Lesson 6: More Linear Relationships

Goals

- Describe (orally and in writing) how the slope and vertical intercept influence the graph of a line.
- Identify and interpret the positive vertical intercept of the graph of a linear relationship.

Learning Targets

- I can interpret the vertical intercept of a graph of a real-world situation.
- I can match graphs to the real-world situations they represent by identifying the slope and the vertical intercept.

Lesson Narrative

The previous lesson looked in depth at an example of a linear relationship that was not proportional and then examined an interpretation of the slope as the rate of change for a linear relationship. In this lesson, slope remains important. In addition, students learn the new term **vertical intercept** or y-intercept for the point where the graph of the linear relationship touches the y-axis.

In the first activity, students match situations to graphs and then interpret different features of the graph (slope and y-intercept) in terms of the situation being modeled (MP2). In the second activity, students analyze a common error, studying what happens when the slope and y-intercept are interchanged. This provides an opportunity to see how the y-intercept and slope influence the shape and location of a line: the y-intercept indicates where the line meets the y-axis while the slope determines how steep the line is.

Interpreting features of a graph or an equation in terms of a real-world context is an important component of mathematical modeling (MP4).

Alignments

Building On

- 5.OA.B.3: Generate two numerical patterns using two given rules. Identify apparent relationships between corresponding terms. Form ordered pairs consisting of corresponding terms from the two patterns, and graph the ordered pairs on a coordinate plane. For example, given the rule “Add 3” and the starting number 0, and given the rule “Add 6” and the starting number 0, generate terms in the resulting sequences, and observe that the terms in one sequence are twice the corresponding terms in the other sequence. Explain informally why this is so.

- 7.RP.A.2.a: Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.

Addressing

- 8.EE.B: Understand the connections between proportional relationships, lines, and linear equations.
- 8.EE.B.5: Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

Instructional Routines

- MLR6: Three Reads
- MLR7: Compare and Connect
- MLR8: Discussion Supports

Required Materials

Pre-printed cards, cut from copies of the blackline master

Required Preparation

Print and cut up slips from the Slopes, Vertical Intercepts, and Graphs blackline master. Prepare 1 set of cards for every 2 students.

> **Student Learning Goals**
>
> Let's explore some more relationships between two variables.

6.1 Growing

Warm Up: 5 minutes

This warm-up encourages students to look for regularity in how the number of tiles in the diagram are growing. This relates naturally to the work that they are doing with understanding the linear relationships as two of the three patterns students are likely to observe are linear and, in fact, proportional.

Building On

- 5.OA.B.3

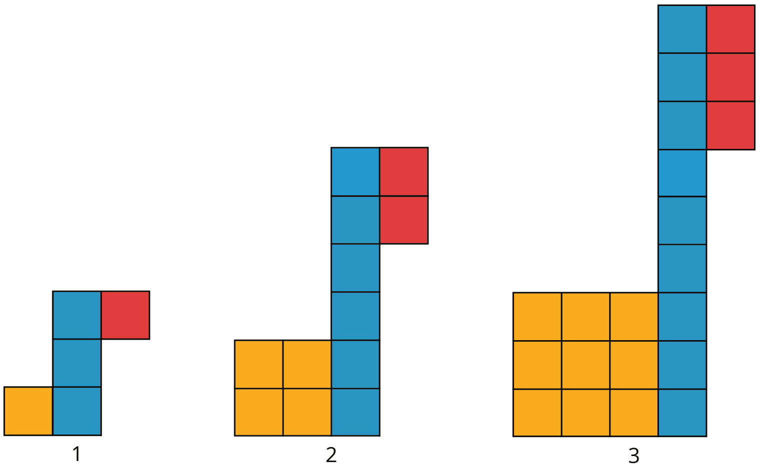
Launch

Arrange students in groups of 2. Display the image for all to see and ask students to look for a pattern in the way the collection of red, blue, and yellow tiles are growing. Ask how many tiles of each color will be in the 4th, 5th, and 10th diagrams if the diagrams keep growing in the same way.

