# Lesson 4: Comparing Proportional Relationships

## Goals

- Compare the rates of change for two proportional relationships, given multiple representations.
- Interpret multiple representations of a proportional relationship in order to answer questions (in writing), and explain the solution method.
- Present a comparison of two proportional relationships (using words and multiple other representations).

## Learning Targets

- I can compare proportional relationships represented in different ways.

## Lesson Narrative

In this fourth lesson on proportional relationships, students expand on the work of the previous lesson by comparing two situations that are represented in different ways. For example, one situation might specify a rate of change, while the other is represented by a table of values, a graph, or an equation. Students move flexibly between representations and consider how to find the information they need from each type. They respond to context-related questions that compare the two situations and solve problems with the information they've garnered from each representation.

### Alignments

### Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

### Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

### Required Materials

**Tools for creating a visual display**
Any way for students to create work that can be easily displayed to the class. Examples: chart paper and markers, whiteboard space and markers, shared online drawing tool, access to a document camera.

**Student Learning Goals**

Let's compare proportional relationships.

# 4.1 What's the Relationship?

**Warm Up: 10 minutes**

The purpose of this warm-up is for students to create a graph and a description from an equation, building on their work in the previous lesson. Students decide on a context and then make the graph, scaling the axes appropriately to the situation. Moving between representations of a proportional relationship here is preparation for the following activity where students compare proportional relationships represented in different ways.

## Addressing

- 8.EE.B

## Launch

Arrange students in groups of 2. Give 2–3 minutes of quiet work time followed by a whole-class discussion.

**Student Task Statement**

The equation $y = 4.2x$ could represent a variety of different situations.

1. Write a description of a situation represented by this equation. Decide what quantities $x$ and $y$ represent in your situation.
2. Make a table and a graph that represent the situation.

## Student Response

1. Answers vary. Sample response: A frog jumps 16.8 feet in 4 seconds.
2. Graph:

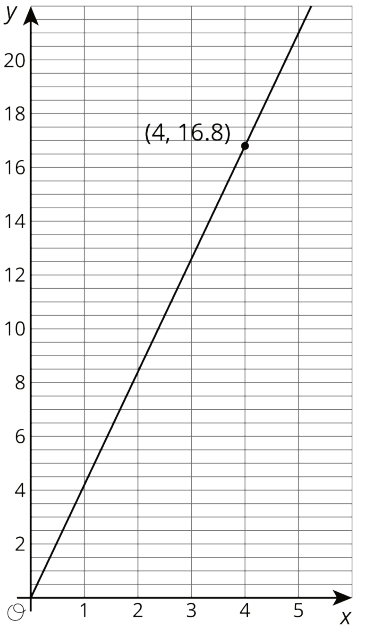


Table:

| $x$ | $y$ |
|---|---|
| 3 | 12.60 |
| 4 | 16.80 |

**Activity Synthesis**

Invite several students to share their situations and display their graphs for all to see. Ask:

- "What does the rate of change represent in this situation?"
- "How did you decide on the scale for your axes?"

# 4.2 Comparing Two Different Representations

**25 minutes**

The purpose of this activity is for students to compare two different proportional relationships represented in different ways using the skills they have worked on over the past three lessons. Working in groups, students compare the relationships, responding to questions about their rate of change, which rate of change is higher, and one other situation-based question. Groups make a visual display for their problem set to explain each of their responses and convince others of their accuracy.

Identify groups using a variety of representations to share during the Activity Synthesis.

### Addressing

- 8.EE.B.5

### Instructional Routines

- Group Presentations
- MLR7: Compare and Connect

### Launch

Remind students that in previous lessons they identified representations of and created representations for a single proportional relationship. In this activity, they will consider representations of two *different* proportional relationships and make comparisons between them.

Arrange students in groups of 2–3. Assign to each group (or ask groups to choose) one of the three question sets. Tell groups that they will make a visual display for their responses to the questions. The display should clearly demonstrate their reasoning and use multiple representations in order to be convincing. Let them know that there will be a gallery walk when they finish for the rest of the class to inspect their solutions' accuracy.

If time allows, ask groups to complete all three problems and make a visual display for just one.

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**Access for Students with Disabilities**

*Action and Expression: Internalize Executive Functions.* To support development of organizational skills in problem-solving, chunk this task into more manageable parts. For example, present one question at a time.
*Supports accessibility for: Memory; Organization*

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### Anticipated Misconceptions

Some students may confuse the values for the rate of change of a situation. For example, Lemonade Recipe 1's equation, $y = 4x$, shows that the rate of change is 4 cups of water per cup of lemonade mix. Students may switch these values and think that the rate of change is 4 cups lemonade mix per cup of water. Ask students who do this to explain where in the original representations they see the rate of change. Students may need to list a few values or sketch a graph in order to see their mix-up between the two quantities.

