

C. $3^4 \cdot 3$

D. $5 \cdot 3$

E. 5^3

Solution

["B", "C"]

Problem 2

Statement

Noah starts with 0 and then adds the number 5 four times. Diego starts with 1 and then multiplies by the number 5 four times. For each expression, decide whether it is equal to Noah's result, Diego's result, or neither.

a. $4 \cdot 5$

b. $4 + 5$

c. 4^5

d. 5^4

Solution

a. Noah's

b. Neither

c. Neither

d. Diego's

Problem 3

Statement

Decide whether each equation is true or false, and explain how you know.

a. $9 \cdot 9 \cdot 3 = 3^5$

b. $7 + 7 + 7 = 3 + 3 + 3 + 3 + 3 + 3 + 3$

c. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{3}{7}$

d. $4^1 = 4 \cdot 1$

e. $6 + 6 + 6 = 6^3$

Solution

a. True. Explanations vary. Sample explanation: The expression on the left is equivalent to $(3 \cdot 3) \cdot (3 \cdot 3) \cdot 3 = 3^5$.

b. True. Explanations vary. Sample explanation: Both sides of the equation are ways of writing $3 \cdot 7$.

c. False. $\frac{1}{7} \cdot \frac{1}{7} \cdot \frac{1}{7} = \frac{1}{7^3}$ or $\frac{1}{343}$ which does not equal $\frac{3}{7}$.

d. True. Both sides equal 4.

e. False. $6^3 = 216$, but $6 + 6 + 6 = 18$.

Problem 4

Statement

a. What is the area of a square with side lengths of $\frac{3}{5}$ units?

b. What is the side length of a square with area $\frac{1}{16}$ square units?

c. What is the volume of a cube with edge lengths of $\frac{2}{3}$ units?

d. What is the edge length of a cube with volume $\frac{27}{64}$ cubic units?

Solution

a. $\frac{9}{25}$ square units ($\frac{3}{5} \cdot \frac{3}{5} = \frac{9}{25}$)

b. $\frac{1}{4}$ units ($\frac{1}{4} \cdot \frac{1}{4} = \frac{1}{16}$)

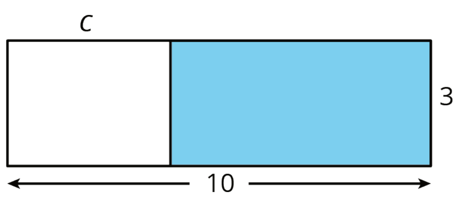
c. $\frac{8}{27}$ cubic units ($\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{8}{27}$)

d. $\frac{3}{4}$ units ($\frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64}$)

Problem 5

Statement

Select **all** the expressions that represent the area of the shaded rectangle.



A. $3(10 - c)$

B. $3(c - 10)$

C. $10(c - 3)$

D. $10(3 - c)$

E. $30 - 3c$

F. $30 - 10c$

Solution

["A", "E"]

(From Unit 6, Lesson 10.)

Problem 6

Statement

A ticket at a movie theater costs \$8.50. One night, the theater had \$29,886 in ticket sales.

a. Estimate about how many tickets the theater sold. Explain your reasoning.

b. How many tickets did the theater sell? Explain your reasoning.

Solution

a. About 3,000. Reasoning varies. Sample reasoning: If there were \$30,000 in sales and the tickets were \$10 each, then it would be 3000. The actual tickets are less than \$10 (while \$30,000 is very close to the total sales), so the actual answer should be more than 3000.

b. 3,516. Reasoning varies. Sample reasoning: The number of tickets sold is $29{,}886 \div 8.5$, and this is 3,516.

(From Unit 5, Lesson 13.)

Problem 7

Statement

A fence is being built around a rectangular garden that is $8\frac{1}{2}$ feet by $6\frac{1}{3}$ feet. Fencing comes in panels. Each panel is $\frac{2}{3}$ of a foot wide. How many panels are needed? Explain or show your reasoning.

Solution

Answers vary. Possible solution (not reusing panel pieces): 46 panels. For the sides of length $8\frac{1}{2}$ feet, Jada needs $8\frac{1}{2} \div \frac{2}{3}$ panels. This is $\frac{51}{4} = 12\frac{3}{4}$ so these will use 13 panels of fencing. The other two sides each use $6\frac{1}{3} \div \frac{2}{3}$ panels of fencing, which is $9\frac{1}{2}$. This is 10 panels each. Possible solution (reusing panel pieces): 45 panels. The sides of length $8\frac{1}{2}$ feet each use $12\frac{3}{4}$ panels of fencing, for a total of $25\frac{1}{2}$. The other two sides each use $9\frac{1}{2}$ pieces of fencing for a total of 19 panels. Jada needs $44\frac{1}{2}$ panels, which means she needs 45 whole panels.

(From Unit 4, Lesson 12.)