Lesson 3: Measuring with Different-Sized Units

Goals

- Generalize (orally and in writing) that it takes more of a smaller unit or fewer of a larger unit to measure the same quantity.
- Given a measurement in one unit, estimate what would be the same amount expressed in a different unit, and explain (orally) the reasoning.

Learning Targets

- When I know a measurement in one unit, I can decide whether it takes more or less of a different unit to measure the same quantity.

Lesson Narrative

This lesson develops students' familiarity with standard units of length, volume, weight, and mass through the tactile experiences of measuring objects. The main idea is that it takes more of a smaller unit and less of a larger unit to measure the same quantity. This idea is an important foundation for converting units of measurement using ratio reasoning in the next lesson (MP7).

Alignments

Building On

- 2.MD.A.2: Measure the length of an object twice, using length units of different lengths for the two measurements; describe how the two measurements relate to the size of the unit chosen.

Addressing

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Building Towards

- 6.RP.A.3.d: Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

Instructional Routines

- MLR8: Discussion Supports

Required Materials

Base-ten blocks
Blank paper
Cuisenaire rods
Gallon-sized jug
Graduated cylinders
Household items
Inch cubes
Internet-enabled device
Liter-sized bottle
Materials assembled from the blackline master
Metal paper fasteners
brass brads

Meter sticks
Paper clips
Pre-assembled polyhedra
Quart-sized bottle
Rulers
Salt
Scale
a digital scale that can output in grams, kilograms, ounces, or pounds

Straightedges
A rigid edge that can be used for drawing line segments. Sometimes a ruler is okay to use as a straightedge, but sometimes it is preferable to use an unruled straightedge, like a blank index card.

Teaspoon
Tray

Required Preparation

For the first activity, prepare to display or distribute 6-cm and 9-cm Cuisenaire rods, which are often colored dark green and blue, respectively. If Cuisenaire rods are not available, small and large paper clips can be substituted.

For the second activity, identify where each station will be and set up the following materials:

For Station 1:

- From the first page of the blackline master, print the net for the 2-in by 2-in by 4-in box onto card stock, cut it out, and assemble it.
- Provide at least twenty inch cubes, one centimeter cube, and thirty 10-cm rods. The centimeter cube and 10-cm rods can come from a set of base-ten blocks or Cuisenaire rods. However, base-ten blocks are preferable so students can see how one rod is composed of ten centimeter cubes. Wooden inch cubes are available inexpensively at craft stores.

For Station 2:

- Identify something in the classroom that is about 20 feet long. Prepare a way to communicate to students that this is the object they are supposed to measure (but do not give away its length).
- Provide rulers and at least 2 meter sticks.

For Station 3:

- Prepare a way for students to be able to watch this video

Video 'Quarts and Liters' available here: https://player.vimeo.com/video/304136597.

- Provide an empty gallon-sized jug, quart-sized bottle, and liter-sized bottle for comparison.

For Station 4 (there are 3 different options):

1. *If students will weigh objects on a real scale:* Set up the scale and provide common household items for students to weigh. Note: The scale must be able to output in grams, kilograms, ounces, and pounds for this option to work.
2. *If students will use the digital scale simulation:* Prepare a way for students to access this widget http://ggbm.at/eQQVYB7D.
3. *If students will use the paper scale simulations:* Print pages 2–13 of the blackline master onto cardstock and cut out the scale images and output wheels. Make sure to cut out the two white windows on the base of each scale where the output wheels are supposed to show through. Assemble the paper scale simulations using metal fasteners so the output wheels can rotate behind the scale images.

For Station 5:

- On a tray for catching spills, provide a 100-ml graduated cylinder, a teaspoon, a straightedge for leveling off the teaspoon, and a small bowl with at least $\frac{1}{2}$ cup of salt.

Student Learning Goals

Let's measure things.

3.1 Width of a Paper

Warm Up: 5 minutes
Students begin by thinking about length in terms of non-standard units—9-cm and 6-cm Cuisenaire rods—and consider how the size of units affects the number of units needed to express a length. If Cuisenaire rods are not available, modify the task to say: Does it take more large paper clips or small paper clips lined up end-to-end to measure the width of a piece of paper?

