Lesson 10: The Distributive Property, Part 2

Goals

- Generate algebraic expressions that represent the area of a rectangle with an unknown length.
- Justify (orally and using other representations) that algebraic expressions that are related by the distributive property are equivalent.

Learning Targets

- I can use a diagram of a split rectangle to write different expressions with variables representing its area.

Lesson Narrative

The purpose of this lesson is to extend the work with the distributive property in the previous lesson to situations where one of the quantities is represented by a variable, as in $2(x+3) = 2x + 2 \cdot 3$. Students use the same rectangle diagrams as before to represent these situations, reinforcing the idea that the work they do with expressions is simply an extension of the work they previously did with numbers. They see that the distributive property can arise out of writing areas of rectangles in two different ways, which emphasizes the idea of equivalent expressions as being two different ways of writing the same quantity.

Alignments

Addressing

- 6.EE.A.2: Write, read, and evaluate expressions in which letters stand for numbers.
- 6.EE.A.3: Apply the properties of operations to generate equivalent expressions. For example, apply the distributive property to the expression $3(2+x)$ to produce the equivalent expression $6+3x$; apply the distributive property to the expression $24x+18y$ to produce the equivalent expression $6(4x+3y)$; apply properties of operations to $y+y+y$ to produce the equivalent expression $3y$.
- 6.EE.A.4: Identify when two expressions are equivalent (i.e., when the two expressions name the same number regardless of which value is substituted into them). For example, the expressions $y+y+y$ and $3y$ are equivalent because they name the same number regardless of which number y stands for.

Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports

Student Learning Goals

Let's use rectangles to understand the distributive property with variables.

10.1 Possible Areas

Warm Up: 5 minutes

Students consider the area of a rectangle to reason about equivalent expressions and review symbolic notation for showing multiplication. This work supports the activities that follow, where students will explore and apply the distributive property by considering expressions for the areas of rectangles.

Addressing

- 6.EE.A.2

Launch

Allow students 2–3 minutes of quiet work time, followed by a whole-class discussion.

Anticipated Misconceptions

If students are struggling but they haven't drawn a diagram of a rectangle, suggest that they do so.

Student Task Statement

1. A rectangle has a width of 4 units and a length of m units. Write an expression for the area of this rectangle.
2. What is the area of the rectangle if m is:

   3 units? 2.2 units? $\frac{1}{5}$ unit?

3. Could the area of this rectangle be 11 square units? Why or why not?

Student Response

1. $4m$ (or equivalent)
2. 12 square units, 8.8 square units, $\frac{4}{5}$ square units
3. Yes, the area could be 11 square units. m would have to be $\frac{11}{4}$ units, since $4 \cdot \frac{11}{4} = 11$.

Activity Synthesis

Select students to share their response to each question. Points to highlight:

- Rectangle areas can be found by multiplying length by width.
- Both $4m$ and $m \cdot 4$ are expressions for the area of this rectangle. These are equivalent expressions.

- Lengths don't have to be whole numbers. Neither do areas.

10.2 Partitioned Rectangles When Lengths are Unknown

10 minutes

In this activity, students use expressions with variables to represent lengths of sides and areas of rectangles. These expressions are used to help students understand the distributive property and its use in creating equivalent expressions.

Addressing

- 6.EE.A.3
- 6.EE.A.4

Instructional Routines

- MLR6: Three Reads

Launch

Arrange students in groups of 2–3. Allow students 5 minutes to work with their groups, followed by a quick whole-class discussion.

Access for Students with Disabilities

Action and Expression: Develop Expression and Communication. Maintain a display of important terms and vocabulary. During the launch take time to review the following terms from previous lessons that students will need to access for this activity: variable, area, length, width, expression.
Supports accessibility for: Memory; Language

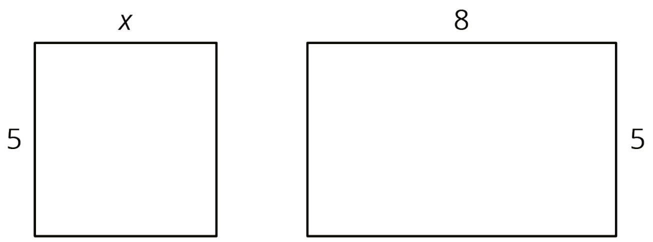
Access for English Language Learners

Reading: MLR6 Three Reads. Use this routine to support reading comprehension. In the first read, students read the situation with the goal of comprehending the text (e.g., the problem is about two rectangles with some dimensions given). In the second read, ask students to analyze the text to understand the mathematical structure (e.g., the width of both rectangles is 5. The length of one rectangle is 8 and the other rectangle's length is x). In the third read, ask students to brainstorm possible strategies to answer the follow-up questions. This routine helps students in reading comprehension and negotiating information in the text with a partner through mathematical language.
Design Principle(s): Support sense-making

Student Task Statement

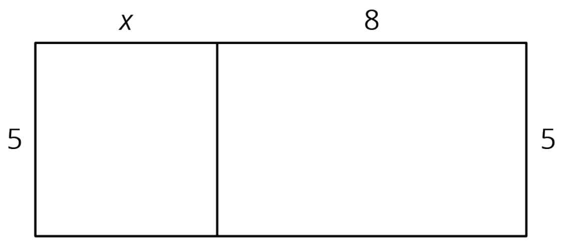
1. Here are two rectangles. The length and width of one rectangle are 8 and 5. The width of the other rectangle is 5, but its length is unknown so we labeled it x.