Tell students to give a signal when they have an answer and strategy. Give students 1 minute of quiet think time, and then time to discuss their responses and reasoning with their partner.

Student Task Statement

Look for a growing pattern. Describe the pattern you see.



1. If your pattern continues growing in the same way, how many tiles of each color will be in the 4th and 5th diagram? The 10th diagram?
2. How many tiles of each color will be in the nth diagram? Be prepared to explain how your reasoning.

Student Response

1. 4th diagram: 16 yellow, 12 blue, 4 red; 5th image: 25 yellow, 15 blue, 5 red; 10th image: 100 yellow, 30 blue, 10 red
2. Yellow: n^2, Blue: $3n$, Red: n.

Activity Synthesis

Invite students to share their responses and reasoning. Record and display the different ways of thinking for all to see. If possible, record the relevant reasoning on or near the images themselves. After each explanation, ask the class if they agree or disagree and to explain alternative ways of thinking, referring back to what is happening in the images each time.

Time permitting, ask students which patterns represent a linear relationship? Which ones represent a proportional relationship? The patterns for the blue and red blocks are both proportional (hence linear) while the pattern for the yellow blocks is neither proportional nor linear.

6.2 Slopes, Vertical Intercepts, and Graphs

20 minutes

In the previous lesson, students analyzed the graph of a linear, non-proportional relationship (number of cups in a stack versus the height of the stack). This task focuses on interpreting the slope of a graph and where it crosses the y-axis in context. Students are given cards describing situations with a given rate of change and cards with graphs. Students match each graph with a situation it could represent, and then use the context to interpret the meaning of the slope. They find where the line crosses the vertical axis, i.e., the **vertical intercept**, and interpret its meaning in each situation. They also decide if the two quantities in each situation are in a proportional relationship.

Make sure that students draw the triangle they use to compute the slope. There are strategic choices that can be made to make the computation easier and more precise. Watch for students who use different triangles for the same slope computation, and ask them to share their reasoning during the whole-group discussion.

In the whole-group discussion at the end of the task, discuss and emphasize the meaning of the terms slope and vertical intercept or y-intercept (in situations where the name of the variable graphed on the vertical axis is y).

You will need the Slopes and Graphs blackline master for this lesson.

Building On

- 7.RP.A.2.a

Addressing

- 8.EE.B

Instructional Routines

- MLR7: Compare and Connect

Launch

Tell students that they will match a set of cards describing different relationships with a set of cards showing graphs of lines. The axes on the graphs are not labeled (since this could be used as an aid in the matching). Instruct students to add labels to the axes as they make their matches.

Arrange students in groups of 2. Distribute a set of 12 cards to each group. 10 minutes of group work and then whole-class discussion.

> ### Student Task Statement
>
> Your teacher will give you 6 cards describing different situations and 6 cards with graphs.
>
> 1. Match each situation to a graph.
> 2. Pick one proportional relationship and one non-proportional relationship and answer the following questions about them.
>
>   a. How can you find the slope from the graph? Explain or show your reasoning.
>
>   b. Explain what the slope means in the situation.

c. Find the point where the line crosses the vertical axis. What does that point tell you about the situation?

Student Response

A:

1. Graph 2.
2. y increases by 10 when x increases by 1.
3. The slope is the cost per month of the streaming service.
4. The line crosses the vertical axis at $(0, 40)$. The tablet costs $40.
5. Not proportional.

B:

1. Graph 6.
2. The graph passes through the points $(0, 0)$ and $(5, 20)$.
3. Every increase of 1 in side length adds 4 to the perimeter.
4. Line crosses vertical axis at $(0, 0)$. The perimeter of a square of side length 0 is 0.
5. Proportional.

C:

1. Graph 1.
2. y increases by 5 when x increases by 1.
3. Diego puts in $5 each week.
4. Line crosses vertical axis at $(0, 10)$. There initially was $10 in the piggy bank.
5. Not proportional.

