Lesson 11: The Distributive Property, Part 3

Goals

- Draw a diagram to justify that two expressions that are related by the distributive property are equivalent.
- Explain (orally) how to use the distributive property to identify or generate equivalent algebraic expressions.
- Use the distributive property to write equivalent algebraic expressions, including where the common factor is a variable.

Learning Targets

- I can use the distributive property to write equivalent expressions with variables.

Lesson Narrative

This is an optional lesson to practice identifying and writing equivalent expressions using the distributive property. If your students don't need additional practice at this point, this lesson can be skipped (or saved for a review days later) without missing any new material.

Alignments

Addressing

- 6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.
- 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2+x)$ to produce the equivalent expression $6+3x$; apply the distributive property to the expression $24x+18y$ to produce the equivalent expression $6(4x+3y)$; apply properties of operations to $y+y+y$ to produce the equivalent expression $3y$.
- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y+y+y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports

Student Learning Goals

Let's practice writing equivalent expressions by using the distributive property.

11.1 The Shaded Region

Warm Up: 5 minutes

Students reflect on equivalent expressions that represent the area of a shaded rectangle which is part of a larger rectangle of unknown width.

Addressing

- 6.EE.A.2
- 6.EE.A.3

Launch

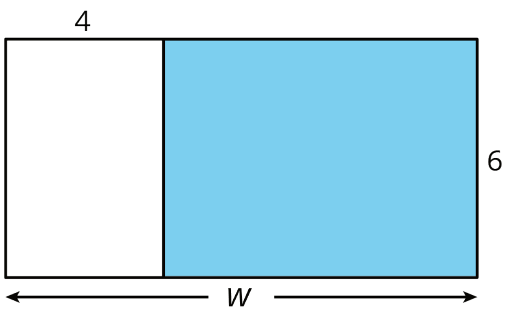
Allow students 2 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

A rectangle with dimensions 6 cm and w cm is partitioned into two smaller rectangles.

Explain why each of these expressions represents the area, in cm^2, of the shaded region.

- $6w - 24$
- $6(w - 4)$



Student Response

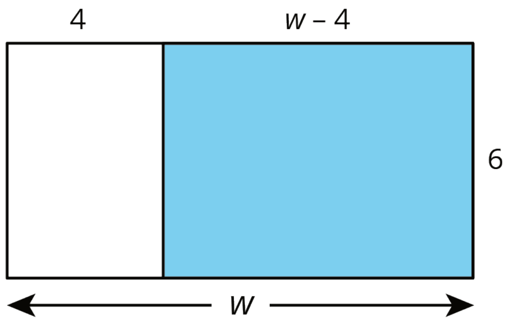
Answers vary. Sample responses:

- $6w - 24$: The area, in cm^2, of the entire rectangle is $6w$. The area of the unshaded rectangle is $6 \cdot 4$ or 24 cm^2. Subtracting the two, $6w - 24$, gives the area of the shaded rectangle.
- $6(w - 4)$: The length of the shaded rectangle is $w - 4$. Its width is 6 cm, so its area, in cm^2, is $6(w - 4)$.

Activity Synthesis

Ask students to share their reasoning. $6w - 24$ should be straighforward: the area of the entire rectangle is $6w$, and the area of the unshaded portion is $6 \cdot 4$ or 24, so the area of the shaded portion is $6w - 24$.

Students may have more trouble with $6(w - 4)$. The key is to understand that the longer side of the shaded portion can be represented by $w - 4$.



11.2 Matching to Practice Distributive Property

Optional: 15 minutes
Students practice finding expressions that are equivalent because of the distributive property.

Addressing

- 6.EE.A.4

Instructional Routines

- MLR7: Compare and Connect

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. Activate or supply background knowledge. Remind students that they can use rectangle diagrams to help them match expressions.
Supports accessibility for: Social-emotional skills; Conceptual processing

Student Task Statement

Match each expression in column 1 to an equivalent expression in column 2. If you get stuck, consider drawing a diagram.

Column 1

1. $a(1 + 2 + 3)$
2. $2(12 - 4)$
3. $12a + 3b$
4. $\frac{2}{3}(15a - 18)$
5. $6a + 10b$
6. $0.4(5 - 2.5a)$
7. $2a + 3a$

Column 2

- $3(4a + b)$
- $12 \cdot 2 - 4 \cdot 2$
- $2(3a + 5b)$
- $(2 + 3)a$
- $a + 2a + 3a$
- $10a - 12$
- $2 - a$

Student Response

1. $a(1 + 2 + 3)$ and $a + 2a + 3a$
2. $2(12 - 4)$ and $12 \cdot 2 - 4 \cdot 2$
3. $12a + 3b$ and $3(4a + b)$
4. $\frac{2}{3}(15a - 18)$ and $10a - 12$
5. $6a + 10b$ and $2(3a + 5b)$
6. $0.4(5 - 2.5a)$ and $2 - a$
7. $2a + 3a$ and $(2 + 3)a$

Activity Synthesis

Invite students to share whether any of the matches were difficult to find and how they worked through the challenge.