   Write an expression for the sum of the areas of the two rectangles.

2. The two rectangles can be composed into one larger rectangle as shown.

   What are the width and length of the new, large rectangle?

3. Write an expression for the total area of the large rectangle as the product of its width and its length.

Student Response

1. $5x + 40$ or $5x + 5 \cdot 8$
2. The length is $x + 8$ and width is 5 (or vice versa).
3. $5(x + 8)$ or $(x + 8) \cdot 5$

Activity Synthesis

Solicit students' responses to the first and third questions. Display two expressions for all to see. Expressions that are equivalent to these are fine; ensure everyone agrees that one is an acceptable response to the first question and the other is an acceptable response to the third question.

$$5 \cdot x + 5 \cdot 8$$

$$5(x + 8)$$

Ask students to look at the two expressions and think of something they notice and wonder. Here are some things that students might notice.

- The 5 appears twice in one expression and only once in the other.
- These expressions must be equivalent to each other, because they each represent the area of the same region.

- These look like an example of the distributive property, but with a letter.

10.3 Areas of Partitioned Rectangles

20 minutes

In this activity students are presented with several partitioned rectangles. They identify the length and width for each rectangle, and then write expressions for the area in two different ways: first as the product of the length times the width, where one of these measurements will be expressed as a sum, and then as the sum of the areas of the smaller rectangles that make up the large rectangle. Students reason that these two expressions must be equal since they both represent the total area of the partitioned rectangle. In this way they see several examples of the distributive property. Students may choose to assign values to the variable in each rectangle to check that their expressions for area are equal.

Addressing

- 6.EE.A.3
- 6.EE.A.4

Instructional Routines

- MLR8: Discussion Supports

Launch

Keep students in the same groups. Allow students 10 minutes to work with their groups, followed by a whole-class discussion.

Access for Students with Disabilities

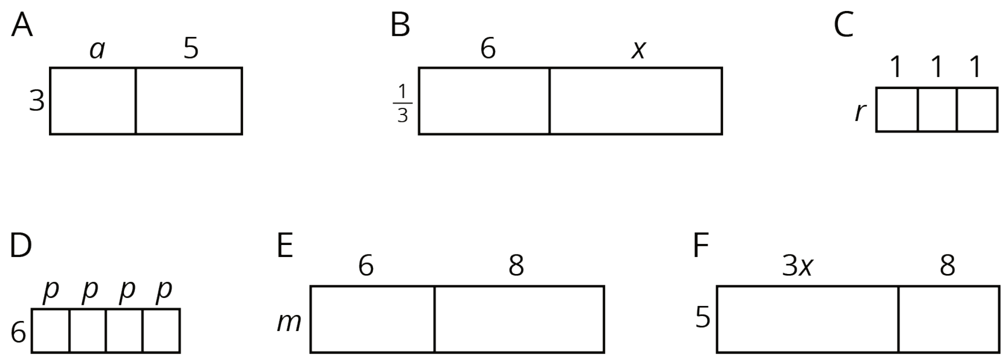
Action and Expression: Internalize Executive Functions. Begin with a small-group or whole-class demonstration and think aloud of how to complete the first row of the table to remind students how to write expressions for the length, width and total area. Keep the worked-out calculations on display for students to reference as they work.
Supports accessibility for: Memory; Conceptual processing

Access for English Language Learners

Listening, Representing: MLR8 Discussion Supports. To develop students' meta-awareness for writing equivalent expressions, demonstrate a think aloud about representing rectangle areas. As you talk, use mathematical language and highlight the connection between the written expressions and the chosen partitioned rectangle. Say, "For Rectangle A, if the width is 3, how can I write an expression for the length (e.g., $a + 5$)? If the length is represented as $a + 5$, then what is one way I can write an expression for the total area (e.g., a product of $3(a + 5)$)? What can be a second way (e.g., $3a + 3 \cdot 5$)?"
Design Principle(s): Maximize meta-awareness

Student Task Statement

For each rectangle, write expressions for the length and width and two expressions for the total area. Record them in the table. Check your expressions in each row with your group and discuss any disagreements.



rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A				
B				
C				
D				
E				
F				

