Lesson 6: Building Polygons (Part 1)

Goals

- Comprehend that two shapes are considered “identical copies” if they can be placed on top of each other and match up exactly.
- Recognize that four side lengths do not determine a unique quadrilateral, but that three side lengths can determine a unique triangle.
- Use manipulatives to create a polygon with given side lengths, and describe (orally) the resulting shape.

Learning Targets

- I can show that the 3 side lengths that form a triangle cannot be rearranged to form a different triangle.
- I can show that the 4 side lengths that form a quadrilateral can be rearranged to form different quadrilaterals.

Lesson Narrative

This lesson is the first in a series of lessons in which students create shapes with given conditions. During these lessons students think about what conditions are needed to determine a unique figure, in preparation for future work with congruence in grade 8 and high school. These lessons continue the language used in grade 6: two polygons are identical if they match up exactly when placed one on top of the other.

In this lesson, students experiment with making polygons of various numbers and combinations of side lengths, using cardboard strips and metal fasteners. The goal of the lesson is to help students see that sometimes lots of different shapes are possible under given constraints about side lengths, and that at other times, with different constraints, it might be that only one shape is possible or that no shape is possible. In this lesson, students do not try to formulate general rules about what side lengths are possible; in the next lesson, they formulate such a rule for triangles.

Alignments

Addressing

- 7.G.A.2: Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
- 7.NS.A.1: Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.

Instructional Routines

- MLR7: Compare and Connect
- MLR8: Discussion Supports
- True or False

Required Materials

Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Metal paper fasteners
brass brads

Pre-printed slips, cut from copies of the blackline master

Required Preparation

For the activities in this lesson and the next, you will need slips cut from copies of the What Can You Build? blackline master. Prepare 1 copy for every 2 students. These slips can be reused from one class to the next. To make the slips sturdier, it is recommended to copy them onto card stock. If card stock is not available, consider gluing each copy to light cardboard, such as a cereal box. Also if possible, copy each set of slips on a different color of paper, so that a stray strip can quickly be put back.

After the slips are cut, punch holes into the endpoints of each segment. A standard hole punch makes holes that are a little larger than needed for the metal paper fasteners, causing the cardboard strips to wiggle around. If possible, find a way to punch holes that are slightly smaller than the size of a standard hole punch.

Put each set of strips in an envelope. Prepare to distribute at least 12 metal paper fasteners (i.e., brass brads) to each group.

Note: If using the digital version of every activity, the strips and fasteners will not be needed.

Student Learning Goals

Let’s build shapes.

6.1 True or False: Signed Numbers

Warm Up: 5 minutes

The purpose of this warm-up is to encourage students to reason about properties of operations without evaluating each expression. Encourage students to think about the meaning of the operations in each question.

Addressing

- 7.NS.A.1

Instructional Routines

- MLR7: Compare and Connect
- True or False

Launch

Display each problem one at a time. Tell students to give a signal when they have a response. Give 30 seconds of quiet think time. Ask students to share their reasoning for each. Record and display their thinking for all to see. Leave each problem displayed as you move onto the next problem.

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Anticipated Misconceptions

Some students may try evaluating each side of each equation. Encourage them to look for patterns or shortcuts that would help them answer each question without doing all the calculations.

Student Task Statement

Decide whether each equation is true or false. Be prepared to explain your reasoning.

$4 \cdot (-6) = (-6) + (-6) + (-6) + (-6)$

$-8 \cdot 4 = (-8 \cdot 3) + 4$

$6 \cdot (-7) = 7 \cdot (-7) + 7$

$-10 - 6 = -10 - (-6)$

Student Response

1. True. Four groups of -6 added together can be thought of as 4 times -6.
2. False. $-8 \cdot 4 = (-8 \cdot 3) - 8$ or $-8 \cdot 4 = (-8 \cdot 3) + (-8)$
3. True. 7 is the additive inverse of -7, so adding 7 at the end makes up for the one additional group of -7 in the multiplication.
4. False. $-10 - 6 = -10 - (+6)$ or $-10 - 6 = -10 + (-6)$

Activity Synthesis

Ask students to share their reasoning. Record and display the responses for all to see. To involve more students in the conversation, use some of the following questions:

- "Do you agree or disagree? Why?"
- "Who can restate ___'s reasoning in a different way?"
- "Did anyone reason about the problem the same way, but would explain it differently?"
- "Did anyone reason about the problem in a different way?"
- "Does anyone want to add on to _____'s reasoning?" If there is time, ask students how they could rewrite the false equations to be true.

