Lesson 8: Equal and Equivalent

Goals

- Draw a diagram to represent the value of an expression for a given value of its variable.
- Explain (in writing) that some pairs of expressions are equal for one value of their variable but not for other values.
- Justify (orally, in writing, and through other representations) whether two expressions are "equivalent", i.e., equal to each other for every value of their variable.

Learning Targets

- I can explain what it means for two expressions to be equivalent.
- I can use a tape diagram to figure out when two expressions are equal.
- I can use what I know about operations to decide whether two expressions are equivalent.

Lesson Narrative

In this lesson students are introduced to the idea of **equivalent expressions**. Two expressions are equivalent if they have the same value no matter what the value of the variable in them. Students use diagrams where the variable is represented by a generic length to decide if expressions are equivalent, and they show that expressions are not equivalent by giving values of the variable that make them unequal. They identify simple equivalent expressions using familiar facts about operations.

Alignments

Addressing

- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y+y+y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.
- 6.EE.B.5: Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.

Instructional Routines

- Algebra Talk
- MLR3: Clarify, Critique, Correct
- MLR8: Discussion Supports

Required Materials

Graph paper

Required Preparation

Graph paper in addition to grids printed with the tasks may or may not be necessary. It is recommended to have some on hand just in case.

Student Learning Goals

Let's use diagrams to figure out which expressions are equivalent and which are just sometimes equal.

8.1 Algebra Talk: Solving Equations by Seeing Structure

Warm Up: 5 minutes

In this algebra talk, students recall how to solve equations by considering what number can be substituted for the variable to make the equation true. (Note: $x^2 = 49$ of course has another solution if we allow solutions to be negative, but students haven't studied negative numbers yet, and don't study operations with negative numbers until grade 7, so it is unlikely to come up.)

Addressing

- 6.EE.B.5

Instructional Routines

- Algebra Talk
- MLR8: Discussion Supports

Launch

Display one problem at a time. Give students 30 seconds of quiet think time for each problem and ask them to give a signal when they have an answer and a strategy. Keep all problems displayed throughout the talk. Follow with a whole-class discussion.

Access for Students with Disabilities

Representation: Internalize Comprehension. To support working memory, provide students with sticky notes or mini whiteboards.
Supports accessibility for: Memory; Organization

Student Task Statement

Find a solution to each equation mentally.

$3 + x = 8$

$10 = 12 - x$

$x^2 = 49$

$\frac{1}{3}x = 6$

Student Response

- 5
- 2
- 7
- 18

Activity Synthesis

Ask students to share their strategies for each problem. Record and display their responses for all to see. To involve more students in the conversation, consider asking:

- “Who can restate ___’s reasoning in a different way?”
- “Did anyone have the same strategy but would explain it differently?”
- “Did anyone solve the problem in a different way?”
- “Does anyone want to add on to _____’s strategy?”
- “Do you agree or disagree? Why?”

Access for English Language Learners

Speaking: MLR8 Discussion Supports.: Display sentence frames to support students when they explain their strategy. For example, "First, I _____ because . . ." or "I noticed _____ so I . . . ." Some students may benefit from the opportunity to rehearse what they will say with a partner before they share with the whole class.
Design Principle(s): Optimize output (for explanation)

8.2 Using Diagrams to Show That Expressions are Equivalent

20 minutes
Students use diagrams to show that expressions can be equivalent or expressions can be equal for only one value of their variable. Working through these tape diagrams with small whole numbers,

where students can count grids and use lengths to check their results, allows students to begin to generalize about equal and equivalent expressions.