### Student Task Statement

1. Elena babysits her neighbor's children. Her earnings are given by the equation $y = 8.40x$, where $x$ represents the number of hours she worked and $y$ represents the amount of money she earned.

   Jada earns $7 per hour mowing her neighbors' lawns.

a. Who makes more money after working 12 hours? How much more do they make? Explain your reasoning by creating a graph or a table.

b. What is the rate of change for each situation and what does it mean?

c. Using your graph or table, determine how long it would take each person to earn $150.

2. Clare and Han have summer jobs stuffing envelopes for two different companies.

Han earns $15 for every 300 envelopes he finishes.

Clare's earnings can be seen in the table.

| number of envelopes | money in dollars |
|---|---|
| 400 | 40 |
| 900 | 90 |

a. By creating a graph, show how much money each person makes after stuffing 1,500 envelopes.

b. What is the rate of change for each situation and what does it mean?

c. Using your graph, determine how much more money one person makes relative to the other after stuffing 1,500 envelopes. Explain or show your reasoning.

3. Tyler plans to start a lemonade stand and is trying to perfect his recipe for lemonade. He wants to make sure the recipe doesn't use too much lemonade mix (lemon juice and sugar) but still tastes good.

Lemonade Recipe 1 is given by the equation $y = 4x$ where $x$ represents the amount of lemonade mix in cups and $y$ represents the amount of water in cups.

Lemonade Recipe 2 is given in the table.

| lemonade mix (cups) | water (cups) |
|---|---|
| 10 | 50 |
| 13 | 65 |
| 21 | 105 |

a. If Tyler had 16 cups of lemonade mix, how many cups of water would he need for each recipe? Explain your reasoning by creating a graph or a table.

b. What is the rate of change for each situation and what does it mean?

c. Tyler has a 5-gallon jug (which holds 80 cups) to use for his lemonade stand and 16 cups of lemonade mix. Which lemonade recipe should he use? Explain or show your reasoning.

### Student Response

1. a. Elena will make $16.80 more than Jada. Answers vary. Sample response: A graph showing hours worked along the $x$-axis and money earned along the $y$-axis. The graph scale is large enough to show the points on the lines representing each situation at 12 hours. The difference between the $y$-coordinates of these two points is $16.80.

   b. Answers vary. Sample response: Elena: $8.40 per hour, Jada: $7 per hour. These rates of change tell us how much money Elena and Jada make for each hour they work.

   c. Answers vary. Sample response: Elena: about 18 hours, Jada: about 21.5 hours, if she gets paid for half hours or 22 hours if she is paid only in whole hour increments.

2. a. Clare earns $150.00 and Han earns $75.00. Answers vary. Sample response: A graph showing hours worked along the $x$-axis and money earned along the $y$-axis. The graph scale is large enough to show the points on the lines representing each situation at 1,500 envelopes.

   b. Answers vary. Sample response: Clare: $0.10 per envelope, Han: $0.05 per envelope. These rates of change tell us how much money they make for each envelope they finish.

   c. Clare earns $75.00 more than Han. Explanations vary. Sample explanation: At 1,500 envelopes, the difference between the $y$-coordinates of these two points is $75.00.

3. a. Recipe 1: 64 cups of water, Recipe 2: 80 cups of water. Answers vary. Sample response: A graph showing cups of lemonade mix along the $x$-axis and cups of water along the $y$-axis. The graph scale is large enough to show the points on the lines representing each situation at 16 cups of lemonade mix.

   b. Recipe 1: 4 cups of water per cup of lemonade mix, Recipe 2: 5 cups of water per cup of lemonade mix. Each rate of change tells how much water is needed per cup of lemonade mix.

   c. Answers vary. Sample response: Either recipe could be used to fill the 5 gallon jug with lemonade if there are 16 cups of mix. Recipe 1 will use all 16 cups of mix and 64 cups of water since there are 4 cups of water per cup of lemonade mix. Recipe 1 would use $13\frac{1}{3}$ cups mix and $16\frac{2}{3}$ cups water since there are 5 cups of water per cup of lemonade mix. We think he should use recipe 2, because then he will have $2\frac{2}{3}$ cup lemonade mix leftover to use at his stand the next day.

### Are You Ready for More?

Han and Clare are still stuffing envelopes. Han can stuff 20 envelopes in a minute, and Clare can stuff 10 envelopes in a minute. They start working together on a pile of 1,000 envelopes.

1. How long does it take them to finish the pile?

2. Who earns more money?

### Student Response

1. Working together they can stuff 30 envelopes per minute, so it takes them $\frac{1000}{30} = 33\frac{1}{3}$ minutes to finish the pile.

