# Lesson 17: Building Prisms

## Goals

- Compare and contrast (orally) triangular prisms, including comparisons of their height, cross sections, surface area, and volume.
- Compose two triangular prisms into a new prism, and describe (orally and in writing) the composite shape.
- Draw and assemble a net of a triangular prism, given two side lengths of the prism's base and one angle measure.

## Learning Targets

- I can build a triangular prism from scratch.

## Lesson Narrative

This lesson is optional. In this culminating lesson, students use what they have learned in this unit to build a triangular prism, given some measures for the angles and sides of the triangular base. There are 4 possible solutions.

This lesson is organized into three activities. First, students draw triangles that could be the base of the prism, given the conditions. They select one of the 4 possible solutions and calculate its area. Then, students create and assemble a net for the prism. They calculate its volume and surface area. In the last activity, students experiment with different ways two prisms could be put together to make one larger prism. They analyze how different configurations affect the volume and surface area of the composed prism.

### Alignments

#### Building On

- 6.G.A.4: Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

#### Addressing

- 7.G.A.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- 7.G.B.6: Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

### Instructional Routines

- MLR1: Stronger and Clearer Each Time
- MLR7: Compare and Connect
- MLR8: Discussion Supports

### Required Materials

**Compasses**
**Copies of blackline master**
**Geometry toolkits**
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

**Rulers marked with centimeters**

### Required Preparation

Print a copy of the blackline master for each student. Rulers should be part of the geometry toolkit, but make sure that rulers provided have markings in centimeters.

#### Student Learning Goals

Let's build a triangular prism from scratch.

# 17.1 Nets

**Warm Up: 5 minutes**
The purpose of this warm-up is for students to reason about prisms formed from various nets. During the partner and whole-group discussions, listen for how students name each prism: pentagonal prism, triangular prism, square prism (but not a cube), and select students who correctly name each prism to share during the whole-group discussion.

### Building On

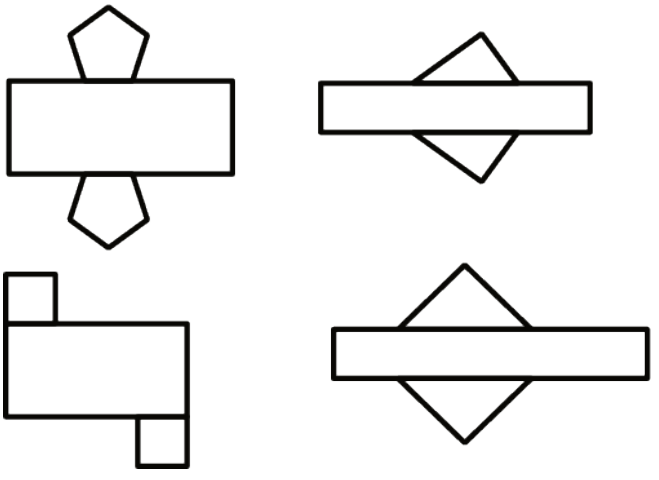
- 6.G.A.4

### Launch

Arrange students in groups of 2. Give students 30 seconds of quiet think time to look at the nets followed by 2 minutes to describe each net with a partner.

#### Student Task Statement

Here are some nets for various prisms.



1. What would each net look like when folded?
2. What do you notice about the nets?

### Student Response

1. From top left: A pentagonal prism, a triangular prism where the base is a right triangle, a square prism (but not a cube), a triangular prism where the base is an isosceles triangle.
2. Answers vary. Sample responses:
    - They all have a long rectangle in the middle.
    - The bases on the bottom are upside down compared to the bases on the top.
    - There's one base on each side of the rectangle.

### Activity Synthesis

Ask selected students to describe the object formed by each net. Record and display their responses for all to see. If a student's description does not include the name of the prism, ask other students to name the object and explain how they know.

Ask students to share what they notice about all of the nets. Record and display their responses for all to see. While students may notice many things, important ideas to highlight during the discussion are:

- They all have a long rectangle in the middle.
- The bases on top and bottom are upside down.
- There's one base on each side of the rectangle.