D:

1. Graph 3.
2. y increases by 15 when x increases by 1.
3. Noah adds $15 each week.
4. Line crosses vertical axis at $(0, 40)$. He started with $40 in the piggy bank.
5. Not proportional.

E:

1. Graph 5.
2. y increases by 0.25 when x increases by 1.
3. The amount of money she added per day was $0.25.
4. Line crosses vertical axis at $(0, 9)$. There were initially $9 in the piggy bank.
5. Not proportional.

F:

1. Graph 4.
2. y increases by 40 when x increases by 1.
3. Lin's mom pays $40 each month for internet service.
4. Line crosses vertical axis at $(0, 0)$. Lin's mom paid no money before the contract started.
5. Proportional.

Activity Synthesis

The slopes of the 6 lines given on the situation card are all different, so the matching part of the task can be accomplished by examining the slopes of the different lines. Invite students who have made strategic choices of slope triangles for calculating the slopes (for example, Graph 6 contains the points $(0, 0)$ and $(5, 20)$, which give a value of $\frac{20}{5}$ for the slope), and ask them to share.

Next, focus the discussion on the interpretation of the point where the line crosses (or touches) the y-axis. For some situations, the contextual meaning of this point is abstract. For example, in Situation B, a square with side length 0 is just a point that has no perimeter and so it "makes sense" that $(0, 0)$ is on the graph. Some students may argue that a point is not a square at all but if we consider it to be a square then it definitely has 0 perimeter. In other situations, the point where the line touches the y-axis has a very natural meaning. For example, in Situation A, it is the amount Lin's dad spent on the tablet and 0 months of service: so this is the cost of the tablet.

Define the **vertical intercept** or y-intercept as the point where a line crosses the y-axis. Note that sometimes "y-intercept" refers to the numerical value of the y-coordinate where the line crosses the y-axis. Go over each of the situations and ask students for the meaning of the vertical intercept in the situation (A: cost of the device, B: perimeter of a square with 0 side length, C: amount of money in Diego's piggy bank before he started adding $5 each week, D: money Noah saved helping his neighbor, E: amount of money in Elena's piggy bank before she started adding money, F: amount Lin's mom has paid for internet service before her service begins.

Access for Students with Disabilities

Representation: Develop Language and Symbols. Create a display of important terms and vocabulary. Include the following terms and maintain the display for reference throughout the unit: vertical intercept, y-intercept. Invite students to suggest language or diagrams to include on the display that will support their understanding of these terms.
Supports accessibility for: Memory; Language

Access for English Language Learners

Representing, Conversing: MLR7 Compare and Connect. As students share their matches with the class, call students' attention to the different ways the vertical intercept is represented graphically and within the context of each situation. Take a close look at Graphs 2 and 3 to distinguish what the 40 represents in each corresponding situation. Wherever possible, amplify student words and actions that describe the correspondence between specific features of one mathematical representation and a specific feature of another representation.
Design Principle(s): Maximize meta-awareness; Support sense-making

6.3 Summer Reading

10 minutes
Students have just learned the meaning of the y-intercept for a line and have been interpreting slope in context. In this activity, they investigate the y-intercept and slope together and investigate what happens when their values are switched.

In contexts like this one, the y-intercept and the slope come with natural units and understanding this can help graph accurately. Specifically, the y-intercept is the number of pages Lin read before she starts gathering data for the graph. The slope, on the other hand, is a rate: it's the number of pages Lin reads *per day*.

Watch for students who understand the source of Lin's error (confusing the y-intercept with the slope) and invite them to share this observation during the discussion.

Addressing

- 8.EE.B

Instructional Routines

- MLR6: Three Reads
- MLR8: Discussion Supports

Launch

Work time followed by whole-class discussion.