See MLR 7 (Connect and Compare) for more examples.

6.2 What Can You Build?

15 minutes (there is a digital version of this activity)
The purpose of this activity is for students to explore a physical representation of polygons and make observations about triangles and quadrilaterals. This introductory activity serves to familiarize students with the tools and definitions they will use in future activities. Students may notice that some sets of 3 strips cannot make a triangle, but formalizing rules about what lengths can and cannot be used to form a triangle is *not* the goal of this lesson.

This is the place to notice that pairs of triangles with 3 matching lengths make identical triangles, whereas pairs of quadrilaterals with 4 matching lengths do not necessarily make identical quadrilaterals. Students will express this observation more formally in the next activity.

As students work, select at least one group's triangle and another group's quadrilateral to recreate for the whole-class discussion.

Addressing

- 7.G.A.2

Instructional Routines

- MLR8: Discussion Supports

Launch

Arrange students in groups of 2. Remind students that a polygon is a closed shape with straight sides. If necessary, demonstrate how to use the fasteners to connect the strips.

Distribute one set of strips and fasteners to each group. Provide access to geometry toolkit, including rulers and protractors. Give students 5–6 minutes of quiet work time followed by a whole-class discussion.

For classes using the digital materials, there is an applet for students to use to build polygons with the given side lengths. If necessary, demonstrate how to create a vertex by overlapping the endpoints of two segments. It may work best for positioning each segment to put the green endpoint in place first and then adjust the yellow endpoint as desired.

Access for Students with Disabilities

Representation: Internalize Comprehension. Begin the activity with concrete or familiar contexts. Remind students that a polygon is a closed figure with straight sides. Demonstrate how to use fasteners to connect the slips.
Supports accessibility for: Conceptual processing; Memory

Anticipated Misconceptions

Some students may try to bend the strips to make shapes with curved sides. Remind them that a polygon has all straight sides.

Student Task Statement

Your teacher will give you some strips of different lengths and fasteners you can use to attach the corners.

1. Use the pieces to build several polygons, including at least one triangle and one quadrilateral.
2. After you finish building several polygons, select one triangle and one quadrilateral that you have made.
    a. Measure all the angles in the two shapes you selected.
    b. Using these measurements along with the side lengths as marked, draw your triangle and quadrilateral as accurately as possible.

Student Response

Answers vary.

Activity Synthesis

The purpose of this discussion is to establish what is meant when we say two shapes are identical copies. While students don't use the word *congruent* until grade 8, they should recognize that two shapes are identical only when they can match perfectly on top of each other by movements that don't change lengths or angles.

Invite a group to share the side lengths they chose for their triangle. Create a copy of the triangle with another set of strips and fasteners. Display it for all to see alongside the group's original triangle, but oriented differently. Ask students, "Are the two shapes identical? How can you tell?" (Yes, because you can put them on top of each other and they match up exactly.) Demonstrate

turning or flipping the triangle, as needed, to place one copy on top of the other and show that they match.

Repeat the demonstration with a group's quadrilateral. Create a copy that has the same side lengths as what they used, but different angles. Demonstrate the "floppiness" of the quadrilateral (that is, the angles can change even though the side lengths remain the same). Make sure students realize that the two quadrilaterals are not necessarily identical copies, even though they have the same side lengths.

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. After a group shares the side lengths they chose for their triangle, ask students to restate or revoice what they heard using mathematical language. Consider providing students time to restate what they heard to a partner, before selecting one or two students to share with the class. Ask the original group if their peer was accurately able to restate their thinking. Encourage students to supplement their explanations multi-modally by using gestures with the polygons.
Design Principle(s): Support sense-making; Maximize meta-awareness

6.3 Building Diego's and Jada's Shapes

10 minutes (there is a digital version of this activity)
The purpose of this activity is to reinforce that some conditions define a unique polygon while others do not. Students build polygons given only a description of their side lengths. They articulate that this is not enough information to guarantee that a pair of quadrilaterals are identical copies. On the other hand, triangles have a special property that three specific side lengths result in a unique triangle. Students should notice that their recreation of Jada's triangle is rigid; the side lengths and angles are all fixed.