## 17.2 Making the Base

**Optional: 10 minutes**

This activity reviews the work students did previously drawing shapes with given conditions. Students draw as many different triangles as they can that could be the base of the triangular prism, given two side lengths and one angle measure for the triangle.

In preparation for calculating surface area and volume in the next activity, students select one of their triangles and find its area. This will require them to draw and measure the height of the triangle. As needed, remind students that the height must be perpendicular to whichever side they are using as the base of their triangle. Also, prompt students to measure the height as precisely as possible, because it will influence the accuracy of their later calculations.

### Addressing

- 7.G.A.2

### Instructional Routines

- MLR7: Compare and Connect

### Launch

Provide access to geometry toolkits and compasses.

### Anticipated Misconceptions

Students may try to multiply two side lengths of the triangle to calculate the area. Remind them that the height must be perpendicular to the base. If necessary, demonstrate using an index card to draw in the height of the triangle.

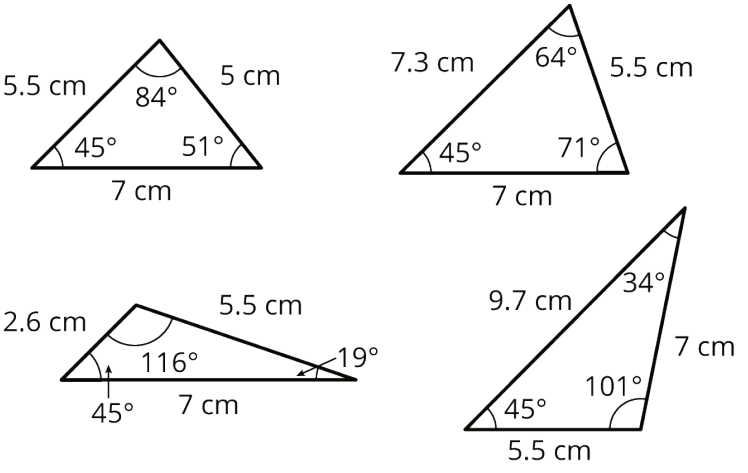
### Student Task Statement

The base of a triangular prism has one side that is 7 cm long, one side that is 5.5 cm long, and one angle that measures $45^\circ$.

1. Draw as many different triangles as you can with these given measurements.
2. Select one of the triangles you have drawn. Measure and calculate to approximate its area. Explain or show your reasoning.

### Student Response

1. There are four possible triangles.




2. Answers vary depending on the selected triangle:
    - The area is approximately 13.6 $cm^2$ for the triangle with the third side of 5.0 cm.
    - The area is approximately 18.2 $cm^2$ for the triangle with the third side of 7.3 cm.
    - The area is approximately 6.3 $cm^2$ for the triangle with the third side of 2.6 cm.
    - The area is approximately 18.9 $cm^2$ for the triangle with the third side of 9.7 cm.

### Activity Synthesis

Ask students to share triangles they drew so that everyone has an opportunity to see all four triangles. If any of the four triangles are not presented by students, demonstrate how to construct it. Ensure the class agrees that 4 unique triangles have the given measurements. The third side length of the triangle could be 5.0 cm, 7.3 cm, 2.6 cm, or 9.7 cm.

Make sure students have calculated the area of their selected triangle correctly, because this will affect their volume and surface area calculations in the next activity.

| If the third side of the triangle is | then the area of the triangle should be about | possible strategies |
|---|---|---|
| 5.0 cm | 13.7 $cm^2$ | $\frac{1}{2} \cdot 7 \cdot (3.9)$ or $\frac{1}{2} \cdot (5.5) \cdot (5.0)$ |
| 7.3 cm | 18.2 $cm^2$ | $\frac{1}{2} \cdot 7 \cdot (5.2)$ or $\frac{1}{2} \cdot (5.5) \cdot (6.6)$ or $\frac{1}{2} \cdot (7.3) \cdot (5.0)$ |
| 2.6 cm | 6.3 $cm^2$ | $\frac{1}{2} \cdot 7 \cdot (1.8)$ or $\frac{1}{2} \cdot (5.5) \cdot (2.3)$ or $\frac{1}{2} \cdot (2.6) \cdot (5.0)$ |
| 9.7 | 18.9 $cm^2$ | $\frac{1}{2} \cdot 7 \cdot (5.4)$ or $\frac{1}{2} \cdot (5.5) \cdot (6.9)$ or $\frac{1}{2} \cdot (9.7) \cdot (3.9)$ |