Access for Students with Disabilities

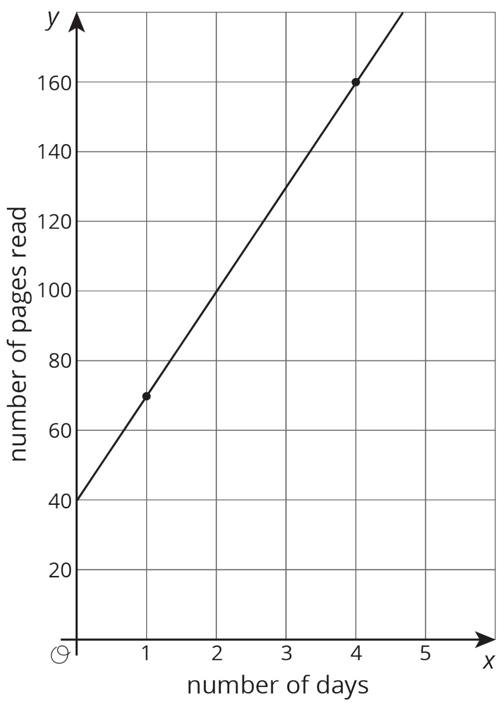
Representation: Internalize Comprehension. Use MLR6 Three Reads to supporting reading comprehension of the word problem. Use the first read to orient students to the situation. Ask students to describe what the situation is about without using numbers (Lin's reading assignment). Use the second read to identify quantities and relationships. Ask students what can be counted or measured without focusing on the values. Listen for and amplify the important quantities that vary in relation to each other in this situation: first 30 pages and 40 pages each day. After the third read, ask students to brainstorm possible strategies to answer the question, "What does the vertical intercept mean in this situation?"
Supports accessibility for: Language; Conceptual processing

Access for English Language Learners

Speaking: MLR8 Discussion Supports. Use this to amplify mathematical uses of language to communicate about vertical intercepts, slope, and constant rate. Invite students to use these words when describing their ideas. Ask students to chorally repeat phrases that include these words in context.
Design Principle(s): Support sense-making, Optimize output (for explanation)

Student Task Statement

Lin has a summer reading assignment. After reading the first 30 pages of the book, she plans to read 40 pages each day until she finishes. Lin makes the graph shown here to track how many total pages she'll read over the next few days.



After day 1, Lin reaches page 70, which matches the point $(1, 70)$ she made on her graph. After day 4, Lin reaches page 190, which does not match the point $(4, 160)$ she made on her graph. Lin is not sure what went wrong since she knows she followed her reading plan.

1. Sketch a line showing Lin's original plan on the axes.
2. What does the **vertical intercept** mean in this situation? How do the vertical intercepts of the two lines compare?
3. What does the slope mean in this situation? How do the slopes of the two lines compare?

Student Response

1. The graph should start at $(0, 30)$ since Lin read 30 pages before monitoring her progress each day. It should go through $(1, 70)$, because she reads 40 pages each day after the first. So the graph will also contain $(2, 110)$, $(3, 150)$, etc.
2. The vertical intercept is the number of pages Lin has read before she monitors her progress. Lin’s graph shows 40 pages but this is not accurate. She had only read 30 pages before beginning to track her progress.
3. The slope is the number of pages Lin reads per day (after she starts to record her progress). The slope of the line Lin drew is 30. This is not correct since she is reading 40 pages per day. The correct slope is 40.

Are You Ready for More?

Jada's grandparents started a savings account for her in 2010. The table shows the amount in the account each year.

If this relationship is graphed with the year on the horizontal axis and the amount in dollars on the vertical axis, what is the vertical intercept? What does it mean in this context?

year	amount in dollars
2010	600
2012	750
2014	900
2016	1050

Student Response

The vertical intercept corresponds to the amount in the year 0. The amount goes up by $150 every two years, which is the same as $75 per year. If you extend the graph backwards to the year zero, it goes down by $2010 \times 75 = 150,750$ from its value in 2010, so the vertical intercept would be 600, the amount in 2010, minus 150,750, which is $-150{,}150$. You could think of this as a balance of the account in the year zero, but it doesn't really make sense to extend the graph back that far because the account was not open then. The intercept has no useful meaning in this context.

