## Lesson 14: Completing the Square (Part 3)

* Let’s complete the square for some more complicated expressions.

### 14.1: Perfect Squares in Two Forms

Elena says, “$(x+3)^{2}$ can be expanded into $x^{2}+6x+9$. Likewise, $(2x+3)^{2}$ can be expanded into $4x^{2}+6x+9$.”

Find an error in Elena’s statement and correct the error. Show your reasoning.

### 14.2: Perfect in A Different Way

1. Write each expression in standard form:
	1. $(4x+1)^{2}$
	2. $(5x−2)^{2}$
	3. $(\frac{1}{2}x+7)^{2}$
	4. $(3x+n)^{2}$
	5. $(kx+m)^{2}$
2. Decide if each expression is a perfect square. If so, write an equivalent expression of the form $(kx+m)^{2}$. If not, suggest one change to turn it into a perfect square.
	1. $4x^{2}+12x+9$
	2. $4x^{2}+8x+25$

### 14.3: When All the Stars Align

1. Find the value of $c$ to make each expression in the left column a perfect square in standard form. Then, write an equivalent expression in the form of squared factors. In the last row, write your own pair of equivalent expressions.

|  |  |
| --- | --- |
| * standard form $(ax^{2}+bx+c)$
 | * squared factors $(kx+m)^{2}$
 |
| * $100x^{2}+80x+c$
 | *
 |
| * $36x^{2}−60x+c$
 | *
 |
| * $25x^{2}+40x+c$
 | *
 |
| * $0.25x^{2}−14x+c$
 | *
 |
| *
 | *
 |

1. Solve each equation by completing the square:
* $25x^{2}+40x=-12$
* $36x^{2}−60x+10=-6$

### 14.4: Putting Stars into Alignment

Here are three methods for solving $3x^{2}+8x+5=0$.

Try to make sense of each method.

Method 1:

$\begin{matrix}3x^{2}+8x+5&=0\\(3x+5)(x+1)&=0\end{matrix}$

$\begin{matrix}x=-\frac{5}{3} or x=-1\end{matrix}$

Method 2:

$\begin{matrix}3x^{2}+8x+5&=0\\9x^{2}+24x+15&=0\\(3x)^{2}+8(3x)+15&=0\\U^{2}+8U+15&=0\\(U+5)(U+3)&=0\end{matrix}$
$\begin{matrix}U=-5 &or U=-3\\3x=-5 &or 3x=-3\\x=-\frac{5}{3} &or x=-1\end{matrix}$

Method 3:

$\begin{matrix}3x^{2}+8x+5&=0\\9x^{2}+24x+15&=0\\9x^{2}+24x+16&=1\\(3x+4)^{2}&=1\end{matrix}$

$\begin{matrix}3x+4=1 &or 3x+4=-1\\x=-1 &or x=-\frac{5}{3}\end{matrix}$

Once you understand the methods, use each method at least one time to solve these equations.

1. $5x^{2}+17x+6=0$
2. $6x^{2}+19x=-10$
3. $8x^{2}−33x+4=0$
4. $8x^{2}−26x=-21$
5. $10x^{2}+37x=36$
6. $12x^{2}+20x−77=0$

#### Are you ready for more?

Find the solutions to $3x^{2}−6x+\frac{9}{4}=0$. Explain your reasoning.

### Lesson 14 Summary

In earlier lessons, we worked with perfect squares such as $(x+1)^{2}$ and $(x−5)(x−5)$. We learned that their equivalent expressions in standard form follow a predictable pattern:

* In general, $(x+m)^{2}$ can be written as $x^{2}+2mx+m^{2}$.
* If a quadratic expression is of the form $ax^{2}+bx+c$, and the value of $a$ is 1, then the value of $b$ is $2m$, and the value of $c$ is $m^{2}$ for some value of $m$.

In this lesson, the variable in the factors being squared had a coefficient other than 1, for example $(3x+1)^{2}$ and $(2x−5)(2x−5)$. Their equivalent expression in standard form also followed the same pattern we saw earlier.

|  |  |
| --- | --- |
| squared factors | standard form |
| $(3x+1)^{2}$ | $(3x)^{2}+2(3x)(1)+1^{2} or 9x^{2}+6x+1$ |
| $(2x−5)^{2}$ | $(2x)^{2}+2(2x)(-5)+(-5)^{2} or 4x^{2}−20x+25$ |

In general, $(kx+m)^{2}$ can be written as:

$(kx)^{2}+2(kx)(m)+m^{2}$

or

$k^{2}x^{2}+2kmx+m^{2}$

If a quadratic expression is of the form $ax^{2}+bx+c$, then:

* the value of $a$ is $k^{2}$
* the value of $b$ is $2km$
* the value of $c$ is $m^{2}$

We can use this pattern to help us complete the square and solve equations when the squared term $x^{2}$ has a coefficient other than 1—for example: $16x^{2}+40x=11$.

What constant term $c$ can we add to make the expression on the left of the equal sign a perfect square? And how do we write this expression as squared factors?

* 16 is $4^{2}$, so the squared factors could be $(4x+m)^{2}$.
* 40 is equal to $2(4m)$, so $2(4m)=40$ or $8m=40$. This means that $m=5$.
* If $c$ is $m^{2}$, then $c=5^{2}$ or $c=25$.
* So the expression $16x^{2}+40x+25$ is a perfect square and is equivalent to $(4x+5)^{2}$.

Let’s solve the equation $16x^{2}+40x=11$ by completing the square!

$\begin{matrix}16x^{2}+40x&=11\\16x^{2}+40x+25&=11+25\\(4x+5)^{2}&=36\\&\\4x+5=6 &or 4x+5=-6\\4x=1 &or 4x=-11\\x=\frac{1}{4} &or x=-\frac{11}{4}\end{matrix}$.



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