### Access for English Language Learners

*Representing, Conversing: MLR7 Compare and Connect.* Invite students to prepare a visual display that shows how they measured and calculated the approximate area for their chosen triangle. Students should consider how to display their calculations so that another student can interpret them. For example, some students may wish to add notes or details to their drawings to help communicate their thinking. Arrange students in groups of 2–4. Give 2–3 minutes of quiet think time for students to read and interpret each other's calculations before they begin to discuss them. Display a list of questions that students can ask each other about their work. For example, "Can you show me how you measured the side lengths?", “How did you calculate the area of the triangle?”, etc. During the whole-class discussion, draw students’ attention to the relationship between measuring and approximating area. Emphasize the language used to make sense of the strategies used to measure and approximate area. These exchanges strengthen students’ mathematical language use and reasoning based on the relationship between measures and area.
*Design Principle(s): Maximize meta-awareness*

# 17.3 Making the Prism

**Optional: 10 minutes**

In this activity, students take the triangle they selected in the previous activity and use it as the base of their triangular prism. After students have drawn their net and before they cut it out and assemble it, make sure they have correctly positioned their bases, opposite from each other on the top and bottom of the rectangle and reflected. It will also make assembling the net easier for students if they draw lines subdividing the large rectangle into the individual rectangular faces and draw tabs where the faces will be glued or taped together.

### Addressing

- 7.G.B.6

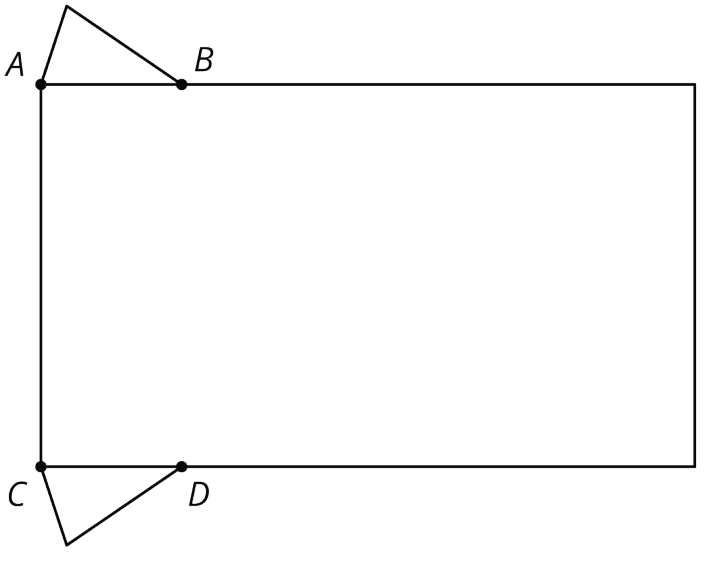
### Instructional Routines

- MLR1: Stronger and Clearer Each Time

### Launch

Demonstrate the positions triangles should attach to the rectangle to form a net by displaying an example and describing the important parts:

- The triangles must have a vertex at $A$ and $C$.
- The triangles must be identical copies with one “upside down” from the other.
- Corresponding sides of each triangle must be along the side of the rectangle.




#### Access for Students with Disabilities

*Engagement: Internalize Self Regulation.* Check for understanding by inviting students to rephrase directions in their own words. Provide a project checklist that chunks the various steps of the activity into a set of manageable tasks.
*Supports accessibility for: Organization; Attention*

### Student Task Statement

Your teacher will give you an incomplete net. Follow these instructions to complete the net and assemble the triangular prism:

1. Draw an identical copy of the triangle you selected in the previous activity along the top of the rectangle, with one vertex on point $A$.
2. Draw another copy of your triangle, flipped upside down, along the bottom of the rectangle, with one vertex on point $C$.
3. Determine how long the rectangle needs to be to wrap all the way around your triangular bases. Pause here so your teacher can review your work.
4. Cut out and assemble your net.