Some students may be able to reason that it takes more of the smaller unit than the larger unit to measure the same length; encourage them to articulate their reasoning. Others may need to visualize the situation by drawing or by measuring with actual rods (or paper clips).

Building On

- 2.MD.A.2

Building Towards

- 6.RP.A.3.d

Launch

This activity is written to use 9-cm and 6-cm Cuisenaire rods, which are often blue and dark green, respectively. If your set of Cuisenaire rods has different colors, or if using small and large paper clips as substitutes, instruct students to modify the task accordingly.

Hold up the two sizes of rods or paper clips for the students to see. Give them quiet think time but not the manipulatives. Later, allow students to use the rods or paper clips to measure the paper if they need or wish to do so.

Anticipated Misconceptions

Some students may assume that it will take more of the longer rods because they are used to associating the idea of "more" with "larger." Encourage them to use the manipulatives to see that it actually takes fewer of the longer rods to reach across the paper.

Student Task Statement

Your teacher will show you two rods. Does it take more green rods or blue rods lined up end to end to measure the width of a piece of printer paper?

Student Response

It takes more green rods, because they are shorter than the blue rods.

Activity Synthesis

Ask students to share their responses and reasoning. Highlight the fact that it takes more of a smaller unit and fewer of a larger unit to measure the same length.

3.2 Measurement Stations

35 minutes (there is a digital version of this activity)
In groups, students rotate through five different stations, where they measure one or more quantities using different units, and answer a series of summary questions afterward. Here are the quantities being measured and the units used at each station:

- **Station 1:** Volume of a box, in cubic inches and cubic centimeters.
- **Station 2:** Length, in meters and feet.
- **Station 3:** Volume of water, in gallons, quarts, and liters.
  (If desired, you can have students measure water with actual containers instead of watching the video https://vimeo.com/illustrativemathematics/water.)
- **Station 4:** Weights and masses of 2–3 objects, in ounces, pounds, grams, and kilograms.
  (You can have students weigh actual objects, use the digital simulation http://ggbm.at/eQQVYB7D, or use the paper simulations from the blackline master. If using one of the simulations instead of a real scale, prepare some real objects labeled with their weight or mass for students to hold and feel the weight of.)

- **Station 5:** Volume of salt, in milliliters and teaspoons.

You will need the blackline master for this activity. Page 1 is a net for the box needed for station 1. If you are using the paper scale simulation instead of a real scale or the applet, pages 2–13 are the parts needed to assemble Station 4.

Addressing

- 6.RP.A.3.d

Instructional Routines

- MLR8: Discussion Supports

Launch

Tell students they will further investigate the idea of using different units to measure the same set of items. Introduce the five stations, what students are expected to do at each, the protocol for rotating through them, and the questions to answer at the end. Then, demonstrate how to use the straightedge to measure a level teaspoon of salt. If students do not use a level teaspoons of salt, they will not be able to answer the last set of questions about volume.

Arrange students into 5 groups and assign a starting station for each group.

If students have devices, Stations 3 and 4 can be digital.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Encourage and support opportunities for peer interactions. Invite students to talk about their ideas with a partner before writing them down. Display sentence frames to support students when they explain their strategy. For example, “I noticed _____ so I think...”
Supports accessibility for: Language; Social-emotional skills

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Access for English Language Learners

Speaking, Representing: MLR8 Discussion Supports. Use this routine to support small-group discussion. As students rotate through stations, encourage students to solidify their own understanding by pressing for details and questioning their peers' explanations. Provide sentence frames for students to use, such as "I agree/disagree because . . .", "How do you know . . .", and “Can you give an example?” This will help students clarify their reasoning about comparing different measurements for the same quantity using different units.
Design Principle(s): Support sense-making; Cultivate conversation

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Anticipated Misconceptions

At Station 1, students may count the number of base-10 centimeter rods rather than the number of centimeter cubes. Remind them that the question prompts for the number of cubes.

At Station 2, students may need reminders about measuring objects at the zero marking on the ruler and about keeping the ruler going straight, both of which will affect the answer. Show them they can measure along the edge of the object to make sure the ruler is not veering off in one direction or another.

