Lesson 17: Rotate and Tessellate

Goals

- Create tessellations and designs with rotational symmetry using rigid transformations.
- Explain (orally and in writing) the rigid transformations needed to move a tessellation or design with rotational symmetry onto itself.

Learning Targets

- I can repeatedly use rigid transformations to make interesting repeating patterns of figures.
- I can use properties of angle sums to reason about how figures will fit together.

Lesson Narrative

In this unit, students have learned how to name different types of rigid motions of the plane and have studied how to move different figures (lines, line segments, polygons, and more complex shapes). They have also used rigid motions to define what it means for figures to be congruent and have used rigid motions to investigate the sum of the angles in a triangle. In this lesson, students use the language of transformations to produce, describe, and investigate patterns in the plane. This is a direct extension of earlier work with triangles

- three triangles were arranged in the plane to show that the sum of the angles in a triangle is 180 degrees
- four copies of a triangle were arranged in a large square, cutting out a smaller square in the middle

Here the focus is more creative. Students will examine and create different patterns of shapes, including *tessellations* (patterns that fill the entire plane), and complex designs that exhibit rotational symmetry (that is, the design is congruent to itself by several rotations). Depending on the time available, students might work on both activities or choose one of the two.

As with many activities in this lesson, MP7 is central as students use the structure of a given set of polygons to produce a tessellation. The side lengths and angles of the polygons are constraints and through experimenting and abstract reasoning students discover a repeating pattern (MP2).

Alignments

Building On

- 4.MD.C: Geometric measurement: understand concepts of angle and measure angles.
- 7.G.B.5: Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.

Addressing

- 8.G.A: Understand congruence and similarity using physical models, transparencies, or geometry software.

Instructional Routines

- Group Presentations
- MLR8: Discussion Supports

Required Materials

Copies of blackline master
Geometry toolkits
For grade 6: tracing paper, graph paper, colored pencils, scissors, and an index card to use as a straightedge or to mark right angles.

For grades 7 and 8: everything in grade 6, plus a ruler and protractor. Clear protractors with no holes and with radial lines printed on them are recommended.

Notes: (1) "Tracing paper" is easiest to use when it's a smaller size. Commercially-available "patty paper" is 5 inches by 5 inches and ideal for this. If using larger sheets of tracing paper, consider cutting them down for student use. (2) When compasses are required in grades 6-8 they are listed as a separate Required Material.

Graph paper
Isometric graph paper

Required Preparation

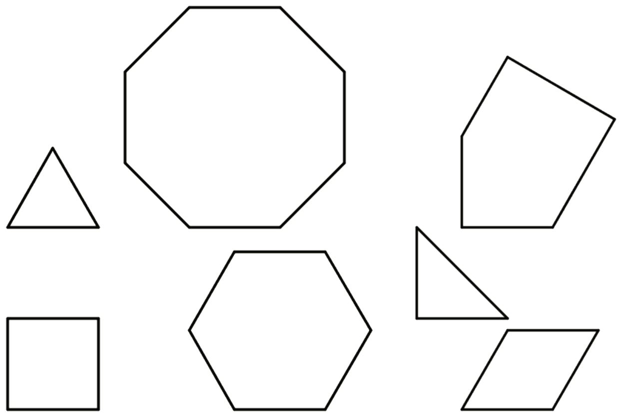
Print the Deducing Angle Measures blackline master. Prepare 1 copy for every 2 students. Cut the copies in half, so that there are enough copies for each student to receive a half-sheet. If possible, make these copies on cardstock so that students will have an easier time tracing shapes after they cut them out. If available, pattern blocks also work well for this.

Students may benefit from using graph paper and isometric graph paper, but these materials are optional.

> **Student Learning Goals**
> Let's make complex patterns using transformations.

17.1 Deducing Angle Measures

Warm Up: 10 minutes
Throughout this lesson, students build different patterns with copies of some polygons. In this activity, they make some copies of each polygon and arrange them in a circle. They calculate some of the angles of the polygons while also gaining an intuition for how the polygons fit together. Here are the figures included in the blackline master:



Students might use a protractor to measure angles, but the measures of all angles can also be deduced. In the first question in the task, students are instructed to fit copies of an equilateral triangle around a single vertex. Six copies fit, leading them to deduce that each angle measures 60° because $360 \div 6 = 60$. For the other shapes, they can reason about angles that sum to 360°, angles that sum to a line, and angles that sum to a known angle.