After you finish assembling your triangular prism, answer these questions. Explain or show your reasoning.

1. What is the volume of your prism?
2. What is the surface area of your prism?
3. Stand your prism up so it is sitting on its triangular base.
    a. If you were to cut your prism in half horizontally, what shape would the cross section be?
    b. If you were to cut your prism in half vertically, what shape would the cross section be?

### Student Response

| If the third side of the triangle is | then the volume of the solid should be about | and the surface area of the solid should be about |
|---|---|---|
| 5.0 cm | 137 $cm^3$ | 202.2 $cm^2$ |
| 7.3 cm | 182 $cm^3$ | 234.4 $cm^2$ |
| 2.6 cm | 63 $cm^3$ | 163.6 $cm^2$ |
| 9.7 cm | 189 $cm^3$ | 259.8 $cm^2$ |

Cross sections:

- A triangle that is identical to the one I drew as my base.
- A rectangle with one side the same length as the height of the prism.

### Activity Synthesis

Select students to share their answers for the cross sections. For cross sections taken in these two ways, all triangular prisms should have the same shapes as answers although the actual size of the cross section will differ based on the size of the base triangle and the height of the prism.

The volume and surface areas will depend on the triangle they have chosen to use as their base.

Select students to share their methods for computing volume and surface area. The base area is important in the calculation of each, so students should use the values they computed in the previous activity.

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**Access for English Language Learners**

*Writing, Speaking, Listening: MLR1 Stronger and Clearer Each Time.* Use this routine to provide students a structured opportunity to refine their methods for calculating the volume of their prisms. Give students time to meet with 2–3 partners, to share and get feedback on their responses. Provide students with prompts for feedback that will help them strengthen their ideas and clarify their language. For example, "What did you do first?", "What was your base?", "Can you draw a picture?" Give students 1–2 minutes to revise their writing based on the feedback they received.
*Design Principle(s): Optimize output (for explanation)*

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# 17.4 Combining Prisms

**Optional: 10 minutes**
Students combine their solid with a partner's and examine the new solid's properties.

If there is time, this activity can be extended to review relationships between angles as well.

- Take two of the prisms and put them together so that their $45^\circ$ angles are adjacent. Ask students what type of angle is created and what the relationship between the two angles creating that angle must be. (a right angle, complementary angles)
- Take three identical prisms and put them together so that a different angle from each prism is adjacent.

Ask students what type of angle is created. (a straight angle) Ask students to identify pairs of angles that are supplementary.

### Addressing

- 7.G.B.6

### Instructional Routines

- MLR8: Discussion Supports

### Launch

Arrange students in groups of 2.

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**Access for Students with Disabilities**

*Engagement: Develop Effort and Persistence.* Provide prompts, reminders, rubrics, or checklists that focus on increasing the length of on-task orientation in the face of distractions. For example, provide students with the printed Student Task Statement to use as a checklist for task completion.
*Supports accessibility for: Attention; Social-emotional skills*

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### Anticipated Misconceptions

Student might struggle trying to measure and calculate the area of the base of their new prism if it is an irregular quadrilateral. Prompt them that they can just add the areas of the bases of their individual prisms together.

### Student Task Statement

1. Compare your prism with your partner's prism. What is the same? What is different?
2. Find a way you can put your prism and your partner's prism together to make one new, larger prism. Describe your new prism.
3. Draw the base of your new prism and label the lengths of the sides.
4. As you answer these questions about your new prism, look for ways you can use your calculations from the previous activity to help you. Explain or show your reasoning.
    a. What is the area of its base?
    b. What is its height?
    c. What is its volume?
    d. What is its surface area?

### Student Response

1. Both prisms have the same height. At least two side lengths of the base are the same. It is possible that both prisms are identical.