Monitor for students who try putting the side lengths together in different orders to build different polygons and invite them to share during the whole-class discussion.

Addressing

- 7.G.A.2

Instructional Routines

- MLR7: Compare and Connect

Launch

Keep students in the same groups. Tell them they will continue to use the strips and fasteners from the previous activity. Encourage students to think about whether there are different shapes that

would fulfill the given conditions. Give students 5–6 minutes of group work time followed by a whole-class discussion.

If using the digital version, students can keep using the same applet that they explored in the previous activity to build shapes with the given conditions.

Anticipated Misconceptions

Students may think that their triangle is different from Jada's because hers is "upside down." Ask the student to turn their triangle around and ask them if it is now a different triangle. While there is a good debate to be had if they continue to insist they are different, let the students know that, for this unit, we will consider shapes that have been turned or flipped or moved as identical copies and thus "not different."

Student Task Statement

1. Diego built a quadrilateral using side lengths of 4 in, 5 in, 6 in, and 9 in.
    a. Build such a shape.
    b. Is your shape an identical copy of Diego's shape? Explain your reasoning.
2. Jada built a triangle using side lengths of 4 in, 5 in, and 8 in.
    a. Build such a shape.
    b. Is your shape an identical copy of Jada's shape? Explain your reasoning.

Student Response

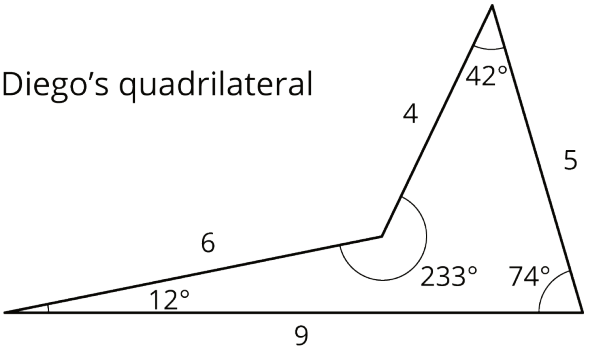
1. No, my quadrilateral is probably not an identical copy of Diego's quadrilateral, because it is floppy; there are lots of different angles I can use to make a quadrilateral with these side lengths.
2. Yes, my triangle should be an identical copy of Jada's triangle, because it is not floppy; there is no way to change the angles to make a different triangle with these side lengths.

Activity Synthesis

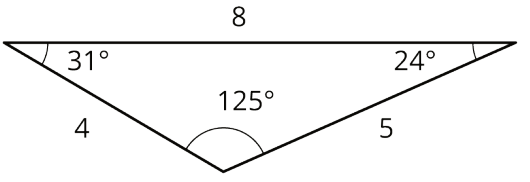
Select previously identified students to share their constructions and explanations. Display each student's example for all to see.

If desired, reveal Diego and Jada's shapes and display for all to see along side students' work.

Diego's quadrilateral



Jada's triangle



Ask students:

- "Is this what you thought Jada and Diego's shapes looked like?"
- "Which shape did you make an identical copy of?" (Jada's triangle.)
- "Why did you not make an identical copy of Diego's shape?" (Because you can make quadrilaterals with the same side lengths but different angle measure.)

Consider explaining to students how this finding is applied in construction projects. For stability, the internal structures of many buildings (and bridges) will include triangles. Rectangles or other polygons with more than three sides often include triangular supports on the inside, to make the construction more rigid and less floppy.

Access for English Language Learners

Speaking: MLR7 Compare and Connect. Use this routine to support whole-class discussion. Ask students to consider what is the same and what is different about their quadrilateral as compared to Diego's. Draw students' attention to the association between the order of side lengths in each quadrilateral and the angle measures in each shape. In this discussion, demonstrate the language used to make sense of the conditions required to make identical figures. Talking about and comparing the quadrilaterals will help draw students' attention to the conditions that define a unique polygon.
Design Principle(s): Maximize meta-awareness

6.4 Building Han's Shape

Optional: 5 minutes (there is a digital version of this activity)
The purpose of this activity is for students to see that sometimes it is impossible to build a polygon with certain conditions, but also that they need to think carefully about the information they are given before making assumptions.