Building On

- 4.MD.C
- 7.G.B.5

Launch

Provide access to geometry toolkits. Distribute one half-sheet (that contains 7 shapes) to each student. It may be desirable to demonstrate how to use tracing paper to position and trace copies of the triangle around a single vertex, as described in the first question.

Anticipated Misconceptions

When deducing angle measures, it is important to know that angles "all the way around" a vertex sum to 360°. It is also important to know that angles that make a line when adjacent sum to 180°. Monitor for students who need to be reminded of these facts.

Student Task Statement

Your teacher will give you some shapes.

1. How many copies of the equilateral triangle can you fit together around a single vertex, so that the triangles' edges have no gaps or overlaps? What is the measure of each angle in these triangles?
2. What are the measures of the angles in the
    a. square?
    b. hexagon?
    c. parallelogram?

d. right triangle?

e. octagon?

f. pentagon?

Student Response

1. 6, 60°
2. Measures of angles:
    a. Square: 90°

    b. Hexagon: 120°

    c. Parallelogram: 120° and 60°

    d. Right triangle: 45° and 90°

    e. Octagon: 135°

    f. Pentagon: 90°, 120°, and 150°

Activity Synthesis

For the remainder of the lesson, it is not so important that the degree measures of the angles are known, so don’t dwell on the answers. Select a few students who deduced angles' measures by fitting pieces together to present their work. Make sure students see lots of examples of shapes fitting together like puzzle pieces.

Recall from the previous lesson that the 3 congruent angles in an equilateral triangle make a line or 180-degree angle, so it makes sense that 6 copies of this angle make a full circle.

17.2 Tessellate This

35 minutes

Each classroom activity in this lesson (this one, creating a tessellation, and the next one, creating a design with rotational symmetry) could easily take an entire class period or more. Consider letting students choose to pursue one of the two activities.

A tessellation of the plane is a regular repeating pattern of one or more shapes that covers the entire plane. Some of the most familiar examples of tessellations are seen in bathroom and kitchen tiles. Tiles (for flooring, ceiling, bathrooms, kitchens) are often composed of copies of the same shapes because they need to fit together and extend in a regular pattern to cover a large surface.

Addressing

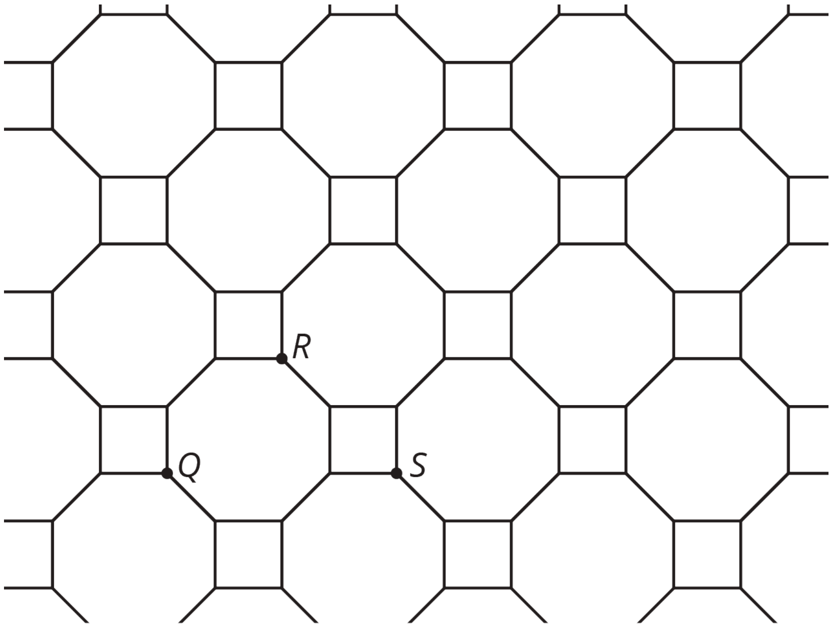
- 8.G.A

Instructional Routines

- Group Presentations

Launch

Share with students a definition of tessellation, like, "A tessellation of the plane is a regular repeating pattern of one or more shapes that covers the entire plane." Consider showing several examples of tessellations. A true tessellation covers the entire plane: While this is impossible to show, we can identify a pattern that keeps going forever in all directions. This is important when we think about tessellations and symmetry. One definition of symmetry is, "You can pick it up and put it down a different way and it looks exactly the same." In a tessellation, you can perform a translation and the image looks exactly the same. In the example of this tiling, the translation that takes point Q to point R results in a figure that looks exactly the same as the one you started with. So does the translation that takes S to Q. Describing one of these translations shows that this figure has translational symmetry.