Student Response

Some answers vary. Width and length can be interchanged. Sample responses:

rectangle	width	length	area as a product of width times length	area as a sum of the areas of the smaller rectangles
A	3	$a+5$	$3(a+5)$	$3a+15$
B	$\frac{1}{3}$	$6+x$ or $x+6$	$\frac{1}{3}(6+x)$	$2+\frac{1}{3}x$
C	r	3 or $1+1+1$	$r(1+1+1)$ or $3r$	$r+r+r$ or $1r+1r+1r$
D	6	$p+p+p+p$ or $4p$ as we saw above	$6(p+p+p+p)$ or $6(4p)$ or $24p$	$6p+6p+6p+6p$
E	m	$6+8$ or 14	$m(6+8)$ or $14m$	$6m+8m$
F	5	$3x+8$	$5(3x+8)$	$15x+40$

Are You Ready for More?

Here is an area diagram of a rectangle.

	y	z
w	A	24
x	18	72

1. Find the lengths w, x, y, and z, and the area A. All values are whole numbers.

2. Can you find another set of lengths that will work? How many possibilities are there?

Student Response

There are four solutions to this problem. The value of A is always 6.

- $w = 1$, $x = 3$, $y = 6$, $z = 24$, $A = 6$
- $w = 2$, $x = 6$, $y = 3$, $z = 12$, $A = 6$
- $w = 3$, $x = 9$, $y = 2$, $z = 8$, $A = 6$
- $w = 6$, $x = 18$, $y = 1$, $z = 4$, $A = 6$

Activity Synthesis

The purpose of the discussion is to help students understand the distributive property and how it can be used to generate equivalent expressions.

Remind students about the term "coefficient" and the convention of writing the coefficient before the variable. Ask students why we consider one expression for area "a sum" and the other "a product" even though both expressions contain sums and products. (For example, we take $6x + 8$ to be a sum because we are adding two terms $6x$ and 8, even though $6x$ is actually a product of 6 and x. Likewise, we take $2(3x + 4)$ to be a product because we note that 2 is multiplied by the quantity $3x + 4$, which contains a sum and a product.)

When appropriate, encourage students to use the word *term* to refer to things like $3a$, $6p$, and $15x$.

Finally, we want to make the point another way that each pair of expressions they wrote are equivalent to each other. For example, we can see that $5(3x + 8)$ and $15x + 40$ are equivalent because they both represent the area of figure G. However, we want to reinforce what equivalent means here. Ask each pair of students to choose any number, and evaluate both $5(3x + 8)$ and $15x + 40$ using the value they chose for x. No matter what value they choose, the expressions will yield the same value. For example, if they choose 2 as the value for x, $5(3 \cdot 2 + 8)$ is 70, and $15 \cdot 2 + 40$ is also 70.

Lesson Synthesis

Ask students to compare the expressions they saw today with the expressions they saw in the last lesson. How are they alike? How are they different? Students should see that their work with expressions containing variables is an extension of the work they did in the last lesson with numbers.

Invite students to share any disagreements that arose within their group and how they were resolved.

10.4 Which Expressions Represent Area?

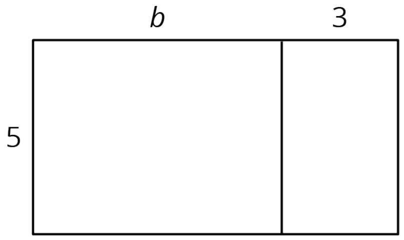
Cool Down: 5 minutes

Addressing

- 6.EE.A.3
- 6.EE.A.4

Student Task Statement

Select **all** the expressions that represent the large rectangle's total area.



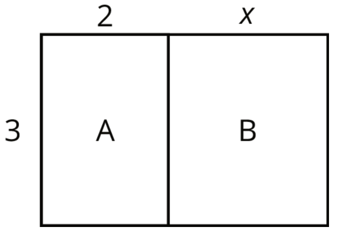
- $3(5+b)$
- $5(b+3)$
- $5b+15$
- $15+5b$
- $3 \cdot 5+3b$

Student Response

$5(b+3)$
$5b+15$
$15+5b$

Student Lesson Summary

Here is a rectangle composed of two smaller rectangles A and B.



Based on the drawing, we can make several observations about the area of the rectangle:

- One side length of the large rectangle is 3 and the other is $2+x$, so its area is $3(2+x)$.
- Since the large rectangle can be decomposed into two smaller rectangles, A and B, with no overlap, the area of the large rectangle is also the sum of the areas of rectangles A and B: $3(2)+3(x)$ or $6+3x$.

- Since both expressions represent the area of the large rectangle, they are equivalent to each other. $3(2+x)$ is equivalent to $6+3x$.

We can see that multiplying 3 by the sum $2+x$ is equivalent to multiplying 3 by 2 and then 3 by x and adding the two products. This relationship is an example of the *distributive property*.