Addressing

- 6.EE.A.4

Instructional Routines

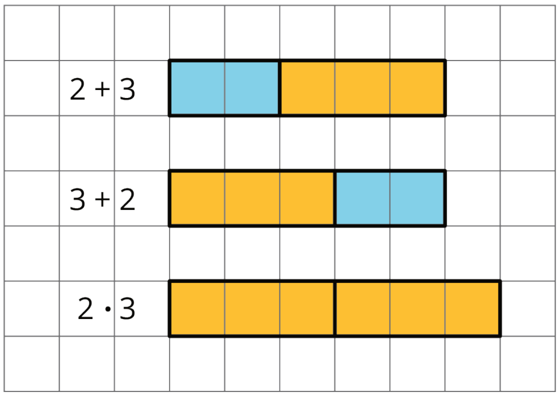
- MLR8: Discussion Supports

Launch

Provide access to graph paper. Ask students to draw diagrams that show:

$2 + 3 = 3 + 2$

$2 + 3$ does not equal $2 \cdot 3$



We can tell that $2 + 3$ and $3 + 2$ are equal because the length of the diagrams represent the value of each expression, and the diagrams are the same length. We can tell that $2 \cdot 3$ is not equal to these because this value is represented by the length of its diagram, and it's not the same length as the others. $2 + 3$ and $3 + 2$ are examples of expressions that are not identical, but are equal. Another example students have seen of this phenomenon are fractions like $\frac{1}{2}$ and $\frac{3}{6}$, which are not identical but equal.

When we start talking about expressions that have letters in them, the language gets more complicated, because expressions can be equal or not equal depending on the value the letter represents.

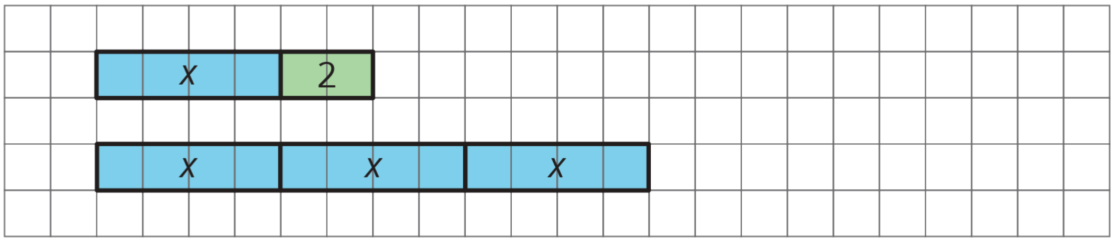
Arrange students in groups of 2. Ask students to work independently on each question and then check in with their partner, discussing and resolving any disagreements. Allow 15 minutes to work and share responses with a partner, followed by a whole-class discussion.

Access for Students with Disabilities

Action and Expression: Internalize Executive Functions. Begin with a small-group or whole-class demonstration and think aloud of the first question to remind students how to draw tape diagrams on grids. Keep the worked-out calculations on display for students to reference as they work.
Supports accessibility for: Memory; Conceptual processing

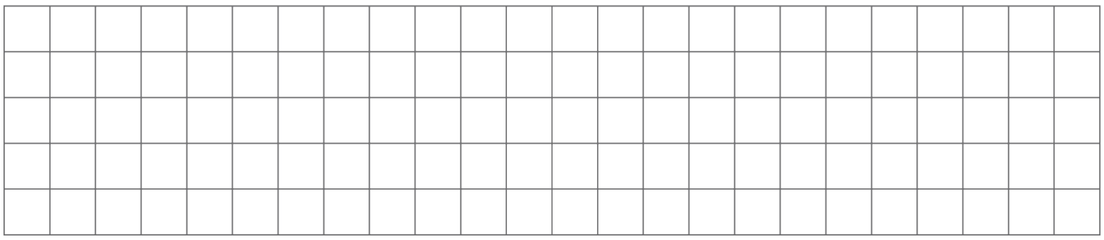
Student Task Statement

Here is a diagram of $x + 2$ and $3x$ when x is 4. Notice that the two diagrams are lined up on their left sides.

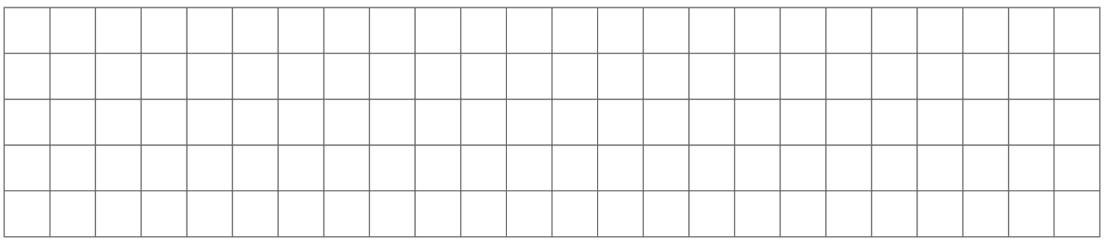


In each of your drawings below, line up the diagrams on one side.

