## Lesson 8: End Behavior (Part 1)

* Let’s investigate the shape of polynomials.

### 8.1: Notice and Wonder: A Different View

What do you notice? What do you wonder?

$y=x^{3}+4x^{2}−x−4$



$y=x^{4}−10x^{2}+9$



### 8.2: Polynomial End Behavior

1. For your assigned polynomial, complete the column for the different values of $x$. Discuss with your group what you notice.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| * $x$
 | * $y=x^{2}+1$
 | * $y=x^{3}+1$
 | * $y=x^{4}+1$
 | * $y=x^{5}+1$
 |
| * -1000
 |  |  |  |  |
| * -100
 |  |  |  |  |
| * -10
 |  |  |  |  |
| * -1
 |  |  |  |  |
| * 1
 |  |  |  |  |
| * 10
 |  |  |  |  |
| * 100
 |  |  |  |  |
| * 1000
 |  |  |  |  |

1. Sketch what you think the **end behavior** of your polynomial looks like, then check your work using graphing technology.

#### Are you ready for more?

Mai is studying the function $p(x)=-\frac{1}{100}x^{3}+25,​422x^{2}+8x+26$. She makes a table of values for $p$ with $x=\pm 1,\pm 5,\pm 10,\pm 20$ and thinks that this function has large positive output values in both directions on the $x$-axis. Do you agree with Mai? Explain your reasoning.

### 8.3: Two Polynomial Equations

Consider the polynomial $y=2x^{5}−5x^{4}−30x^{3}+5x^{2}+88x+60$.

1. Identify the degree of the polynomial.
2. Which of the 6 terms, $2x^{5}$, $5x^{4}$, $30x^{3}$, $5x^{2}$, $88x$, or $60$, is greatest when:
	1. $x=0$
	2. $x=1$
	3. $x=3$
	4. $x=5$
3. Describe the end behavior of the polynomial.

### Lesson 8 Summary

We know that if the expression for a polynomial function $f$ written in factored form has the factor $(x−a)$, then $a$ is a zero of $f$ (that is,$f(a)=0$) and the point $(a,0)$ is on the graph of the function. But what about other values of $x$? In particular, as we consider values of $x$ that get larger and larger in either the negative or positive direction, what happens to the values of $f(x)$?

The answer to this question depends on the degree of the polynomial, because any negative real number raised to an even power results in a positive number. For example, if we graph $y=x^{2}$, $y=x^{3}$ and $y=x^{4}$ and zoom out, we see the following:

$y=x^{2}$



$y=x^{3}$



$y=x^{4}$



For both $y=x^{2}$ and $y=x^{4}$, large positive values of $x$ or large negative values of $x$ each result in large positive values of $y$. But for $y=x^{3}$, large positive values of $x$ result in large positive values of $y$, while large negative values of $x$ result in large negative values of $y$.

Consider the polynomial $P(x)=x^{4}−30x^{3}−20x^{2}+1000$. The leading term, $x^{4}$, almost seems smaller than the other 3 terms. For certain values of $x$, this is even true. But, for values of $x$ far away from zero, the leading term will always have the greatest value. Can you see why?

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| $x$ | $x^{4}$ | $-30x^{3}$ | $-20x^{2}$ | $1000$ | $P(x)$ |
| -500 | 62,500,000,000 | 3,750,000,000 | -5,000,000 | 1,000 | 66,245,001,000 |
| -100 | 100,000,000 | 30,000,000 | -200,000 | 1,000 | 129,801,000 |
| -10 | 10,000 | 30,000 | -2,000 | 1,000 | 39,000 |
| 0 | 0 | 0 | 0 | 1,000 | 1000 |
| 10 | 10,000 | -30,000 | -2,000 | 1,000 | -21,000 |
| 100 | 100,000,000 | -30,000,000 | -200,000 | 1,000 | 69,801,000 |
| 500 | 62,500,000,000 | -3,750,000,000 | -5,000,000 | 1,000 | 58,745,001,000 |

The value of the leading term $x^{4}$ determines the **end behavior** of the function, that is, how the outputs of the function change as we look at input values farther and farther from 0. In the case of $P(x)$, as $x$ gets larger and larger in the positive and negative directions, the output of the function gets larger and larger in the positive direction.



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