Access for English Language Learners

Speaking: MLR7 Compare and Connect. Ask students to consider what is the same and what is different between the representations of expressions in each column. Highlight and demonstrate mathematical language used (e.g., distributive property, distribute, product, sum, difference, coefficient) to make connections among the matched expressions. These exchanges strengthen students' mathematical language use and reasoning about equivalent expressions in general and the distributive property specifically.
Design Principle(s): Maximize meta-awareness

11.3 Writing Equivalent Expressions Using the

Distributive Property

Optional: 15 minutes
Students practice working back and forth writing equivalent expressions with the distributive property.

Addressing

- 6.EE.A.3

Instructional Routines

- MLR8: Discussion Supports

Launch

Allow students 10 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

The distributive property can be used to write equivalent expressions. In each row, use the distributive property to write an equivalent expression. If you get stuck, consider drawing a diagram.

product	sum or difference
$3(3+x)$	
	$4x-20$
$(9-5)x$	
	$4x+7x$
$3(2x+1)$	
	$10x-5$
	$x+2x+3x$
$\frac{1}{2}(x-6)$	
$y(3x+4z)$	
	$2xyz-3yz+4xz$

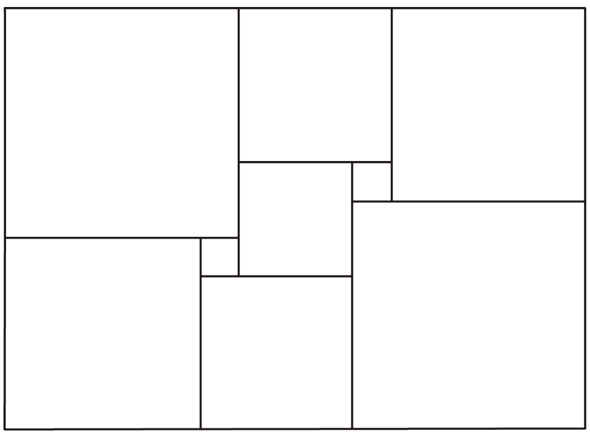
Student Response

product	sum or difference
$3(3+x)$	$9+3x$
$4(x-5)$	$4x-20$
$(9-5)x$	$9x-5x$
$(4+7)x$	$4x+7x$
$3(2x+1)$	$6x+3$
$5(2x-1)$	$10x-5$
$x(1+2+3)$	$x+2x+3x$
$\frac{1}{2}(x-6)$	$\frac{1}{2}x-3$
$y(3x+4z)$	$3xy+4zy$
$z(2xy-3y+4x)$	$2xyz-3yz+4xz$

Note that in cases where factoring happens, expressions equivalent to these are also acceptable. For example, for $4x-20$, equivalent expressions are $2(2x-10)$ and $20(\frac{1}{5}x-1)$ in addition to $4(x-5)$.

Are You Ready for More?

This rectangle has been cut up into squares of varying sizes. Both small squares have side length 1 unit. The square in the middle has side length x units.



1. Suppose that x is 3. Find the area of each square in the diagram. Then find the area of the large rectangle.

2. Find the side lengths of the large rectangle assuming that x is 3. Find the area of the large rectangle by multiplying the length times the width. Check that this is the same area you found before.

3. Now suppose that we do not know the value of x. Write an expression for the side lengths of the large rectangle that involves x.

Student Response

1. Answers are given in a sequence in which they can be derived:
   Small squares: 1 square unit each
   Center square: 9 square units
   Top center: 16 square units
   Top right: 25 square units
   Bottom right: 36 square units
   Bottom center: 16 square units
   Bottom left: 25 square units
   Top left: 36 square units
   The area of the large rectangle is the sum of the these numbers: 165 square units.

2. 11 units by 15 units. $11 \cdot 15 = 165$.

3. Answers vary, depending on how much they rewrite their expressions or on whether they combine like terms. Sample response: the length is $2x + (2x - 1)$, or $4x - 1$ units, and the width is $2x + (1 + x) + (2 + x)$, or $4x + 3$ units.

Activity Synthesis

Invite students to explain how they knew, when working backwards, what to put in front of the parentheses and what remained inside.