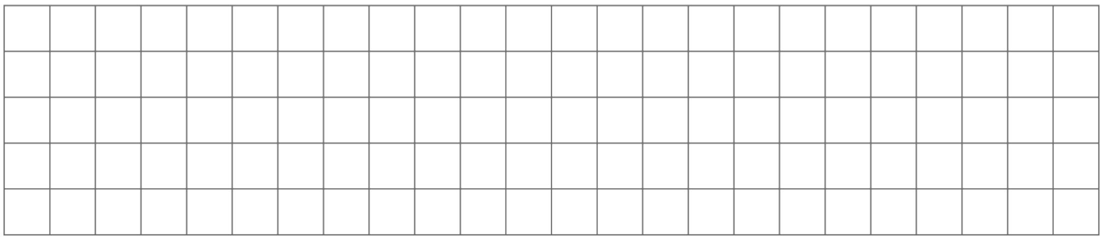
1. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 3.



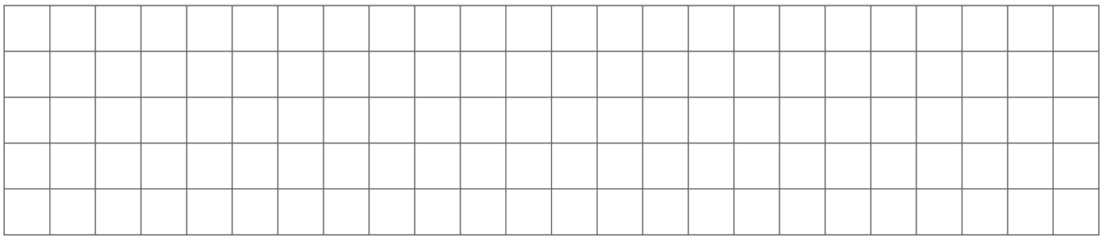
2. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 2.



3. Draw a diagram of $x + 2$, and a separate diagram of $3x$, when x is 1.



4. Draw a diagram of $x+2$, and a separate diagram of $3x$, when x is 0.



5. When are $x+2$ and $3x$ equal? When are they not equal? Use your diagrams to explain.
6. Draw a diagram of $x+3$, and a separate diagram of $3+x$.
7. When are $x+3$ and $3+x$ equal? When are they not equal? Use your diagrams to explain.

Student Response

1. Diagram shows a length of 5 for $x+2$ and 9 for $3x$.
2. Diagram shows a length of 4 for $x+2$ and 6 for $3x$.
3. Diagram shows a length of 3 for $x+2$ and 3 for $3x$.
4. Diagram shows a length of 2 for $x+2$ and 0 for $3x$.
5. They are equal when $x=1$, not equal for other values. Explanations vary. Sample response: we can check the number of boxes or the lengths to see that the expressions have equal value when $x=1$.
6. Answers vary. Diagrams should be the same length regardless of choice of x.
7. They are always equal. Answers vary. Sample response: The lengths will always be the same, even though one shows the 3 first and one shows x first.

Activity Synthesis

For the first sets of diagrams, if we consider $x+2=3x$, we can see that this is true when x is 1, but not for the other values of x that we tried. For the second set of diagrams, if we consider $x+3=3+x$ we can see that this equation is always going to be true no matter what the value of x is. We call $x+3$ and $3+x$ *equivalent expressions*, because their values are equal no matter what the value of x is.