Activity Synthesis

Ask students:

- "How did your graph compare to Lin's?" (It is steeper but starts off at 30 instead of 40 pages.)
- "Which point does your graph have in common with Lin's?" (The point $(1, 70)$, 70 pages read after one day.)

Invite selected students to share what's the likely source of Lin's error (confusing the y-intercept with the slope). In this context, the y-intercept is the number of pages Lin read before she starts counting the days (30), and the slope is the number of pages Lin reads per day (40).

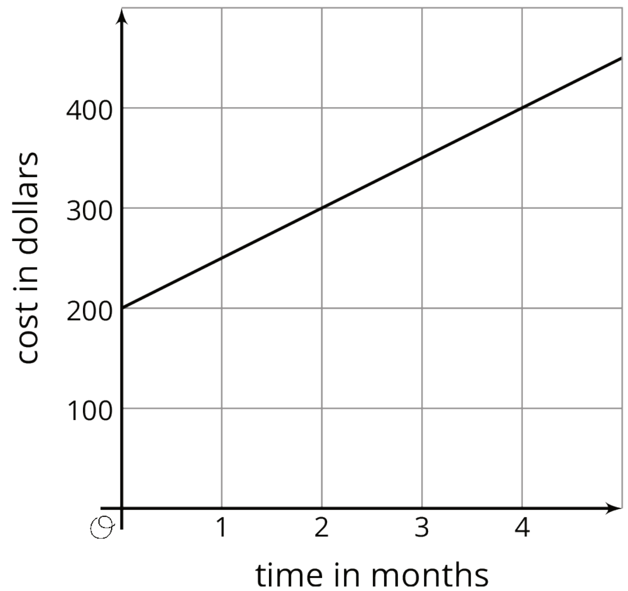
Emphasize how the y-intercept and slope influence the graph of a line.

- The y-intercept indicates where the line touches or crosses the y-axis.
- The slope indicates how steep the line is.

Lesson Synthesis

Lines have a slope and **vertical intercept**. The vertical intercept indicates where the line meets the y-axis. For example, a line represents a proportional relationship when the vertical intercept is 0.

Here is a graph of a line showing the amount of money paid for a new cell phone and monthly plan:



- “What is the vertical intercept for the graph?” ($(0, 200)$)
- “What does it mean?” (There was an initial cost of $200 for the phone.)

The slope of the line is 50 (draw a slope triangle connecting the points such as $(0, 200)$ and $(2, 300)$). This means that the phone service costs $50 per month in addition to the initial $200 for the phone.