At Station 4, students may be unclear about how to change the output unit on the scale for each object. Consider showing the class ahead of time. Students who are able to distinguish between weight and mass might say they cannot accurately compare their measurements. Clarify that we are talking only about the weight of the objects on Earth's surface.

At Station 5, some students may consistently use under-filled or rounded teaspoons of salt, so their data will not reveal the $5:1$ ratio of milliliters to teaspoons. Repeat the demonstration of how to measure a level teaspoon for them.

Students may answer 3 milliliters for the question about 15 teaspoons because they divided by 5 instead of multiplying by 5. Encourage them to pay attention to which unit is bigger and ask what that tells them about which numerical value should be larger.

Student Task Statement

Station 1

- Each large cube is 1 cubic inch. Count how many cubic inches completely pack the box without gaps.
- Each small cube is 1 cubic centimeter. Each rod is composed of 10 cubic centimeters. Count how many cubic centimeters completely fill the box.

	cubic inches	cubic centimeters
volume of the box		

Station 2

Your teacher showed you a length.

- Use the meter stick to measure the length to the nearest meter.
- Use a ruler to measure the length to the nearest foot.

	meters	feet
length of _____		

Station 3

If not using real water, open https://vimeo.com/illustrativemathematics/water.

- Count how many times you can fill the quart bottle from the gallon jug.
- Count how many times you can fill the liter bottle from the gallon jug.

	quarts	liters
1 gallon of water		

Station 4

If not using a real scale, open http://ggbm.at/eQQVYB7D.

- Select 2–3 different objects to measure on the scale.
- Record the weights in ounces, pounds, grams, and kilograms.

object	ounces	pounds	grams	kilograms

Station 5

- Count how many level teaspoons of salt fill the graduated cylinder to 20 milliliters, 40 milliliters, and 50 milliliters.
- Pour the salt back into the original container.

	milliliters	teaspoons
small amount of salt	20	
medium amount of salt	40	
large amount of salt	50	

After you finish all five stations, answer these questions with your group.

1. a. Which is larger, a cubic inch or a cubic centimeter?

   b. Did more cubic inches or cubic centimeters fit in the cardboard box? Why?

2. Did it take more feet or meters to measure the indicated length? Why?
3. Which is larger, a quart or a liter? Explain your reasoning.
4. Use the data from Station 4 to put the units of weight and mass in order from smallest to largest. Explain your reasoning.

5. a. About how many teaspoons of salt would it take to fill the graduated cylinder to 100 milliliters?

   b. If you poured 15 teaspoons of salt into an empty graduated cylinder, about how many milliliters would it fill?

   c. How many milliliters per teaspoon are there?

   d. How many teaspoons per milliliter are there?

Student Response

	cubic inches	cubic centimeters
volume of the box	16	250

	meters	feet
length of _____	6	20

	quarts	liters
1 gallon of water	4	a little less than 4

object to weigh	ounces	pounds	grams	kilograms
Answers vary				

	milliliters	teaspoons
small amount of salt	20	4
medium amount of salt	40	8
large amount of salt	50	10

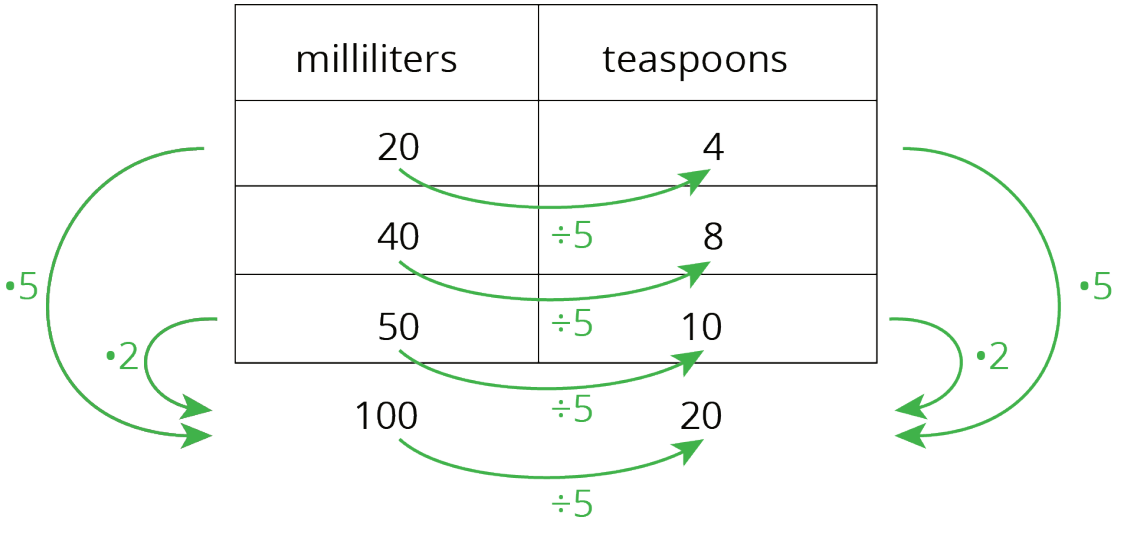
1. A cubic inch is larger. More cubic centimeters fit in the box because they are smaller.