Access for English Language Learners

Representing, Conversing, Listening: MLR8 Discussion Supports. As students share their explanations for “When are $x+2$ and $3x$ equal? When are they not equal?,” offer a sentence frame such as, “I know these expressions are equal (or not equal) when ______ because ...” Highlight diagrams that show the connection to the expressions. This will help students use mathematical language as they connect the representations of equal and not equal values of expressions.
Design Principle(s): Maximize meta-awareness

8.3 Identifying Equivalent Expressions

10 minutes
In this activity, students apply what they know about the meaning of operations and their properties to understand what is meant by “equivalent expressions.” The focus is more on building that understanding than it is about doing all the types they eventually need to be able to do.

It is expected that students will reason using what they know about operations on numbers and potentially use diagrams. For example they learned earlier this year that something $\div \frac{1}{3}$ is equivalent to that same thing $\cdot 3$. They can also reason that they know for example that $4+4+4=3\cdot 4$, so $a+a+a=3a$.

Addressing

- 6.EE.A.4

Instructional Routines

- MLR3: Clarify, Critique, Correct

Launch

Allow students 5 minutes of quiet work time, followed by a whole-class discussion.

Student Task Statement

Here is a list of expressions. Find any pairs of expressions that are equivalent. If you get stuck, try reasoning with diagrams.

$a+3$	$a\div\frac{1}{3}$	$\frac{1}{3}a$	$\frac{a}{3}$	a
$a+a+a$	$a\cdot 3$	$3a$	$1a$	$3+a$

Student Response

- $a + 3$ and $3 + a$
- $a \div \frac{1}{3}$ and $a \cdot 3$
- $a + a + a$ and $3a$ (these are also equivalent to $a \div \frac{1}{3}$ and $a \cdot 3$)
- $\frac{1}{3}a$ and $\frac{a}{3}$
- $1a$ and a

Are You Ready for More?

Below are four questions about equivalent expressions. For each one:

- Decide whether you think the expressions are equivalent.
- Test your guess by choosing numbers for x (and y, if needed).

1. Are $\dfrac{x \cdot x \cdot x \cdot x}{x}$ and $x \cdot x \cdot x$ equivalent expressions?
2. Are $\dfrac{x + x + x + x}{x}$ and $x + x + x$ equivalent expressions?
3. Are $2(x + y)$ and $2x + 2y$ equivalent expressions?
4. Are $2xy$ and $2x \cdot 2y$ equivalent expressions?

Student Response

1. Yes
2. No
3. Yes
4. No

Activity Synthesis

Invite students to share their pairs and reasoning. Include students who used diagrams.

Access for Students with Disabilities

Engagement: Develop Effort and Persistence. Break the class into small discussion groups and then invite a representative from each group to report back to the whole class. This will provide students with additional opportunities to compare strategies and hear from others.
Supports accessibility for: Language; Social-emotional skills; Attention

---

Access for English Language Learners

Conversing: MLR3 Clarify, Critique, Correct. Use this routine to give students an opportunity to clarify a possible misunderstanding from the class. Display incorrect statement, “$2xy$ and $2x \cdot 2y$ are equivalent expressions because there are two x's and two y's.” Ask students to clarify and critique this statement with a partner. Ask, "What error did this student make? Come up with a counterexample to show that these expressions are not equivalent.” This will help students make sense of and define equivalent expressions.
Design Principle(s): Optimize output (for generalization); Cultivate conversation

Lesson Synthesis

The purpose of the discussion is to ensure students understand what is meant by equivalent expressions and how they are different from expressions that are just equal for a given value of their variable. Consider giving them some equivalent expressions, and ask if they can explain why they are equivalent without drawing diagrams. Examples:

- x and $x \cdot 1$
- $x + 1$ and $1 + x$
- $x \cdot 3$ and x
- x and $x + 0$
- $x + x + x$ and $3x$
- $x \div 4$ and $\frac{1}{4}x$

8.4 Decisions About Equivalence

Cool Down: 5 minutes
Addressing

- 6.EE.A.4

Student Task Statement

Decide if the expressions in each pair are equivalent. Explain how you know.

