## Lesson 18: Graphs of Rational Functions (Part 2)

* Let’s learn about horizontal asymptotes.

### 18.1: Rewritten Equations

Decide if each of these equations is true or false for  values that do not result in a denominator of 0. Be prepared to explain your reasoning.

### 18.2: Publishing a Paperback

Let be the function that gives the average cost per book , in dollars, when using an online store to print copies of a self-published paperback book. Here is a graph of

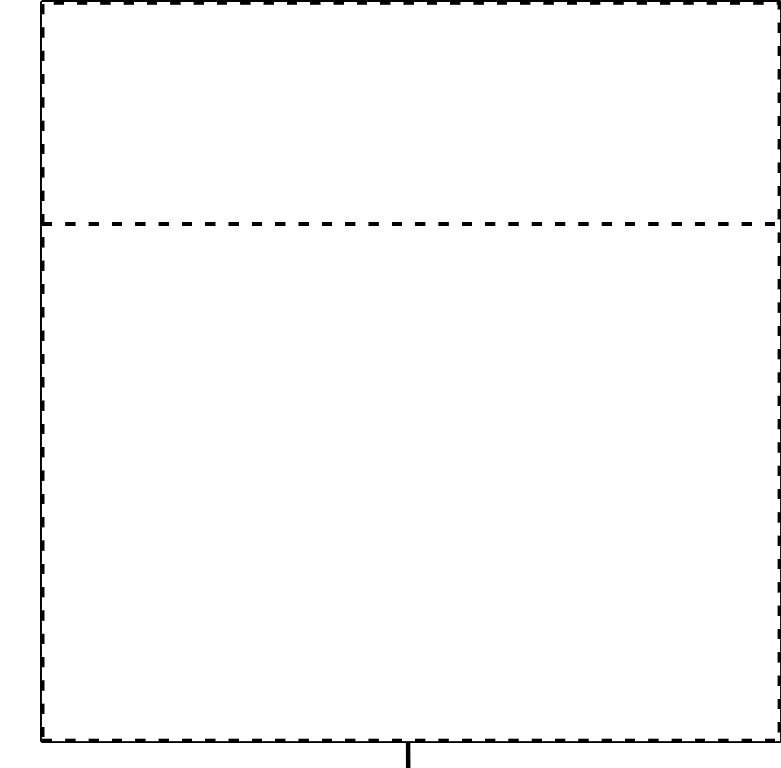


1. What is the approximate cost per book when 50 books are printed? 100 books?
2. The author plans to charge $8 per book. About how many should be printed to make a profit?
3. What is the value of when ? How does this relate to the context?
4. What does the end behavior of the function say about the context?

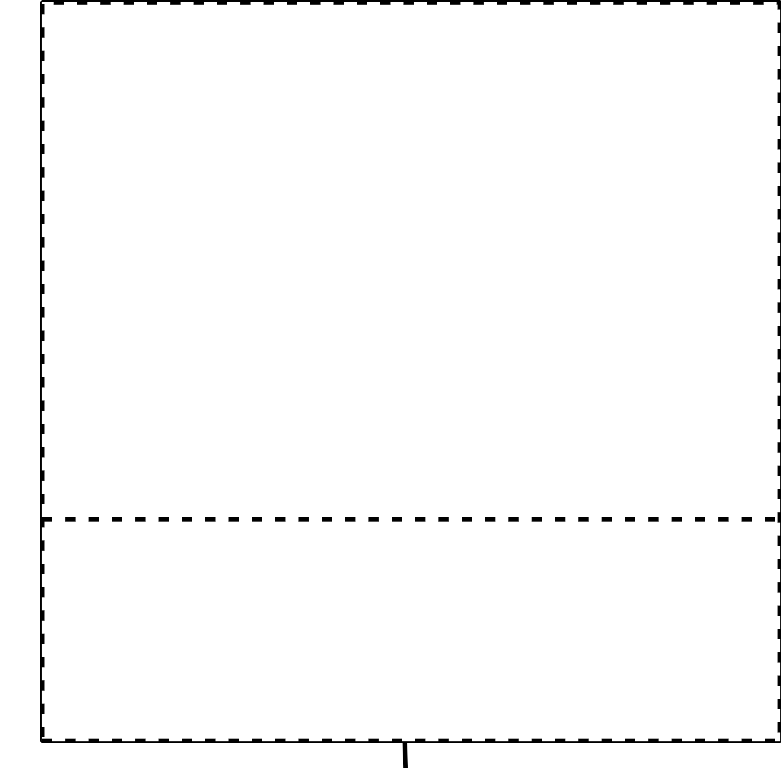
### 18.3: Horizontal Asymptotes

Here are four graphs of rational functions.