Provide access to geometry toolkits. Suggest to students that if they cut out a shape, it is easy to make many copies of the shape by tracing it. Encourage students to use the shapes from the previous activity (or pattern blocks if available) and experiment putting them together. They do not need to use all of the shapes, so if students are struggling, suggest that they try using copies of a couple of the simpler shapes.

Access for Students with Disabilities

Representation: Internalize Comprehension. Provide a range of examples and counterexamples of shapes to use in creating a tessellation. For example, show 1–2 shapes that do not quite fit together that create gaps or overlaps. Then show 1–2 shapes that correctly create a tessellation. Consider providing step-by-step directions for students to create a shape that will make a repeating pattern.
Supports accessibility for: Conceptual processing

Anticipated Misconceptions

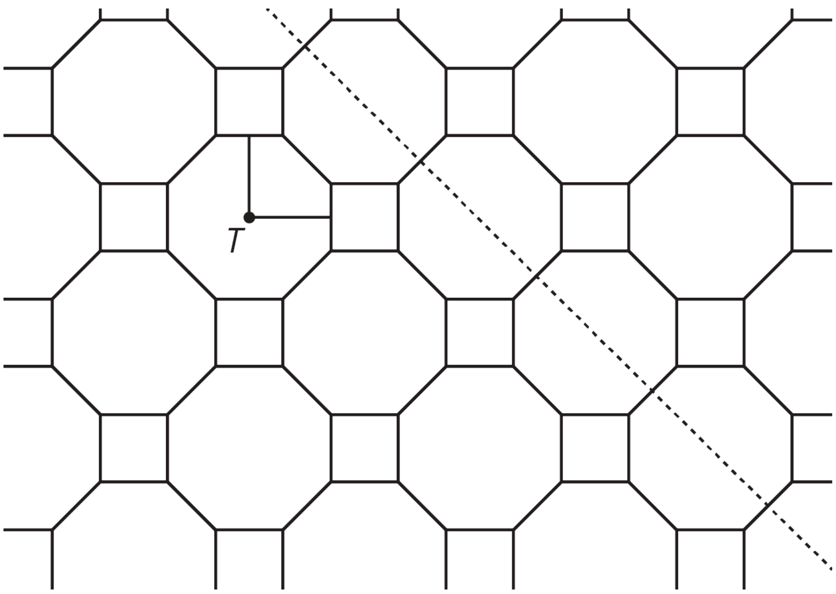
Watch out for students who choose shapes that almost-but-don't-quite fit together. Reiterate that the pattern has to keep going forever—often small gaps or overlaps become more obvious when you try to continue the pattern.

Student Task Statement

1. Design your own **tessellation**. You will need to decide which shapes you want to use and make copies. Remember that a tessellation is a repeating pattern that goes on forever to fill up the entire plane.
2. Find a partner and trade pictures. Describe a transformation of your partner's picture that takes the pattern to itself. How many different transformations can you find that take the pattern to itself? Consider translations, reflections, and rotations.
3. If there's time, color and decorate your tessellation.

Student Response

1. Answers vary.
2. Answers vary. For example, in the tessellation given previously, we could reflect across the dashed line, or rotate 90 degrees clockwise around the point marked T.



Activity Synthesis

Invite students to share their designs and also describe a transformation that takes the design to itself. Consider decorating your room with their finished products.

17.3 Rotate That

35 minutes

Each classroom activity in this lesson (the previous one, creating a tessellation, and this one, creating a design with rotational symmetry) could easily take an entire class period or more. Consider letting students choose to pursue one of the two activities.

In this activity, using their geometry toolkits, students can make their own design that has rotational symmetry. They then share designs and find the different rotations (and possibly reflections) that make the shape match up with itself.