2. It took more feet because feet are smaller than meters.

3. A liter is bigger than a quart because the gallon filled fewer of them.

4. From least to greatest, the units are gram, ounce, pound, and kilogram, because each object's weight was the largest number of grams, fewer ounces, even fewer pounds, and the smallest number of kilograms.

5. a. About 20 teaspoons for 100 milliliters of salt



b. About 75 milliliters for 15 teaspoons of salt

c. About 5 milliliters per teaspoon

d. About $\frac{1}{5}$ teaspoons per milliliter

Are You Ready for More?

People in the medical field use metric measurements when working with medicine. For example, a doctor might prescribe medication in 10 mg tablets.

Brainstorm a list of reasons why healthcare workers would do this. Organize your thinking so it can be followed by others.

Student Response

Answers vary and may include:

- Unit conversions are simpler
- Calculations are often simpler

Activity Synthesis

Though much of the discussion will take place within groups, spend a few minutes ensuring that everyone understands the answers to the five questions. To conclude the activity, invite students to share anything that surprised them from the measuring work.

Lesson Synthesis

If you measure the same quantity with different units, it will take more of the smaller unit and less of the larger one to express the measurement. For example, a jug that holds 2 gallons of liquid also holds 8 quarts of liquid. Quarts are four times smaller than gallons, so it takes four times as many quarts to measure the same volume of liquid.

To reinforce this idea, ask students questions such as:

- "What do quarts and gallons measure?" (Volume of a liquid)
- "Which is bigger: 1 quart, or 1 gallon?" (1 gallon. There are 4 quarts in 1 gallon.)
- "How many quarts are in 8 gallons?" (32. Since a quart is less than a gallon, you need more quarts to measure the same amount.)
- "How many gallons are in 8 quarts?" (2. Since a gallon is bigger than a quart, you need fewer gallons to measure the same amount.)

3.3 Which Measurement is Which?

Cool Down: 5 minutes

Addressing

- 6.RP.A.3.d

Anticipated Misconceptions

If students seem to be guessing on the first two questions, you can have them hold objects from the previous lesson that weigh close to 1 pound, 1 kilogram, 1 ounce, and 1 gram.

Student Task Statement

1. Lin has a pet German Shepherd that weighs 38 when measured in one unit and 84 when measured in a different unit. Which measurement is in pounds, and which is in kilograms?

   38_____ 84_____

2. Elena has a pet parakeet that weighs 6 when measured in one unit and 170 when measured in a different unit. Which measurement is in ounces, and which is in grams?

   6_____ 170_____

3. Behind Lin's house there is a kiddie pool that holds 180 or 680 units of water, depending on which unit you are using to measure. Which measurement is in gallons, and which is in liters?

   180_____ 680_____

4. Behind Elena's house there is a portable storage container that holds 29 or 1024 units, depending on which unit you are using to measure. Which measurement is in cubic feet, and which is in cubic meters?