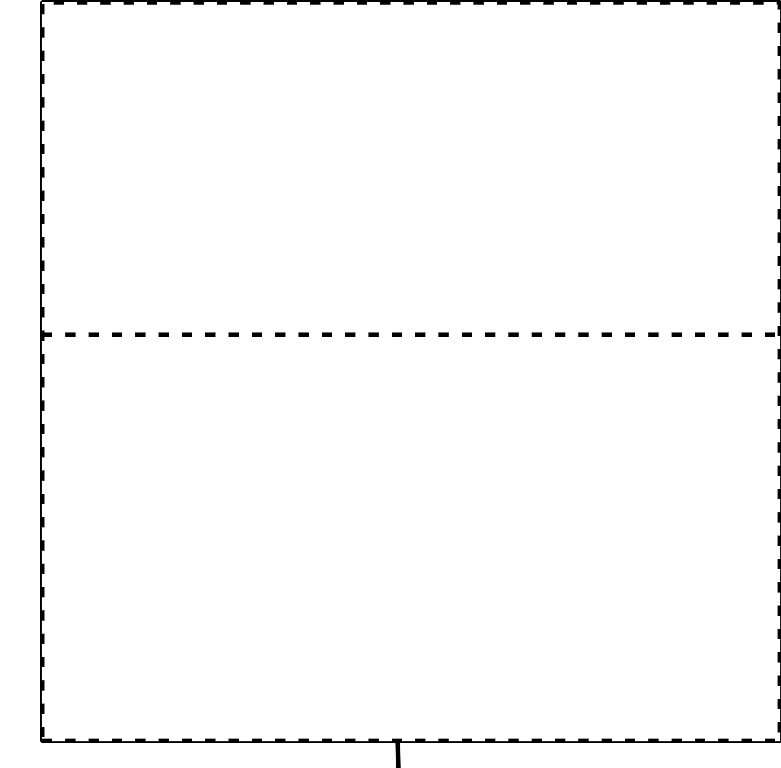
A



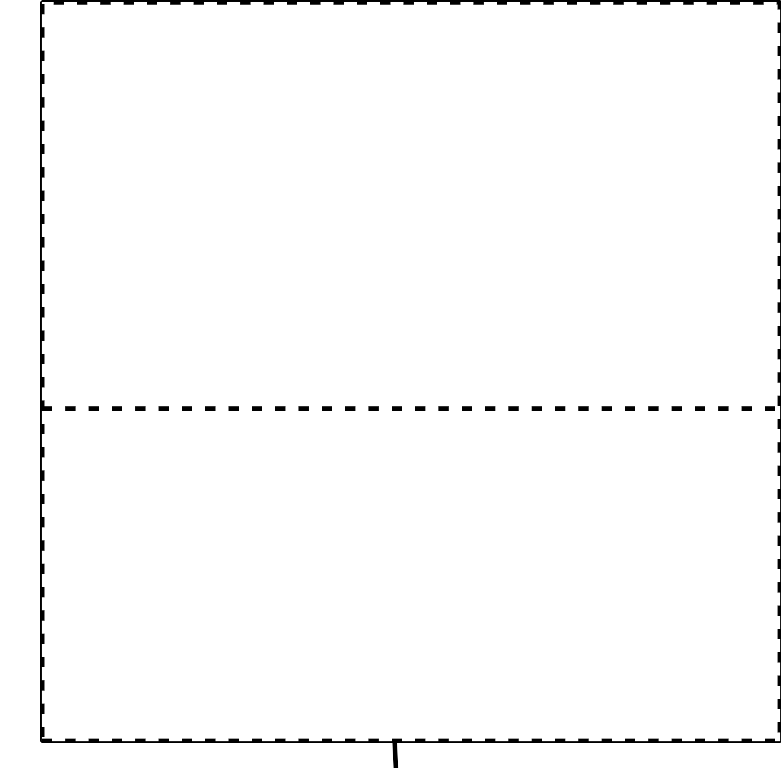
B



C



D



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1. Match each function with its graphical representation.
2. Where do you see the **horizontal asymptote** of the graph in the expressions for the functions?

#### Are you ready for more?

Consider the function .

1. Predict where you think the vertical and horizontal asymptotes of will be. Explain your reasoning.
2. Use graphing technology to check your prediction.

### Lesson 18 Summary

Consider the rational function . Written this way, we can tell that the graph of the function has a vertical asymptote at by reading the denominator and identifying the value that would cause division by zero. But what can we tell about the value of for values of far away from the vertical asymptote?

One way we can think about these values is to rewrite the expression for by breaking up the fraction:

Written this way, it’s easier to see that as gets larger and larger in either the positive or negative direction, the term will get closer and closer to 0. Because of this, we can say that the value of the function will get closer and closer to 3.

More generally, if a rational function can be rewritten as , where is a constant, and and are polynomial expressions where gets closer and closer to zero as gets larger and larger in both the positive and negative directions, then will get closer and closer to .

Rational functions of this type have a **horizontal asymptote** at the constant value. The line   is a horizontal asymptote for  if gets closer and closer to as the magnitude of increases.



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