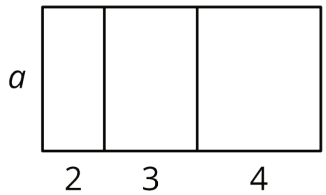
$$3(2+x) = 3 \cdot 2 + 3 \cdot x$$

Lesson 10 Practice Problems

Problem 1

Statement

Here is a rectangle.



a. Explain why the area of the large rectangle is $2a+3a+4a$.

b. Explain why the area of the large rectangle is $(2+3+4)a$.

Solution

a. The large rectangle is made up of three smaller rectangles whose areas are $2a$, $3a$, and $4a$.

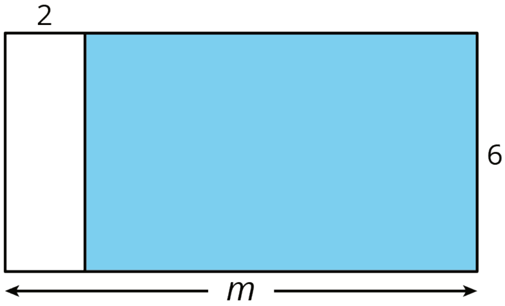
b. The large rectangle has height a and length $2+3+4$, so its area is $(2+3+4)a$.

Problem 2

Statement

Is the area of the shaded rectangle $6(2-m)$ or $6(m-2)$?

Explain how you know.



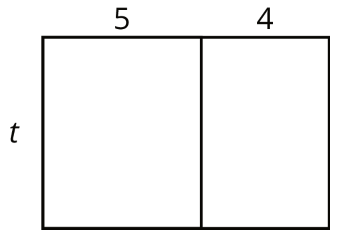
Solution

$6(m-2)$. The width of the shaded rectangle is 6. The length is what is left over if 2 is removed from m, so $m-2$. So the area of the rectangle is $6(m-2)$.

Problem 3

Statement

Choose the expressions that do *not* represent the total area of the rectangle. Select **all** that apply.



A. $5t+4t$

B. $t+5+4$

C. $9t$

D. $4 \cdot 5 \cdot t$

E. $t(5+4)$

Solution

["B", "D"]

Problem 4

Statement

Evaluate each expression mentally.

a. $35 \cdot 91 - 35 \cdot 89$

b. $22 \cdot 87 + 22 \cdot 13$

c. $\frac{9}{11} \cdot \frac{7}{10} - \frac{9}{11} \cdot \frac{3}{10}$

Solution

a. 70, Sample reasoning: $35 \cdot 91 - 35 \cdot 89 = 35 \cdot (91 - 89) = 35 \cdot 2 = 70)$

b. 2,200, Sample reasoning: $22 \cdot 87 + 22 \cdot 13 = 22 \cdot (87 + 13) = 22 \cdot 100 = 2,200$

c. $\frac{36}{110}$, Sample reasoning: $\frac{9}{11} \cdot \frac{7}{10} - \frac{9}{11} \cdot \frac{3}{10} = \frac{9}{11}\left(\frac{7}{10} - \frac{3}{10}\right) = \frac{9}{11} \cdot \frac{4}{10} = \frac{36}{110}$

(From Unit 6, Lesson 9.)

Problem 5

Statement

Select **all** the expressions that are equivalent to $4b$.

A. $b+b+b+b$

B. $b+4$

C. $2b+2b$

D. $b \cdot b \cdot b \cdot b$

E. $b \div \frac{1}{4}$

Solution

["A", "C", "E"]

(From Unit 6, Lesson 8.)

Problem 6

Statement

Solve each equation. Show your reasoning.

$111 = 14a$

$13.65 = b + 4.88$

$c + \frac{1}{3} = 5\frac{1}{8}$

$\frac{2}{5}d = \frac{17}{4}$

$5.16 = 4e$

Solution

a. $a = \frac{111}{14}$ (or equivalent)

b. $b = 8.77$

c. $c = 4\frac{19}{24}$ (or equivalent)

d. $d = \frac{85}{8}$ (or equivalent)

e. $e = 1.29$ (or equivalent)

(From Unit 6, Lesson 4.)

Problem 7

Statement

Andre ran $5\frac{1}{2}$ laps of a track in 8 minutes at a constant speed. It took Andre x minutes to run each lap. Select **all** the equations that represent this situation.

A. $\left(5\frac{1}{2}\right)x = 8$

B. $5\frac{1}{2} + x = 8$

C. $5\frac{1}{2} - x = 8$

D. $5\frac{1}{2} \div x = 8$

E. $x = 8 \div \left(5\frac{1}{2}\right)$

F. $x = \left(5\frac{1}{2}\right) \div 8$

Solution

["A", "E"]

(From Unit 6, Lesson 2.)