6.4 Savings

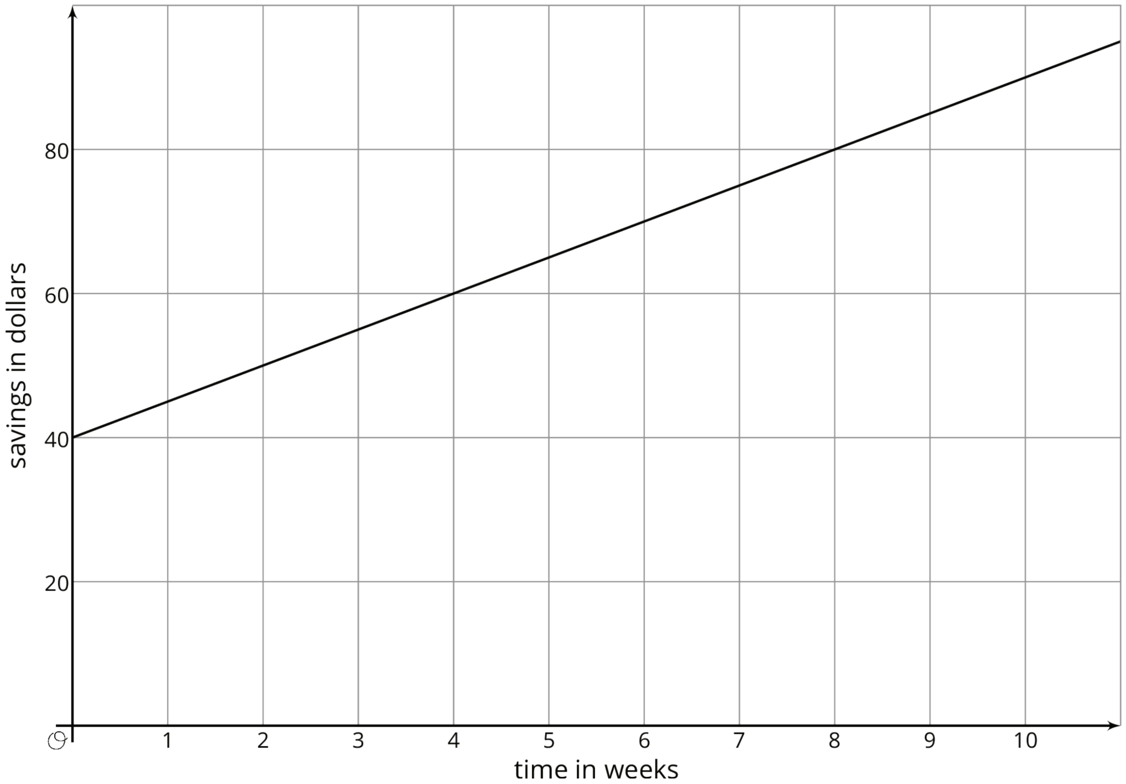
Cool Down: 5 minutes

Addressing

- 8.EE.B.5

Student Task Statement

The graph shows the savings in Andre’s bank account.



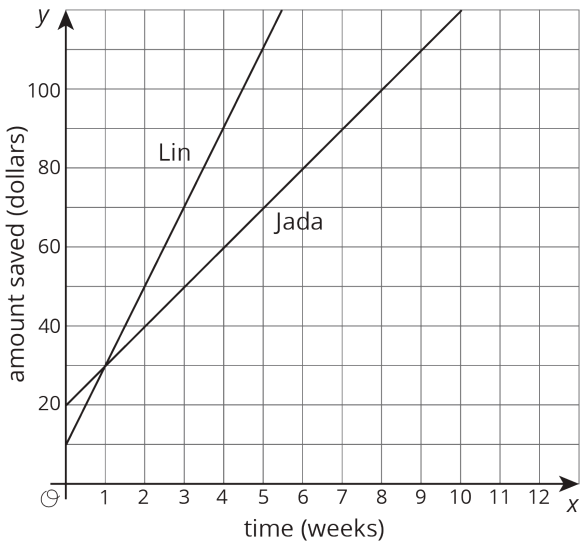
1. Explain what the slope represents in this situation.
2. Explain what the vertical intercept represents in this situation.

Student Response

1. The slope is 5 in this situation. That means that Andre saves 5 dollars every week.
2. The vertical intercept is 40. That means that Andre initially has $40 in his bank account.

Student Lesson Summary

At the start of summer break, Jada and Lin decide to save some of the money they earn helping out their neighbors to use during the school year. Jada starts by putting \$20 into a savings jar in her room and plans to save \$10 a week. Lin starts by putting \$10 into a savings jar in her room plans to save \$20 a week. Here are graphs of how much money they will save after 10 weeks if they each follow their plans:



The value where a line intersects the vertical axis is called the **vertical intercept**. When the vertical axis is labeled with a variable like y, this value is also often called the y-*intercept*. Jada's graph has a vertical intercept of \$20 while Lin's graph has a vertical intercept of \$10. These values reflect the amount of money they each started with. At 1 week they will have saved the same amount, \$30. But after week 1, Lin is saving more money per week (so she has a larger rate of change), so she will end up saving more money over the summer if they each follow their plans.