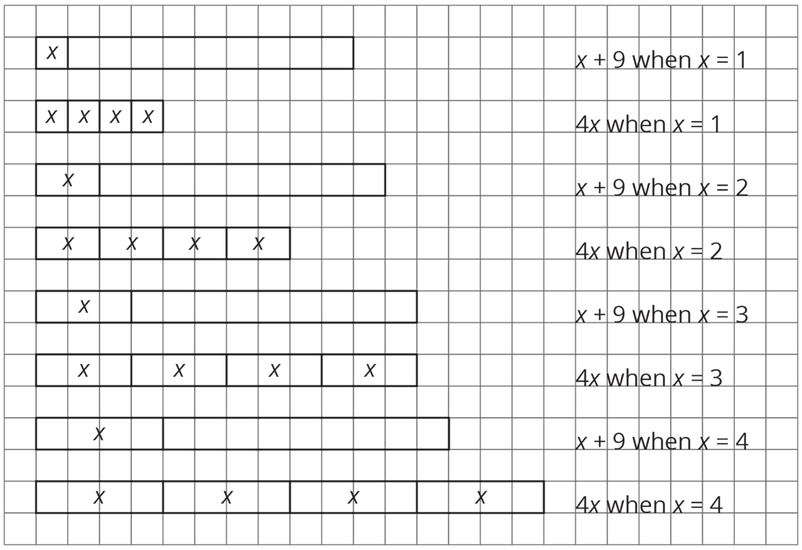
1. $x + x + x + x$ and $4x$
2. $5x$ and $x + 5$

Student Response

1. Equivalent, because the diagrams representing these expressions would have the same length for any value of x.
2. Not equivalent. For example, if $x = 1$, $5x = 5$ and $x + 5 = 6$, so they do not have the same value.

Student Lesson Summary

We can use diagrams showing lengths of rectangles to see when expressions are equal. For example, the expressions $x + 9$ and $4x$ are equal when x is 3, but are not equal for other values of x.



Sometimes two expressions are equal for only one particular value of their variable. Other times, they seem to be equal no matter what the value of the variable.

Expressions that are always equal for the same value of their variable are called **equivalent expressions**. However, it would be impossible to test every possible value of the variable. How can we know for sure that expressions are equivalent? We use the meaning of operations and properties of operations to know that expressions are equivalent. Here are some examples:

- $x + 3$ is equivalent to $3 + x$ because of the commutative property of addition.
- $4 \cdot y$ is equivalent to $y \cdot 4$ because of the commutative property of multiplication.
- $a + a + a + a + a$ is equivalent to $5 \cdot a$ because adding 5 copies of something is the same as multiplying it by 5.
- $b \div 3$ is equivalent to $b \cdot \frac{1}{3}$ because dividing by a number is the same as multiplying by its reciprocal.

In the coming lessons, we will see how another property, the distributive property, can show that expressions are equivalent.

Glossary

- equivalent expressions

Lesson 8 Practice Problems

Problem 1

Statement

a. Draw a diagram of $x + 3$ and a diagram of $2x$ when x is 1.

b. Draw a diagram of $x + 3$ and of $2x$ when x is 2.

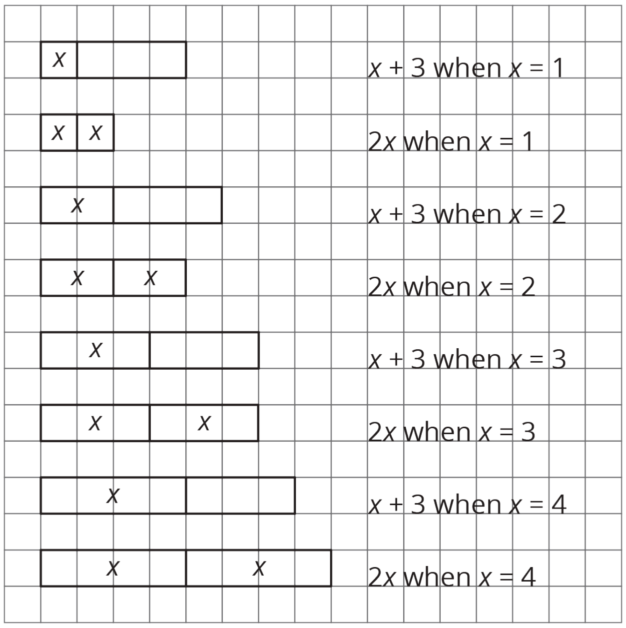
c. Draw a diagram of $x + 3$ and of $2x$ when x is 3.

d. Draw a diagram of $x + 3$ and of $2x$ when x is 4.

e. When are $x + 3$ and $2x$ equal? When are they not equal? Use your diagrams to explain.