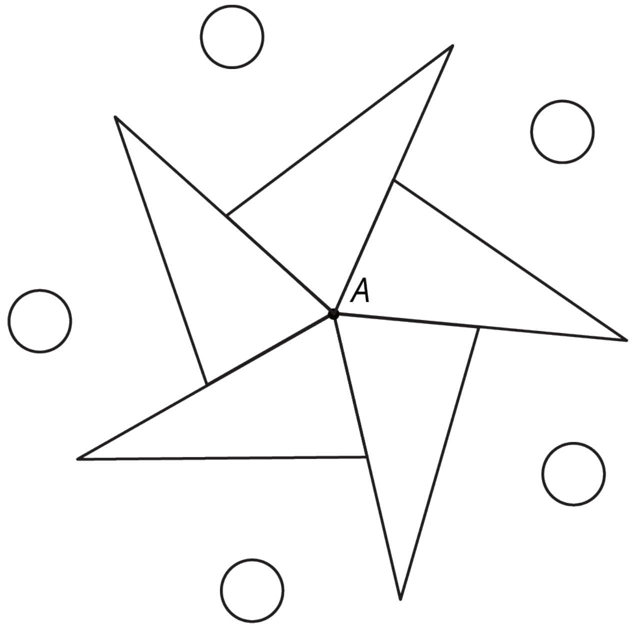
Addressing

- 8.G.A

Instructional Routines

- Group Presentations
- MLR8: Discussion Supports

Launch



Ask students what transformation they could perform on the figure so that it matches up with its original position. There are a number of rotations using A as the center that would work: 72° or any multiple of 72°. Make sure students understand that the 5 triangles in this pattern are congruent and that $5 \cdot 72 = 360$: This is why multiples of 72° with center A match this figure up with itself. They need to be careful in selecting angles for the shapes in their pattern. If they struggle, consider asking them to use pattern tiles or copies of the shapes from the previous activity to help build a pattern.

If possible, show students several examples of figures that have rotational symmetry.

Provide access to geometry toolkits. If possible, provide access to square graph paper or isometric graph paper.

Access for Students with Disabilities

Action and Expression: Provide Access for Physical Action. Provide students with access to square graph paper, isometric graph paper, and/or pattern blocks or shape cut-outs for making a design with rotational symmetry.
Supports accessibility for: Visual-spatial processing; Organization

Anticipated Misconceptions

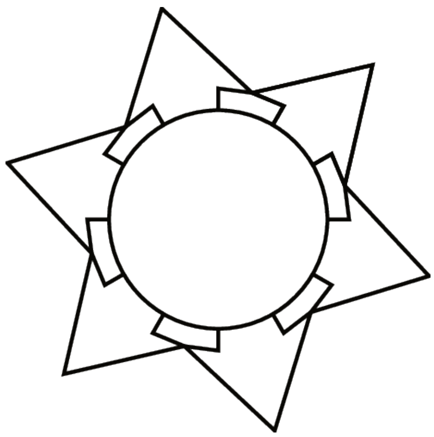
Before now, students may think that reflection symmetry is the only kind of symmetry. Because of this, they may create a design that has reflection symmetry but not rotational symmetry. Steer students in the right direction by asking them to perform a rotation that takes the figure to itself. Acknowledge that reflection symmetry is a type of symmetry, but the task here is to create a design with rotational symmetry.

Student Task Statement

1. Make a design with rotational symmetry.
2. Find a partner who has also made a design. Exchange designs and find a transformation of your partner's design that takes it to itself. Consider rotations, reflections, and translations.
3. If there's time, color and decorate your design.

Student Response

Answers vary. An example shape is below.



Activity Synthesis

Invite students to share their designs and also describe a transformation that takes the design to itself. Consider decorating your room with their finished products.

Access for English Language Learners

Speaking, Listening: MLR8 Discussion Supports. Use this routine to support whole-class discussion. For each design and transformation that is shared, ask students to restate what they heard using precise mathematical language. Consider providing students time to restate what they heard to a partner before selecting one or two students to share with the class. Ask the original speaker if their peer was accurately able to restate their thinking. Call students' attention to any words or phrases that helped to clarify the original statement. This provides more students with an opportunity to produce language as they interpret the reasoning of others.
Design Principle(s): Support sense-making

Glossary

- tessellation