Glossary

- vertical intercept

Lesson 6 Practice Problems

Problem 1

Statement

Explain what the slope and intercept mean in each situation.

a. A graph represents the perimeter, y, in units, for an equilateral triangle with side length x units. The slope of the line is 3 and the y-intercept is 0.

b. The amount of money, y, in a cash box after x tickets are purchased for carnival games. The slope of the line is $\frac{1}{4}$ and the y-intercept is 8.

c. The number of chapters read, y, after x days. The slope of the line is $\frac{5}{4}$ and the y-intercept is 2.

d. The graph shows the cost in dollars, y, of a muffin delivery and the number of muffins, x, ordered. The slope of the line is 2 and the y-intercept is 3.

Solution

Answers vary. Sample responses:

a. The slope of 3 shows that the triangle has 3 sides. For each increase of 1 unit of the side length, the perimeter increases by 3 units. The intercept of 0 shows that the relationship is proportional—a triangle with sides of length 0 has a perimeter of length 0.

b. The slope of $\frac{1}{4}$ means that each ticket is \$0.25. The intercept of 8 represents the \$8 already in the cash box.

c. The slope of $\frac{5}{4}$ shows that 5 chapters are read every 4 days. The intercept might show that 2 chapters were read before beginning to read 5 chapters every 4 days, or it show that an additional 2 chapters were read on the first day.

d. The slope shows that \$2 are added for each muffin ordered. The intercept of 3 probably represents a \$3 delivery fee or tip for the order.

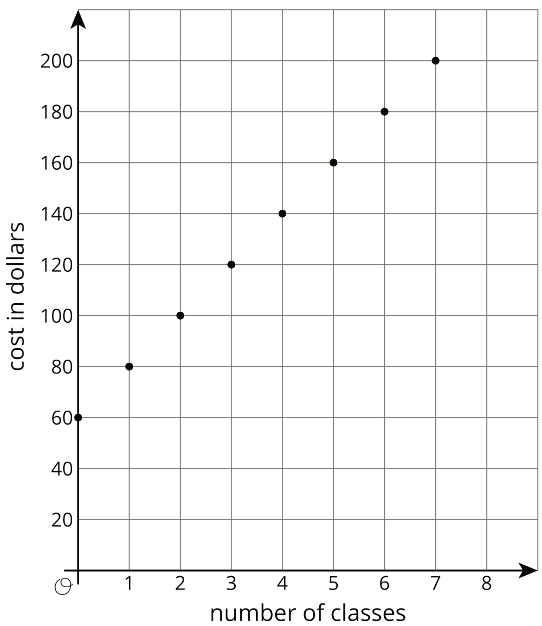
Problem 2

Statement

Customers at the gym pay a membership fee to join and then a fee for each class they attend. Here is a graph that represents the situation.

a. What does the slope of the line shown by the points mean in this situation?

b. What does the vertical intercept mean in this situation?



Solution

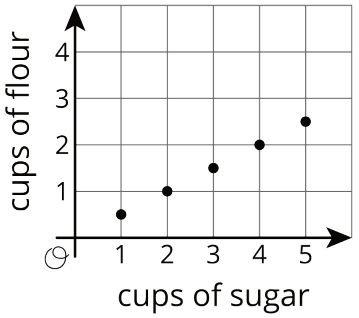
a. The cost for each class, which is \$20

b. The membership fee to join, which is \$60

Problem 3

Statement

The graph shows the relationship between the number of cups of flour and the number of cups of sugar in Lin's favorite brownie recipe.



The table shows the amounts of flour and sugar needed for Noah's favorite brownie recipe.

cups of sugar	cups of flour
$\frac{3}{2}$	1
3	2
$4\frac{1}{2}$	3

a. Noah and Lin buy a 12-cup bag of sugar and divide it evenly to make their recipes. If they each use all their sugar, how much flour do they each need?

b. Noah and Lin buy a 10-cup bag of flour and divide it evenly to make their recipes. If they each use all their flour, how much sugar do they each need?

Solution

a. Lin: 3 cups, Noah: 4 cups

b. Lin: 10 cups, Noah: $7\frac{1}{2}$ cups

(From Unit 3, Lesson 4.)