Solution

1 through 4



5. When $x = 3$ both expressions are 6. When x is less than 3, $3x$ is less than $x + 3$ and when x is larger than 3, $3x$ becomes larger than $x + 3$.

Problem 2

Statement

a. Do $4x$ and $15 + x$ have the same value when x is 5?

b. Are $4x$ and $15 + x$ equivalent expressions? Explain your reasoning.

Solution

a. Yes, they both have the value of 20.

b. No. Equivalent expressions have the same value no matter what number is used in place of the variable. Reasoning varies. Sample reasoning For example, when x is 1, $4x$ has the value 4 but $15 + x$ has the value 16.

Problem 3

Statement

a. Check that $2b + b$ and $3b$ have the same value when b is 1, 2, and 3.

b. Do $2b + b$ and $3b$ have the same value for all values of b? Explain your reasoning.

c. Are $2b + b$ and $3b$ equivalent expressions?

Solution

a. When $b = 1$, they both take the value 3, when $b = 2$ they are both 6, and when $b = 3$ they both have the value 9.

b. Yes, $2b + b$ is the same as $3b$. Both can be written as $b + b + b$.

c. Yes, for any value of b, both $2b + b$ and $3b$ give 3 times the value of b.

Problem 4

Statement

80% of x is equal to 100.

a. Write an equation that shows the relationship of 80%, x, and 100.

b. Use your equation to find x.

Solution

a. $0.8x = 100$

b. $x = 125$

(From Unit 6, Lesson 7.)

Problem 5

Statement

For each story problem, write an equation to represent the problem and then solve the equation. Be sure to explain the meaning of any variables you use.

a. Jada's dog was $5\frac{1}{2}$ inches tall when it was a puppy. Now her dog is $14\frac{1}{2}$ inches taller than that. How tall is Jada's dog now?

b. Lin picked $9\frac{3}{4}$ pounds of apples, which was 3 times the weight of the apples Andre picked. How many pounds of apples did Andre pick?

Solution

a. $t - 14\frac{1}{2} = 5\frac{1}{2}$ or equivalent, where t represents the height of Jada's dog now. Jada's dog is 20 inches tall.

b. $9\frac{3}{4} = 3p$ or equivalent, where p represents the weight in pounds of the apples Andre picked. Andre picked $3\frac{1}{4}$ pounds of apples.

(From Unit 6, Lesson 5.)

Problem 6

Statement

Find these products.

a. $(2.3) \cdot (1.4)$

b. $(1.72) \cdot (2.6)$

c. $(18.2) \cdot (0.2)$

d. $15 \cdot (1.2)$

Solution

a. 3.22

b. 4.472

c. 3.64

d. 18

(From Unit 5, Lesson 8.)

Problem 7

Statement

Calculate $141.75 \div 2.5$ using a method of your choice. Show or explain your reasoning.

Solution

56.7. Sample reasonings:

- Multiply the dividend and divisor by 10 and calculate $1417.5 \div 25$.
- Multiply the dividend and divisor by 100 and calculate $14175 \div 250$. (Note that the fraction $\frac{14175}{250}$ can be simplified to $\frac{567}{10}$ as both the numerator and denominator are divisible by 25, but simplifying the fraction does not save time because finding $14175 \div 25$ is essentially equivalent to the problem of finding $141.75 \div 2.5$.
- Write equivalent expression using fractions ($\frac{14175}{100} \div \frac{25}{10}$) and solve by finding $\frac{14175}{100} \cdot \frac{10}{25}$, which equals $\frac{14175}{250}$.

(From Unit 5, Lesson 13.)