In this case, students are given 3 side lengths that cannot form a triangle and are told to build a polygon. Students may assume that because they were given 3 side lengths, their shape is

supposed to be a triangle; however, they will only succeed in building a polygon using the specified side lengths if it has more than 3 sides. Formalizing rules about what lengths can and cannot be used to form a triangle is *not* the goal of this activity.

As students work on the task, monitor for students who realize that the shape cannot be a triangle and for students who realize it can be a polygon with more than 3 sides.

Addressing

- 7.G.A.2

Launch

Keep students in the same groups. Students keep using the strips and fasteners from the previous activity. Encourage students to read the question carefully. Give students 3–4 minutes of group work time, followed by a whole-class discussion.

If using the digital lesson, the applet is the same as the previous activities.

Anticipated Misconceptions

Students may say that there is no way Han could have built this shape, because they are assuming it must be a triangle. Ask students if the question specifies that the shape is a triangle. If needed, remind students of the definition of *polygon* and prompt them to consider what they could do to finish building a closed shape with all straight sides.

Student Task Statement

Han built a polygon using side lengths of 3 in, 4 in, and 9 in.

1. Build such a shape.
2. What do you notice?

Student Response

Answers vary. Sample response: I notice that this shape cannot be a triangle. I had to include a fourth side to make a quadrilateral.

Activity Synthesis

Select previously identified students to share their explanations. Make sure students realize that this shape cannot be built as a triangle; however, it is possible to build a polygon with more than 3 sides. Tell students, “As the unit progresses, you will be asked to create or draw shapes that include some conditions, but there may be some flexibility with the pieces that are not mentioned. Be aware of what must be included and what is not mentioned.”

If time permits, consider asking some of the following questions:

- “What length did you choose to use for your fourth side? Would another choice have worked?”

- "If another group used the same length for their fourth side as you, does their polygon have to be an identical copy of yours? How do you know?"
- "Did any group choose to have more than 4 sides on their polygon? Does such a shape fulfill the given conditions?"

Lesson Synthesis

- What kinds of shapes could you build with side lengths 4 inches, 4 inches, and 4 inches? (triangle, square, another quadrilateral, pentagon, etc.)
- What kinds of shapes could you **not** build with this set of side lengths and fasteners? (circle, oval, a 9 inch square, etc.)
- How is building a triangle with three given side lengths different from building a quadrilateral with four given side lengths? (the triangle must be a specific one, but the quadrilateral might be a lot of different things by changing the angles)

6.5 An Equilateral Quadrilateral

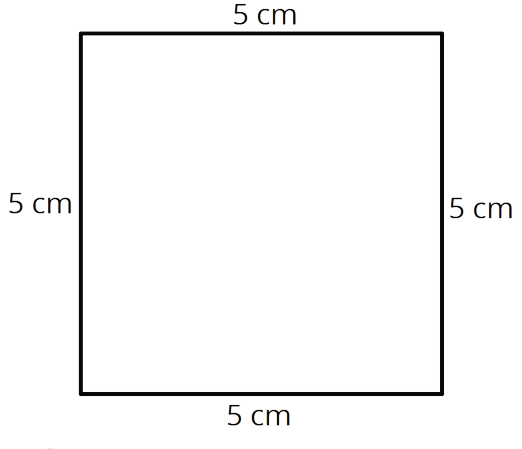
Cool Down: 5 minutes

Addressing

- 7.G.A.2

Student Task Statement

When asked to draw a quadrilateral with all four sides measuring 5 cm, Jada drew a square.



