## Lesson 16: The Quadratic Formula

* Let’s learn a formula for finding solutions to quadratic equations.

### 16.1: Evaluate It

Each expression represents two numbers. Evaluate the expressions and find the two numbers.

### 16.2: Pesky Equations

Choose one equation to solve, either by rewriting it in factored form or by completing the square. Be prepared to explain your choice of method.

### 16.3: Meet the Quadratic Formula

Here is a formula called the **quadratic formula**.

The formula can be used to find the solutions to any quadratic equation in the form of , where , , and are numbers and is not 0.

This example shows how it is used to solve , in which , , and .

Here are some quadratic equations and their solutions. Use the quadratic formula to show that the solutions are correct.

1. . The solutions are and .
2. . The solutions are and .
3. . The solutions are .
4. . The solutions are .
5. . The solution is .
6. . The solutions are and .

#### Are you ready for more?

1. Use the quadratic formula to solve . Let’s call the resulting equation P.
2. Solve the equation in two ways, showing your reasoning for each:
   * Without using any formulas.
   * Using equation P.
3. Check that you got the same solutions using each method.
4. Use the quadratic formula to solve . Let’s call the resulting equation Q.
5. Solve the equation in two ways, showing your reasoning for each:
   * Without using any formulas.
   * Using equation Q.
6. Check that you got the same solutions using each method.

### Lesson 16 Summary

We have learned a couple of methods for solving quadratic equations algebraically:

* by rewriting the equation as and using the zero product property
* by completing the square

Some equations can be solved quickly with one of these methods, but many cannot. Here is an example: . The expression on the left cannot be rewritten in factored form with rational coefficients. Because the coefficient of the squared term is not a perfect square, and the coefficient of the linear term is an odd number, completing the square would be inconvenient and would result in a perfect square with fractions.

The **quadratic formula** can be used to find the solutions to any quadratic equation, including those that are tricky to solve with other methods.

For an equation of the form , where , , and are numbers and , the solutions are given by:

For the equation , we see that , , and . Let’s solve it!

A calculator gives approximate solutions of 0.84 and -0.24 for and .

We can also use the formula for simpler equations like , but it may not be the most efficient way. If the quadratic expression can be easily rewritten in factored form or made into a perfect square, those methods may be preferable. For example, rewriting as immediately tells us that the solutions are 1 and 8.



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