2. Han stuffs twice as many envelopes as Clare, but he only earns half as much, so they both earn the same amount of money.

### Activity Synthesis

Begin with a gallery walk for students to see how other groups answered the same set of questions they did and how students answered questions about the other two contexts.

Invite groups to share the strategies they used with the various representations. Consider asking groups the following questions:

- "What representations did you choose to answer the questions? Why did you pick them?"
- "What representation did you not use? Why?"
- "How did you decide what scale to use when you made your graph?"
- "Now that you have seen the work of other groups, is there anything about your display you would change if you could?"

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#### Access for English Language Learners

*Representing and Conversing: MLR7 Compare and Connect.* During the gallery walk, invite students to discuss "what is the same and what is different?" about the representations on the posters and then share with the whole class. Look for opportunities to highlight representations that helped students answer the questions and decide which scales to use for the graph. This will help students make connections and describe the usefulness of each type of representation. *Design Principle(s): Optimize output*

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## Lesson Synthesis

This lesson asked students to take a single piece of information about a proportional relationship, such as an equation, and use what they know about proportional relationships, rates of change, and representing relationships to compare it with a second proportional relationship in context.

Consider asking some of the following questions. Tell students to use, if possible, examples from today's lesson when responding:

- "What do you need in order to compare two proportional relationships?"

- "What type of wording in a problem statement or description of a situation tells you that you have a rate of change?"
- "How did you decide which representation to use to solve the different types of problems?"

# 4.3 Different Salt Mixtures

**Cool Down: 5 minutes**

**Addressing**

- 8.EE.B.5

### Student Task Statement

Here are recipes for two mixtures of salt and water that taste different.

Information about Salt Mixture A is shown in the table.

| salt (teaspoons) | water (cups) |
|---|---|
| 4 | 5 |
| 7 | $8\frac{3}{4}$ |
| 9 | $11\frac{1}{4}$ |

Salt Mixture B is defined by the equation $y = 2.5x$, where $x$ is the number of teaspoons of salt and $y$ is the number of cups of water.

1. If you used 10 cups of water, which mixture would use more salt? How much more? Explain or show your reasoning.
2. Which mixture tastes saltier? Explain how you know.

### Student Response

1. Mixture A uses 4 more teaspoons of salt than Mixture B. Mixture A would use 8 teaspoons of salt because I can double the row with 4 and 5 to get 8 and 10. Mixture B would use 4 teaspoons of salt because considering if $10 = 2.5x$, the value of $x$ must be 4.
2. Mixture A tastes saltier because it uses more salt for the same amount of water. Mixture A uses 1.25 cups of water per teaspoon of salt while Mixture B uses 2.5 cups of water per teaspoon of salt.

## Student Lesson Summary

When two proportional relationships are represented in different ways, we compare them by finding a common piece of information.

For example, Clare's earnings are represented by the equation $y = 14.5x$, where $y$ is her earnings in dollars for working $x$ hours.

The table shows some information about Jada's pay.

| time worked (hours) | earnings (dollars) |
|---|---|
| 7 | 92.75 |
| 4.5 | 59.63 |
| 37 | 490.25 |

Who is paid at a higher rate per hour? How much more does that person have after 20 hours?

In Clare's equation we see that the rate of change (how many dollars she earns every hour) is 14.50.

We can calculate Jada's rate of change by dividing a value in the earnings column by the value in the same row in the time worked column. Using the last row, the rate of change for Jada is 13.25, since $490.25 \div 37 = 13.25$. An equation representing Jada's earnings is $y = 13.25x$. This means she earns $13.25 per hour.

So Clare is paid at a higher rate than Jada. Clare earns $1.25 more per hour than Jada. After 20 hours of work, she earns $25 more than Jada because $20 \cdot (1.25) = 25$.

# Lesson 4 Practice Problems

## Problem 1

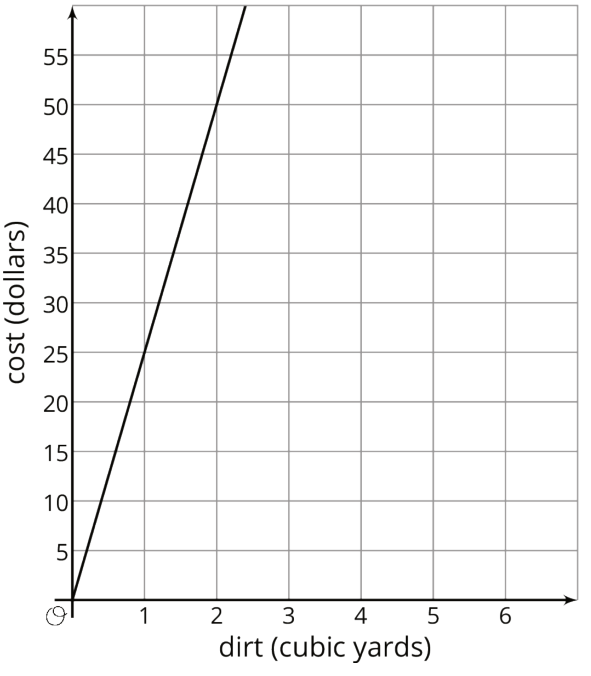
### Statement

A contractor must haul a large amount of dirt to a work site. She collected information from two hauling companies.