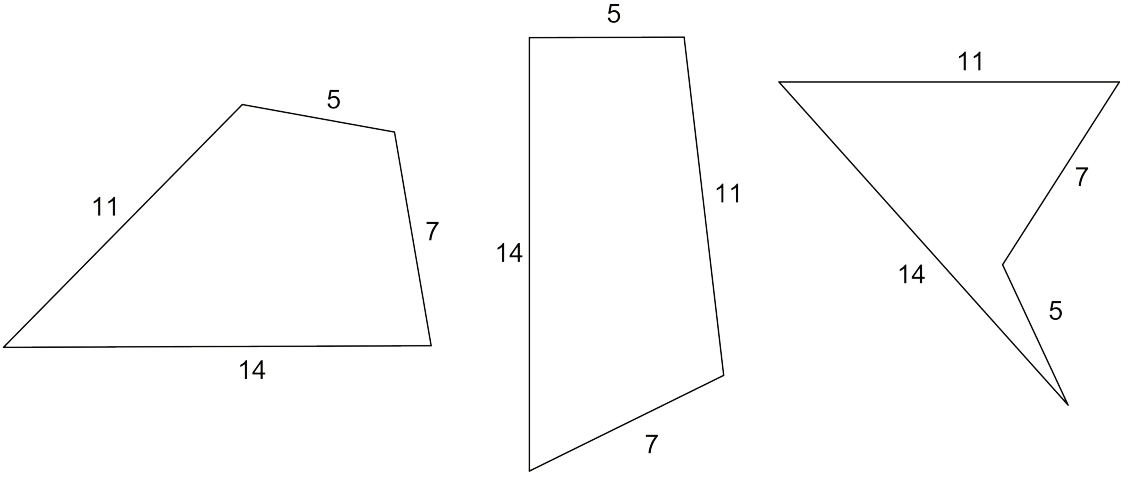
1. Do you agree with Jada's answer?
2. Is there a different shape Jada could have drawn that would answer the question? Explain your reasoning.

Student Response

1. Yes, Jada's shape has 4 sides, all measuring 5 cm.
2. A rhombus could be made with all four sides the same length, but without right angles.

Student Lesson Summary

Sometimes we are given a polygon and asked to find the lengths of the sides. What options do you have if you need to build a polygon with some side lengths? Sometimes, we can make lots of different figures. For example, if you have side lengths 5, 7, 11, and 14, here are some of the many, many quadrilaterals we can make with those side lengths:



Sometimes, it is not possible to make a figure with certain side lengths. For example, 18, 1, 1, 1 (try it!).

We will continue to investigate the figures that can be made with given measures.

Lesson 6 Practice Problems

Problem 1

Statement

A rectangle has side lengths of 6 units and 3 units. Could you make a quadrilateral that is not identical using the same four side lengths? If so, describe it.

Solution

Yes, you could make a parallelogram or a kite using the side lengths 3, 3, 6, and 6.

Problem 2

Statement

Come up with an example of three side lengths that can not possibly make a triangle, and explain how you know.

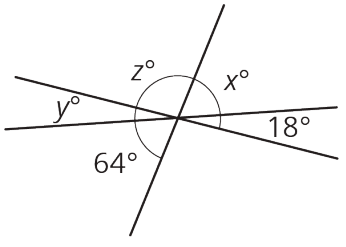
Solution

Answers vary. Sample response: the lengths 1 foot, 1 inch, and 1 inch can not possibly make a triangle, because if you attach the 1 inch lengths to either end of the 1 foot length, the 1 inch lengths are too short to connect at their other ends.

Problem 3

Statement

Find x, y, and z.



Solution

$x = 64, y = 18, z = 98$

(From Unit 7, Lesson 3.)

Problem 4

Statement

How many right angles need to be put together to make:

a. 360 degrees?

b. 180 degrees?

c. 270 degrees?

d. A straight angle?

Solution

a. 4

b. 2

c. 3

d. 2

(From Unit 7, Lesson 1.)

Problem 5

Statement

Solve each equation.

$\frac{1}{7}(x + \frac{3}{4}) = \frac{1}{8}$

$\frac{9}{2} = \frac{3}{4}(z + \frac{2}{3})$

$1.5 = 0.6(w + 0.4)$

$0.08(7.97 + v) = 0.832$

Solution

a. $\frac{1}{8}$

b. $\frac{16}{3}$

c. 2.1

d. 2.43

(From Unit 6, Lesson 8.)

Problem 6

Statement

a. You can buy 4 bottles of water from a vending machine for $7. At this rate, how many bottles of water can you buy for $28? If you get stuck, consider creating a table.

b. It costs $20 to buy 5 sandwiches from a vending machine. At this rate, what is the cost for 8 sandwiches? If you get stuck, consider creating a table.

Solution

a. 16

b. $32

(From Unit 4, Lesson 3.)