Access for English Language Learners

Listening, Representing: MLR8 Discussion Supports. To develop students' meta-awareness, think-aloud as you write an equivalent expression using the distributive property. As you talk, demonstrate mathematical language about the relationship between expression on the same row of the table. For the second row, ask, "What number do I need that is a factor of both $4x$ and 20 (e.g., it's 4)?. Because $4x$ equals 4 times x and since 20 equals 4 times 5, we can write $4x - 20$ as a product $4(x - 5)$."
Design Principle(s): Maximize meta-awareness

Lesson Synthesis

You might want to have students work on a creative visual display for the classroom that shows their understanding of the distributive property in both directions.

11.4 Writing Equivalent Expressions

Cool Down: 5 minutes

Addressing

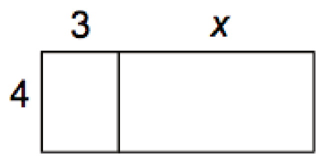
- 6.EE.A.3
- 6.EE.A.4

Student Task Statement

1. Use the distributive property to write an expression that is equivalent to $12 + 4x$.
2. Draw a diagram that shows the two expressions are equivalent.

Student Response

1. Answers vary. Sample response: $4(3 + x)$
2. Answers vary. Sample response:



Student Lesson Summary

The distributive property can be used to write a sum as a product, or write a product as a sum. You can always draw a partitioned rectangle to help reason about it, but with enough practice, you should be able to apply the distributive property without making a drawing.

Here are some examples of expressions that are equivalent due to the distributive property.

$$9 + 18 = 9(1 + 2)$$
$$2(3x + 4) = 6x + 8$$
$$2n + 3n + n = n(2 + 3 + 1)$$
$$11b - 99a = 11(b - 9a)$$
$$k(c + d - e) = kc + kd - ke$$

Lesson 11 Practice Problems

Problem 1

Statement

For each expression, use the distributive property to write an equivalent expression.

a. $4(x + 2)$

b. $(6 + 8) \cdot x$

c. $4(2x+3)$

d. $6(x+y+z)$

Solution

a. $4x+4\cdot 2$

b. $6x+8x$

c. $8x+4\cdot 3$

d. $6x+6y+6z$

Expressions that are equivalent to these are also acceptable, for example, $4x+8$ for the first one.

Problem 2

Statement

Priya rewrites the expression $8y-24$ as $8(y-3)$. Han rewrites $8y-24$ as $2(4y-12)$. Are Priya's and Han's expressions each equivalent to $8y-24$? Explain your reasoning.

Solution

Yes, the distributive property shows that each expression is equivalent to $8y-24$.

Problem 3

Statement

Select **all** the expressions that are equivalent to $16x+36$.

A. $16(x+20)$

B. $x(16+36)$

C. $4(4x+9)$

D. $2(8x+18)$

E. $2(8x+36)$

Solution

["C", "D"]

Problem 4

Statement

The area of a rectangle is $30 + 12x$. List at least 3 possibilities for the length and width of the rectangle.

Solution

Answers vary. Sample responses:

length	width
$10 + 4x$	3
$5 + 2x$	6
$15 + 6x$	2
$60 + 24x$	$\frac{1}{2}$
$3 + 1.2x$	10

Problem 5

Statement

Select **all** the expressions that are equivalent to $\frac{1}{2}z$.

A. $z + z$

B. $z \div 2$

C. $z \cdot z$

D. $\frac{1}{4}z + \frac{1}{4}z$

E. $2z$

Solution

["B", "D"]

(From Unit 6, Lesson 8.)

Problem 6

Statement

a. What is the perimeter of a square with side length:

3 cm? 7 cm? s cm?

b. If the perimeter of a square is 360 cm, what is its side length?

c. What is the area of a square with side length:

3 cm? 7 cm? s cm?

d. If the area of a square is 121 cm^2, what is its side length?

Solution

a. 12 cm $(3 \cdot 4 = 12)$, 28 cm$(7 \cdot 4 = 28)$, $4s$ cm

b. 90 cm $(360 \div 4 = 90)$

c. 9 cm^2 $(3 \cdot 3 = 9)$, 49 $cm^2$$(7 \cdot 7 = 49)$, s^2 cm^2

d. 11 cm $(11 \cdot 11 = 121)$

(From Unit 6, Lesson 6.)

Problem 7

Statement

Solve each equation.

$10 = 4a$ $5b = 17.5$ $1.036 = 10c$

$0.6d = 1.8$ $15 = 0.1e$

Solution

a. $a = 2.5$

b. $b = 3.5$

c. $c = 0.1036$

d. $d = 3$

e. $e = 150$

(From Unit 6, Lesson 5.)