   29_____ 1024_____

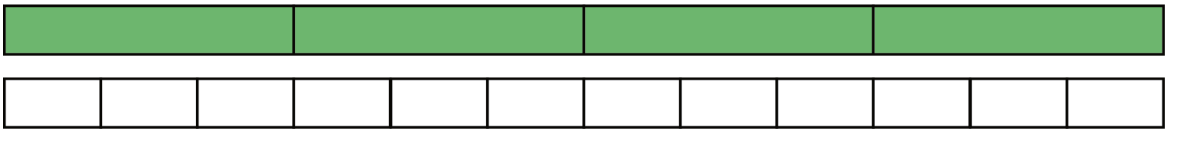
Student Response

1. 38 is in kilograms and 84 is in pounds because a kilograms is heavier than a pound, so you need fewer kilograms to measure the same quantity.
2. 6 is in ounces and 170 is in grams because an ounce weighs more than a gram.
3. 180 is in gallons and 680 is in liters because gallons are a larger unit than liters.
4. 29 is in cubic meters and 1024 is in cubic feet because cubic meters are larger than cubic feet, so you need fewer of them to measure the same quantity.

Student Lesson Summary

The size of the unit we use to measure something affects the measurement.

If we measure the same quantity with different units, it will take more of the smaller unit and fewer of the larger unit to express the measurement. For example, a room that measures 4 yards in length will measure 12 feet.



There are 3 feet in a yard, so one foot is $\frac{1}{3}$ of a yard.

- It takes 3 times as many feet to measure the same length as it does with yards.
- It takes $\frac{1}{3}$ as many yards to measure the same length as it does with feet.

Lesson 3 Practice Problems

Problem 1

Statement

Decide if each is a measurement of length, area, volume, or weight (or mass).

a. How many centimeters across a handprint

b. How many square inches of paper needed to wrap a box

c. How many gallons of water in a fish tank

d. How many pounds in a bag of potatoes

e. How many feet across a swimming pool

f. How many ounces in a bag of grapes

g. How many liters in a punch bowl

h. How many square feet of grass in a lawn

Solution

a. Length

b. Area

c. Volume

d. Weight (or mass)

e. Length

f. Weight (or mass)

g. Volume

h. Area

(From Unit 3, Lesson 2.)

Problem 2

Statement

Clare says, “This classroom is 11 meters long. A meter is longer than a yard, so if I measure the length of this classroom in yards, I will get less than 11 yards.” Do you agree with Clare? Explain your reasoning.

Solution

Clare is incorrect. Explanations vary. Sample explanation: Since yards are shorter than meters, more yards than meters are needed to measure the same length.

Problem 3

Statement

Tyler’s height is 57 inches. What could be his height in centimeters?

A. 22.4

B. 57

C. 144.8

D. 3,551

Solution

C

Problem 4

Statement

A large soup pot holds 20 quarts. What could be its volume in liters?

A. 7.57

B. 19

C. 21

D. 75.7

Solution

B

Problem 5

Statement

Clare wants to mail a package that weighs $4\frac{1}{2}$ pounds. What could this weight be in kilograms?

A. 2.04

B. 4.5

C. 9.92

D. 4,500

Solution

A

Problem 6

Statement

Noah bought 15 baseball cards for $9.00. Assuming each baseball card costs the same amount, answer the following questions.

a. At this rate, how much will 30 baseball cards cost? Explain your reasoning.

b. At this rate, how much will 12 baseball cards cost? Explain your reasoning.

c. Do you think this information would be better represented using a table or a double number line? Explain your reasoning.

Solution

a. $18.00, because 30 is twice as much as 15 and 18 is twice as much as 9.

b. $7.20, because each baseball card costs 60 cents, and 0.6 times 12 is 7.2.

c. Answers vary. Sample response: A table would be more convenient, because the rows of the table can be listed in any order, and not all values between the ones needed have to be filled in.

(From Unit 2, Lesson 13.)

Problem 7

Statement

Jada traveled 135 miles in 3 hours. Andre traveled 228 miles in 6 hours. Both Jada and Andre traveled at a constant speed.

a. How far did Jada travel in 1 hour?

b. How far did Andre travel in 1 hour?

c. Who traveled faster? Explain or show your reasoning.

Solution

a. Jada traveled 45 miles per hour because $135 \div 3 = 45$.

b. Andre traveled 38 miles per hour because $228 \div 6 = 38$.

c. Jada traveled faster because she covered a greater distance in the same amount of time.

(From Unit 2, Lesson 9.)