2. Answers vary based on which triangles and faces are used. Sample responses: A taller triangular prism, a prism with a parallelogram base, a wider triangular prism.
3. Answers for the base vary based on which triangles and faces are used. Sample responses: A triangle, a parallelogram.
4. 
    a. If the students glue two sides together, the area of the base should be the sum of the two individual triangle base areas. If the students glue identical bases together, the area of the base is the same as the original area of one of the bases.
    b. Possible answers: 10 cm, 20 cm. The height of the prism will depend on the faces used to glue the parts together.
    c. The volume of the combined object is the sum of the volume of the two individual triangular prisms.
    d. The surface area of the combined object is the sum of the two individual objects minus twice the area of the shared face.

### Are You Ready for More?

How many identical copies of your prism would it take you to put together a new larger prism in which every dimension was twice as long?

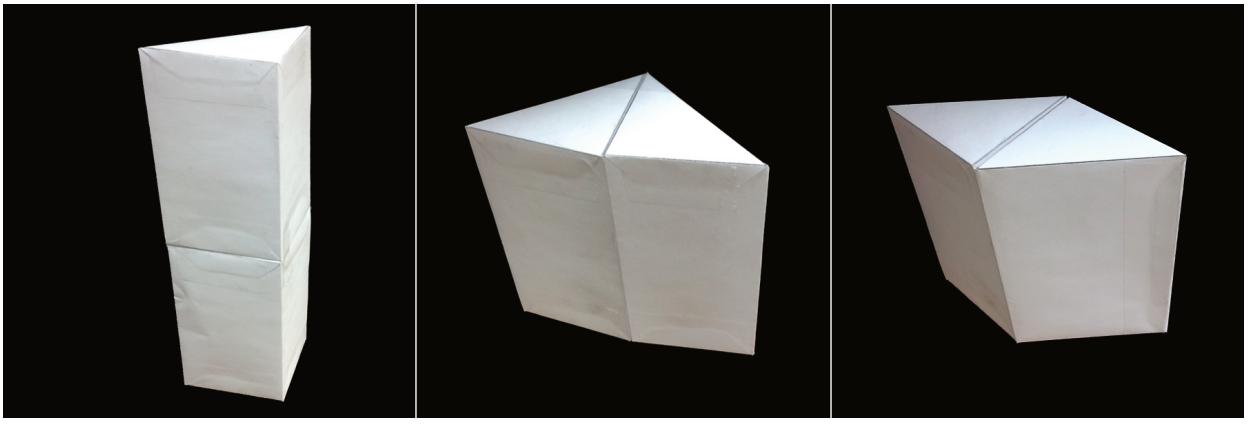
### Student Response

You would need 8 identical prisms (or 7 copies in addition to your original).

### Activity Synthesis

Discuss which attributes of the larger prisms were easiest to determine based on the original prisms and which were hardest.

Using two prisms that are identical, demonstrate putting them together against a matching side various ways.



In the first configuration,

- The area of the base is the same as in the original prism.
- The height is twice the height of the original prism.
- The volume is twice the volume of the original prism.
- The surface area is less than twice the surface area of the original prism (because of the sides that are put together).

In the second and third configurations,

- The area of the base is twice the original.
- The height is the same as the original.
- The volume is twice the original.
- The surface area is less than twice the original.

How could you put these two prisms together to make the largest surface area possible for the new prism? The smallest surface area possible?

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**Access for English Language Learners**

*Speaking: MLR8 Discussion Supports.* When students share which attributes were easier or more challenging to determine, revoice student ideas to demonstrate mathematical language use. Encourage students to consider what details are important to share, and to think about how they will explain their reasoning using mathematical language. Invite other students to challenge an idea, elaborate on an idea, or give an additional example. This will help students to produce and make sense of the language needed to communicate their own ideas.
*Design Principle(s): Support sense-making; Optimize output (for explanation)*

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## Lesson Synthesis

Ask students to reflect on what they have learned in this unit, either in writing or by talking to a partner. Here are some suggested prompts:

- "What is something you learned in this unit that surprised you?"
- "What is a new mathematical word you learned in this unit, and what does it mean?"
- "What is an idea that you learned about in this unit that is useful in the real world?"
- "Describe something that you were confused about at first, but understand now."
- "Describe something that you found challenging, but understood with some effort."
- "What is something from this unit that you are still wondering about?"

- “What was your favorite activity, and what did you learn from it?”