EZ Excavation gives its prices in a table.

| dirt (cubic yards) | cost (dollars) |
|---|---|
| 8 | 196 |
| 20 | 490 |
| 26 | 637 |

Happy Hauling Service gives its prices in a graph.




a. How much would each hauling company charge to haul 40 cubic yards of dirt? Explain or show your reasoning.

b. Calculate the rate of change for each relationship. What do they mean for each company?

c. If the contractor has 40 cubic yards of dirt to haul and a budget of $1000, which hauling company should she hire? Explain or show your reasoning.

## Solution

a. Assuming that both pricing plans are proportional relationships, EZ Excavation: $980, Happy Hauling Service: $1000.

b. EZ Excavation: $24.50/cu yd, Happy Hauling Service: $25/cu yd.

c. EZ Excavation. It would cost $980 and be under budget.

# Problem 2

## Statement

Andre and Priya are tracking the number of steps they walk. Andre records that he can walk 6000 steps in 50 minutes. Priya writes the equation $y = 118x$, where $y$ is the number of steps and $x$ is the number of minutes she walks, to describe her step rate. This week, Andre and Priya each walk for a total of 5 hours. Who walks more steps? How many more?

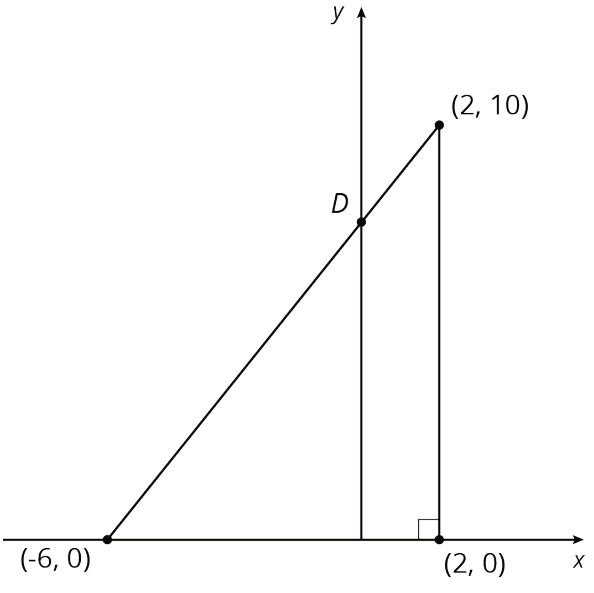
## Solution

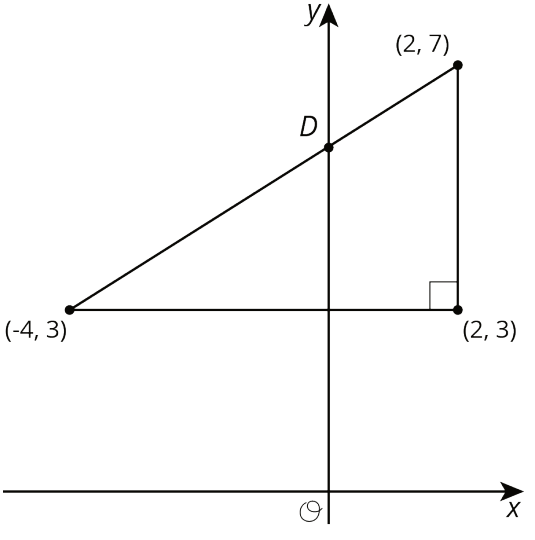
Andre walks 600 more steps than Priya.

## Problem 3

### Statement

Find the coordinates of point $D$ in each diagram:




## Solution

$(0, 7\frac{1}{2})$, $(0, 5\frac{2}{3})$

(From Unit 2, Lesson 11.)

## Problem 4

### Statement

Select **all** the pairs of points so that the line between those points has slope $\frac{2}{3}$.

A. $(0, 0)$ and $(2, 3)$

B. $(0, 0)$ and $(3, 2)$

C. $(1, 5)$ and $(4, 7)$

D. $(-2, -2)$ and $(4, 2)$

E. $(20, 30)$ and $(-20, -30)$

## Solution

["B", "C", "D"]

(From Unit 2, Lesson 11.)